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Seismic moment tensor and focal mechanism for earthquake of February 22, 2024 in Eastern Slovakia (12:54:15 UTC, 21.75°E, 49.03°N, depth 9 km, ML3.0)

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The accuracy of the focal mechanism solution can depend very significantly on the number of stations used, especially in the case of weak earthquakes and sparse networks. We describe the procedure for retrieving the seismic moment tensor for the earthquake of February 22, 2024, which happened in Eastern Slovakia, using a limited number of seismic stations. We use records from only two stations of the Slovakian network: sk19 (49.25 °N, 21.93 °E) and sk20 (49.21 °N, 21.61 °E). The moment tensor inversion of high-frequency seismogram data in this study is based on a point source approach using the matrix method for the direct waves. The process involves generating records in displacements using the frequency and wave-number integration technique for an elastic horizontally layered medium. A method is presented for moment tensor inversion of only direct *P*- and *S*-waves, which is less sensitive to path effects modelling than reflected and converted waves, significantly improving the method's accuracy and reliability. The location and origin time of the event are considered known. Based on forward modelling, a numerical technique is developed for the inversion of observed waveforms for the of moment tensor $\mathbf{M}(t)$ components obtained by generalized inversion.

Key words: focal mechanism, waveform inversion, seismic moment tensor, direct waves, seismic stations, earthquakes.

Introduction. Determining the mechanism of an earthquake source is probably one of the most difficult problems in seismological research today. To be solved as stated, solutions to a number of other and not less difficult problems must be known first. Originally, the source mechanisms were determined from the polarities of the first *P*-waves at the stations [Aki, Richards, 2002]. Also, it was necessary to know the exact location of the source and to have an adequate velocity model between the source and the stations

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to calculate the emergence angles of the first *P*-waves from the source. Usually, a sufficient number of reliable polarities is only available for large (M>4) earthquakes occurring in areas with dense seismological networks [Dziewonski et al., 1981]. Unlike polarities, waveforms contain much more information about the source, which makes it possible to circumvent the above limitations, use fewer stations, and determine the mechanisms of smaller earthquakes [Dreger, Helmberger, 1993; Malytskyy, 2010, 2016; Malytskyy, D'Amico, 2015; Malytskyy et al., 2024]. This is especially important in Eastern Slovakia [Schlomer et al., 2024].

In the current study, the moment tensor components of an earthquake with a reported magnitude of ML3 that occurred on February 22, 2024 (12:54:15 UTC, 21.75 °E, 49.03 °N, depth 9 km, *ML* 3.0) in Eastern Slovakia are determined. They were obtained using a method for inverting the moment tensor of only the direct *P*- and *S*-waves, which are much less sensitive to path effects than reflected and converted ones and contain a much less distorted imprint of the source [Malytskyy, 2010, 2016; Malytskyy, D'Amico, 2015]. The use of only direct waves significantly improves the accuracy and reliability of the inversion. For direct modelling, i.e., calculation of synthetic seismograms, the matrix method was used, which enables the analytical isolation of only direct waves.

Theory: waveform inversion. The method presented here enables obtaining the seismic tensor solution by inverting waveforms recorded at a limited number of seismic stations. We consider the propagation of seismic waves in a vertically inhomogeneous media and develop a version of the matrix method for calculating synthetic seismograms on the upper surface of a horizontally layered isotropic medium. The point source is located inside a layer and is represented by a seismic moment tensor. The displacements on the upper surface are presented in matrix form in the frequency and wave number domain, separately for far-field and near-field [Malytskyy, 2016]. Subsequently, only the far-field displacements are considered, and the wave-field from only direct *P*- and *S*-waves is isolated using eigenvector analysis, which reduces the problem to a system of linear equations [Malytskyy, 2016]. Subsequently (inverse modelling), the spectra of the moment tensor components are calculated using the generalized inversion solution and transformed into the time domain by applying the inverse Laplace transform. As a result, the following expressions have been obtained by [Malytskyy, 2010, 2016] in cylindrical coordinates for the displacements $u_z^{(0)}(t,r,\varphi)$, $u_r^{(0)}(t,r,\varphi)$ and $u_\varphi^{(0)}(t,r,\varphi)$, on the upper surface of the half-space at z=0:

$$\begin{pmatrix} u_z^{(0)} \\ u_r^{(0)} \end{pmatrix} = \sum_{i=1}^{3} \int_0^\infty k^2 \mathbf{I}_i L^{-1}\left[m_i \mathbf{g}_i\right] dk ,$$

$$u_\varphi^{(0)} = \sum_{i=5}^{6} \int_0^\infty k^2 J_i L^{-1}\left[m_i g_{\varphi i}\right] dk , \qquad (1)$$

in which

$$m_1=M_{xz}\cos\varphi+M_{yz}\sin\varphi,\ m_2=M_{zz},$$
$$m_3=\cos^2\varphi\cdot M_{xx}+\sin^2\varphi\cdot M_{yy}+\sin^2\varphi\cdot M_{xy},$$
$$m_4=-\cos2\varphi\cdot M_{xx}+\cos2\varphi\cdot M_{yy}-2\sin2\varphi\cdot M_{xy},$$
$$m_5=M_{yz}\cos\varphi-M_{xz}\sin\varphi, \qquad (2)$$
$$m_6=\sin2\varphi\cdot M_{xx}-\sin2\varphi\cdot M_{yy}-2\cos2\varphi\cdot M_{xy},$$

M_{xx}, M_{xy}, M_{zz} are the Cartesian components of the moment rate tensor **M** in the frequency domain representing the source located at r=0, axis x points North and y points East, φ is the station's azimuth, k is the horizontal wave number, functions $\mathbf{g}_i=(g_{zi},g_{ri})^T$, and $g_{\varphi i}$ contain propagation effects between the source and the receiver, $\mathbf{I}_1=\begin{pmatrix} J_1 & 0 \\ 0 & J_0 \end{pmatrix}$, $\mathbf{I}_2=\begin{pmatrix} J_0 & 0 \\ 0 & J_1 \end{pmatrix}$, $\mathbf{I}_3=\mathbf{I}_2$; $J_5=J_0$, $J_6=J_1$ are the Bessel functions of $k\cdot r$, and L^{-1} is the inverse Laplace transform (from frequency into time domain).

Further, only the far-field displacements are considered, and the wave-field from only direct *P*- and *S*-waves is isolated with eigenvector analysis, reducing the problem to a system of linear equations [Malytskyy, 2016]. Eq. 1 is then expressed in matrix form for only the direct *P*- and *S*-waves on the upper surface of the half-space in frequency and wave number domain (ω, k) [Malytskyy, 2010]:

$$\mathbf{U}^{(0)} = \mathbf{K} \cdot \mathbf{M}, \qquad (3)$$

in which vector $\mathbf{U}^{(0)} = \left(U_x^{(0)P}, U_x^{(0)S}, U_y^{(0)P}, U_y^{(0)S}, U_z^{(0)P}, U_z^{(0)S}\right)^T$ contains the six Cartesian displacement components of direct *P*- and *S*-waves, vector $\mathbf{M} = \left(M_{xz}, M_{yz}, M_{zz}, M_{xx}, M_{yy}, M_{xy}\right)^T$ consists of the six independent Cartesian components of the moment rate tensor **M**, matrix **K** is accounting for path effects and transformations between the Cartesian and cylindrical coordinates. The least-squares solution to the over-determined system of Eq. 3 for **M** can be obtained by generalized inversion [Aki, Richards, 2002]:

$$\mathbf{M} = \left(\mathbf{K} * \mathbf{K}\right)^{-1} \mathbf{K} * \mathbf{U}^{(0)} \qquad (4)$$

in which the star denotes complex conjugation and transposition.

Thus, since all the six independent components of moment tensor **M** contribute to the waveforms $\mathbf{U}^{(0)}$ in the over-determined system of Eq. 3, the inversion scheme of Eq. 4 should make it possible, at least theoretically, to obtain a unique solution for each of them. Within the limitations of current source presentation and path effects modeling, the solution is exact, and convergence is reached after a single iteration. The inverse problem, in this case, consists of determining the parameters of the point source under the condition that the source location and origin time are known, as well as the distribution of velocities of seismic waves between the source and the station.

Application. In this section, the efficiency of the proposed inversion method is tested by applying it to the earthquake in Eastern Slovakia. For this research, we selected a weak crustal earthquake that occurred on February 22, 2024 (12:54:15 UTC, 21.75 °E, 49.03 °N, depth 9 km, ML3.0, Catalog Service: EMSC) (http://eida.gfz.de/webdc3/). To test our method, we use records from only two Slovakia network stations: sk19 (49.25 °N, 21.93 °E) and sk20 (49.21 °N, 21.61 °E) (Fig. 1).

The 1D crustal model used in the inversion is listed in Table [Málek et al., 2023]. The source is located at a depth of 9 km. Note that the stationssk19 and sk20 are located in different quadrants relative to the earthquake's epicenter.

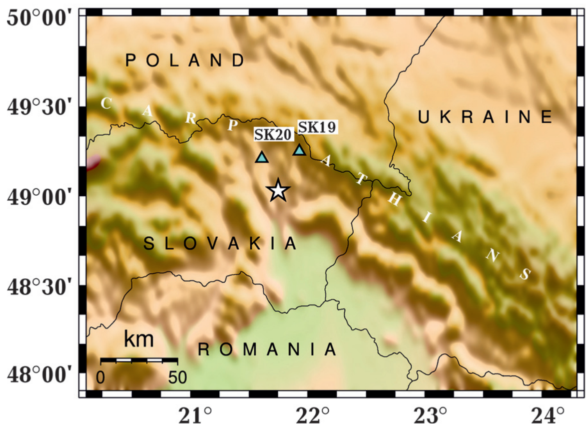


Fig. 1. Location of the Slovakia network stations sk19 (49.25 °N, 21.93 °E) and sk20 (49.21 °N, 21.61 °E) (triangles) and the epicenter of the earthquake of February 22, 2024 (12:54:15 UTC, 21.75 °E, 49.03 °N, depth 9 km, ML3.0) (star).

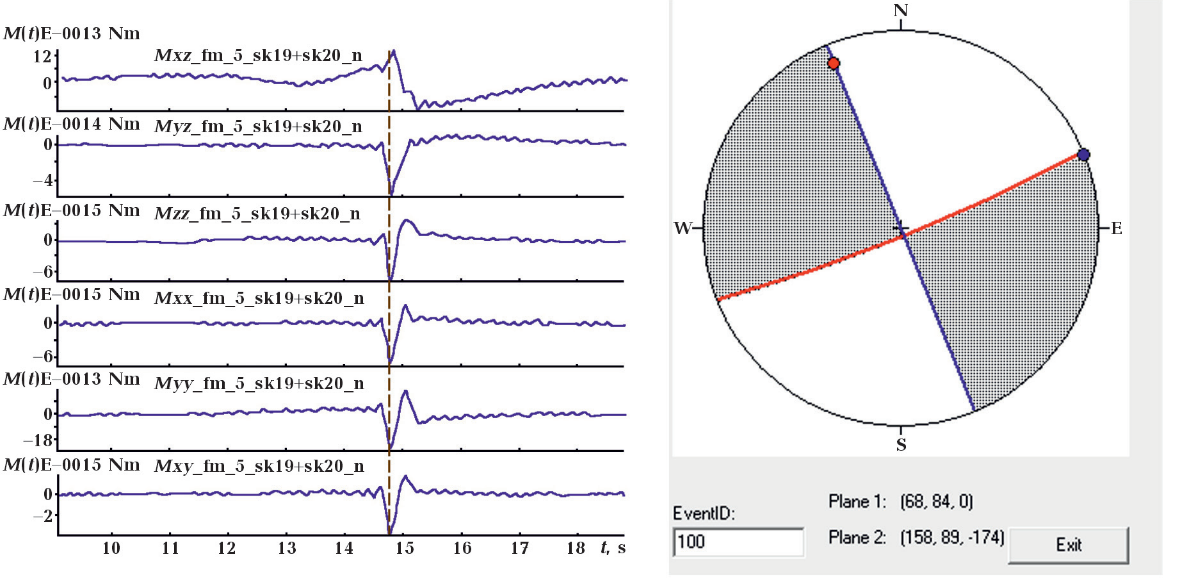
Components of the seismic moment tensor $\mathbf{M}(t)$ and a version of the focal mechanism calculated by the inversion of waveforms recorded at stationssk19 and sk20 using Eq. 4 are shown in Fig. 2.

Discussion and conclusion. The paper presents a method for moment tensor inversion of only direct *P*- and *S*-wave forms registered at only two stations. It is theoretically shown that the displacements at every single point on the upper surface of a horizontally layered and perfectly elastic half-space generated by a point source represented by a time-varying symmetric moment tensor depend on all six of its components.

The method is based on the inversion approach described in [Malytskyy, 2010, 2016],

The 1D crustal model used in the inversion

h_s, km	V_P, km/s	V_S, km/s	c, g/cm^3
1.0	2.5	1.445	2.2
3.0	3.7	2.139	2.44
3.0	5.2	3.006	2.74
10	5.9	3.41	2.88
3	6.4	3.699	2.98
4	6.75	3.902	3.05
—	8.0	4.624	3.3



a *b*

Fig. 2. The components of seismic moment tensor obtained for the earthquake of 2024-02-22 by inversion of its direct waveforms at the stations sk19 and sk20 (*a*); *b* — version of the focal mechanism solution corresponding to the tensor presented in (*a*). The vertical dashed line in (*a*) refers to the origin time at 12:54:14.8UTC.

where a version of the matrix method was developed for calculating direct waves in a horizontally layered half-space from a point source represented by its moment tensor. Using the method presented in the current paper, the focal mechanism of the *ML* 3.0 earthquake on February 22, 2024 (12:54:15 UTC, 21.75 E°, 49.03 °N, depth 9 km) in Eastern Slovakia is retrieved. Choosing to invert only the direct waves calculated by the matrix method instead of the full field makes it possible to reduce the effects of the half-space model inaccuracy since the reflected and converted phases are much more distorted by it. The assumption of horizontally layered half-space, as well as the distribution of seismic velocities in it, may, however, turn out grossly incorrect in fact. Combined with inaccurate knowledge of the source location and origin time, as well as a number of other uncertainties, such as those introduced by seismic noise in the observed seismograms, etc., it can almost completely obscure the source imprint in the seismograms, and especially in those originating, as it is, from only a limited number of seismic stations, turn the moment inversion ill-defined and lead to an intractable solution. This is why some independent controls for the inversion results are needed. However, such studies are very important in regions with low local seismicity, where records from only a limited number of stations can be used to determine the parameters of the earthquake source, which is important for seismology and geophysics in general.

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Тензор сейсмічного моменту і фокальний механізм землетрусу, який відбувся 22 лютого 2024 р. у Східній Словаччині (12:54:15 UTC, 21,75°E, 49,03°N, *H*=9 км, *ML*3,0)

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Точність визначення фокального механізму залежить від кількості сейсмічних станцій, особливо у разі слабких землетрусів. Описано процедуру відновлення тензора сейсмічного моменту для землетрусу, який стався 22 лютого 2024 р. в Східній Словаччині, використовуючи обмежену кількість сейсмічних станцій, а саме записи лише на двох станціях Словацької мережі: sk19 (49.25 °N, 21.93 °E) і sk20 (49.21 °N, 21.61 °E). Отримання тензора сейсмічного моменту для високочастотної сейсмогра-

ми в цьому дослідженні базується на підході точкового джерела з використанням матричного методу для прямих хвиль. Процес передбачає створення сейсмічних записів у переміщеннях за допомогою інтегрування за частотою та хвильовим числом у випадку пружного горизонтально-шаруватого середовища. Представлено метод отримання сейсмічного тензора лише для прямих *P*- та *S*-хвиль, який є менш чутливим до поширення відбитих і заломлених хвиль, що значно покращує точність і надійність методу. Місце і час виникнення події вважаються відомими. На підставі прямого моделювання розроблено методику інверсії хвильових форм для компонент тензора моменту $\mathbf{M}(t)$, отриманих шляхом узагальненої інверсії.

Ключові слова: механізм вогнища, інверсія хвилі, тензор сейсмічного моменту, прямі хвилі, сейсмічні станції, землетруси.