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Geodynamic modeling of geoid changes and true polar migration in the geological past using spherical harmonic analysis

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The results of paleogeoid reconstruction for different Phanerozoic (0—540 million years) geological epochs using spherical harmonic analysis are presented. The method is based on an assumption of the relative stability of the relationship between topography and geoid over geological time for low- and medium-degree of spherical harmonics. The basis for this assumption is that most of the geoid topography (about 9/10) must originate from inhomogeneities in the distribution of subcrustal density, which are more stable in geological time. Knowing the modern relationship between topography and the geoid and having reconstructions of paleotopography, it is possible to estimate the ancient shape of the geoid by removing the contribution of the modern topography of the lithosphere to the heights of the geoid and reconstructing the geoid (paleogeoid) based on paleo-reconstructions of topography in past geological periods. We used modern models of the geoid EGM2008, topography ETOPO1, and paleotopographic reconstructions PaleoDEM. An algorithm is proposed that includes determining the transfer function between topography and geoid, removing the influence of modern topography, and restoring the paleogeoid based on reconstructed paleotopography. The modeling results demonstrate that the main contribution to the formation of the global geoid (about 90 %) is due the mantle density inhomogeneities to depths of ~1000 km, while the contribution of surface topography is secondary. Analysis of second-order harmonic coefficients allowed us to reconstruct the trajectory of the Earth's rotational pole migration, with a maximum deviation of about 2600 m during the Phanerozoic. The results are consistent with current ideas about the dominant role of mantle convection in shaping the Earth's global figure and confirm the connection between tectonic processes, mass redistribution, and changes in the planet's inertia tensor.

Key words: global geodynamics, mantle convection, plate tectonics, geoid, paleogeoid, spherical harmonics, true polar wander, dynamic topography, Phanerozoic.

Introduction. The integration of modern achievements in geodesy, geophysics, tectonics, and paleogeography makes it possible to develop new approaches to reconstructing the Earth's gravitational field over geological time. The reconstruction of the geoid in geological time differs significantly in its formulation from classical problems of physical geodesy [Terkot, Schubert, 2002; Hofmann-Wellenhof, Moritz, 2006; Tserklevich et al., 2025], which are focused on the current state of the Earth. Over intervals of hundreds of millions of years, the main goal is not to reproduce the detailed structure of the gravitational field with reference to specific implementations of reference systems, but rather to identify and trace the evolution of stable long-wave components of the geoid

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capable of reflecting the integral patterns of mass redistribution in the mantle-lithosphere system and their connection with pole migration. Changes in mass distribution, according to the equations of hydrostatic equilibrium of the mantle and lithosphere associated with tectonic processes, subduction, and mantle plumes, are reflected in changes in global harmonic coefficients, primarily low degrees [Hager et al., 1985; Wen, Anderson, 1997; Hofmann-Wellenhof, Moritz, 2006; Tserklevich et al., 2022, 2025]. Harmonic coefficients of low orders (primarily n=2) are directly related to the Earth's moment of inertia tensor, and thus to the orientation of the Earth's axis of rotation and the phenomenon of the pole wobble [Hager et al., 1985; Wen, Anderson, 1997; Tsai, Stevenson, 2007; Fu et al., 2022; Wang, Mitchell, 2023; Ghelichkhan et al., 2025].

True Polar Wander (TPW) is generally understood to refer to the rotation of the entire solid silicate mantle and crust around the liquid outer core, while the position of the axis of rotation in space remains constant. Unlike plate tectonic drift, which describes the relative movements of lithospheric plates on the surface of a spheroid, TPW is associated with a change in the orientation of the entire Earth relative to its moment of inertia. Tectonic processes, particularly the movement of lithospheric plates, alter the distribution of mass within the Earth's crust and result in local deviations from the equatorially symmetric geoid. Any vertical surface movements (uplifts or subsidence) cause a change in the Earth's shape — not only does the height of the geoid change, but so do the planet's rotational parameters due to the redistribution of the moment of inertia [Tserklevich et al., 2022]. Thus, from a geophysical perspective, TPW results from the long-term evolution of large anomalous masses within the Earth. The key factors here are: surface mass redistributions (for example, part of the observed pole migration can be explained by glacial melting and changes in water distribution [Mitrovica et al., 2015; Ghelichkhan et al., 2025]); mantle processes (convection cells create long-wavelength components of the geoid and alter the principal axes of the inertia tensor, setting the stage for TPW [Hager, Richards, 1989; Forte et al., 2010]); thermo-chemical heterogeneities in the lower mantle (LLSVP) (long-lived stable thermo-chemical «domes» in the lower mantle beneath Africa and the Pacific Ocean can act as «anchors» for global convection, causing persistent geoid anomalies and influencing the position of the axis of maximum inertia, and thus on the possible trajectories of the TPW [Wen, Anderson, 1997; Shi et al., 2024]).

Thus, changes in the ellipsoidal shape of the figure and in the orientation of the principal axes of the inertia tensor, caused by the redistribution of masses in the interior and on the surface, lead to a transformation of the equilibrium figure, which is reflected in changes in the harmonic coefficients C_{20}, C_{21}, S_{21}, C_{22}, and S_{22} [Terkot, Schubert, 2002; Tserklevich et al., 2025; Ghelichkhan et al., 2025]. It is the coefficients C_{21} and S_{21} that are responsible for the redistribution of masses in the plane perpendicular to the axis of rotation and are directly related to the movement of the Earth's rotation pole.

Thus, harmonic analysis plays a key role in modeling the evolution of the geoid in geological time and reconstructing pole migration, which expands the possibilities for understanding problematic issues of global geodynamics, including the mechanisms of lithospheric plate movement, dynamic topography in geological time, TPW, and changes in the gravitational field on spatial and temporal scales.

This paper presents the results of research related to modeling the evolution of the geoid and pole migration in the geological past (0—540 million years ago) using planetary-scale paleo-reconstructions. In this regard, the relationship between TPW and the cycle of supercontinent formation and breakup is of particular interest. According to one theory, the long-term existence of a supercontinent (Gondwana, Pangea, Rodinia) leads to an asymmetric load on the mantle: a large thermal and anomalous volume of heterogeneous density forms beneath the continental block, while oceanic regions are characterized by intense subduction of cold plates. This creates an unbalanced distribution of mass, which contributes to the development of TPW episodes [Evans, 1998, 2003]. In this interpretation, TPW can be viewed as a mechanism for the global restructuring of the supercontinent—mantle system, facilitating a transition to a new configuration that is closer to an «energetically more favorable» state. Episodes of rapid TPW may coincide with key tectonic events — the breakup of supercontinents, the formation of new spreading axes, the activation of large magmatic provinces, and changes in the geoid — as reflected in paleogeographic reconstructions and paleomagnetic data [Raub et al., 2007; Fu et al., 2022].

Description of input data. The study uses three main sets of digital models: the modern ETOPO1 relief model (https://www.ngdc.noaa.gov/mgg/global/global.html), the global EGM2008 gravity potential model [Pavlis et al., 2012], and PaleoDEM paleotopographic reconstructions [Scotese, Wright, 2018] for different epochs of the Phanerozoic. Each of these sets has its own data structure: ETOPO1 provides heights with a resolution of 1′, showing both land topography and bathymetry; EGM2008 contains a spectral representation of gravitational potential in the form of harmonic coefficients up to degree and order 2160; PaleoDEM provides reconstructions with a resolution of 1°×1°, based on paleomagnetic, stratigraphic, paleoclimatic, and tectonic models, all referenced to the WGS 84 coordinate system.

Given that the listed datasets had different structures, the primary task was to align these models on a common spatial and mathematical basis that would allow harmonic analysis and the assessment of the relationship between topography and the geoid. Therefore, all data sets needed to be transformed into a single geospatial grid, since harmonic decomposition requires a uniform distribution of nodes on the sphere. A global regular grid with a 1°×1° step was used to determine the spherical harmonics, ensuring complete correspondence between modern topography and paleo-reconstructions.

The first step was to convert ETOPO1 elevations with a resolution of 1′ to average values within each degree block. PaleoDEM paleotopographic models were also converted to the same coordinate system and grid. Since reconstructions for different geological periods are of uneven quality and contain local artifacts, it was necessary to eliminate them first. Such situations arose in areas of sharp transitions between oceanic and continental regions, due to the specifics of oceanic crust reconstructions. For this purpose, we applied local filters with a weighting-by-latitude function, since the density of the nodes of the spherical grid decreases with increasing latitude. In particular, the weighting coefficient was set to $(\varphi)=\cos\varphi$, which ensured a uniform influence of data at different latitudes during smoothing.

An important part of the preprocessing was also the coordination of vertical topography readings. Modern models are presented relative to sea level, but in the geological past, ocean basins did not always align with the current sea level. In this regard, paleotopographic models contained heights that reflect topography in a generalized sense, i.e., not only the relief of the land surface, but also the reconstructed configuration of the seabed. This required bringing all values to a single conditional reference level (WGS84 ellipsoid), which made it possible to avoid shifts in spectral coefficients and obtain correct estimates of the correlation between topography and the geoid.

The next step was to identify and eliminate artifacts at the land-ocean boundary characteristic of PaleoDEM models. In Phanerozoic reconstructions, it is the continental boundaries that are the areas of greatest uncertainty, as they depend on complex models of transgressions and regressions, reconstructions of sea basin depths, and tectonic movements.

It should be noted that spherical harmonics approximate large-scale structures well but are sensitive to noise and local anomalies. Therefore, before performing harmonic analysis, all local artifacts were excluded, including inconsistent values at facies boundaries, isolated outliers, and local depressions or peaks characteristic of PaleoDEM of early geological periods. For this purpose, an adaptive median filter was applied to smooth out selective anomalies while preserving large-scale topographic structures that are most important for harmonic correlation with the geoid.

The final stage of preliminary data preparation was the formation of consistent input arrays for further computational procedures used in the spectral method of paleogeoid reconstruction. For harmonic analysis, each data array had a uniform structure: a regular global grid, identical array sizes, and no gaps or anomalous values. Thanks to preprocessing, grids with a resolution of 180×360 pixels were generated for all PaleoDEM time slices, fully matching the format used for calculating spherical harmonics.

Methodology for constructing a paleogeoid for different geological stages of paleoreconstruction using spherical harmonic analysis. Our method for constructing a paleogeoid for different geological stages of paleoreconstruction assumes that the relationship between modern global topography and the geoid remains constant over time. At the same time, it should be noted that the structure of the spatial correlation between topography and the geoid is neither linear nor homoscalar; it is shaped by the complex interaction of surface masses, deep convection, isostatic mechanisms, and various rheological properties of the lithosphere and asthenosphere [Dovbnych, 2007; Tserklevych et al., 2022]. However, in a generalized sense, the basis for this assumption is that most of the geoid topography (about 9/10) must originate from inhomogeneities in the distribution of subcrustal density, which are more stable in geological time [Sandwell, Schubert, 1980; Burke, Torsvik, 2004; Torsvik et al., 2010; Liu, Zhong, 2015]. Knowing the modern relationship between topography and the geoid and having reconstructions of paleotopography, it is possible to estimate the ancient shape of the geoid by removing the contribution of the modern topography of the lithosphere to the heights of the geoid and reconstructing the geoid (paleogeoid) based on paleo-reconstructions of topography in past geological periods.

In spherical harmonic form, the geoid and topography are described by sets of coefficients C_{nm} and S_{nm}, which contain information about the spatial variations in the distribution of geoid and topography heights on the sphere. This representation allows us to estimate the global interaction between surface and deep structures. The independence of the coefficients at different degrees allows us to study the correlation between the geoid and topography in spectral space. It is also important to note that the correlation between the geoid and topography is not only a diagnosis of the current state of interaction between the distribution of masses, but also the basis for reconstructing the gravitational field in the geological past. If we determine the degrees for which topography is the most informative indicator of geoid structure, we can apply the corresponding spectral correlation coefficients as weighting factors in paleotopographic models. It can also be assumed that the correlation spectrum over time indicates that the geoid and topography not only reflect the distribution of Earth's mass, but also preserve the historical consequences of tectonic processes in their structure.

The method of constructing a paleogeoid using spherical harmonic analysis is based on the decomposition of the geoid and topography into spherical functions, which allows analyzing their relationship in spectral space (by degree and order). This approach allows us to separate deep mass effects from surface relief and use this relationship to reconstruct the paleogeoid from paleotopography datasets.

In the first stage of our study, modern geoid heights (EGM2008) and topography (ETOPO1) were presented in the form of spherical harmonic expansions to a given degree (n=20 and n=70 inclusive) in accordance with the formula:

$$h = R\sum\nolimits_{n=1}^{N}\sum\nolimits_{m=0}^{n}\frac{h_{nm}^{G}}{h_{nm}^{T}}\Lambda_{nm},$$

$$\Lambda_{nm} = \begin{Bmatrix}\cos m\lambda \\ \sin m\lambda\end{Bmatrix}\bar{P}_{nm}(\sin\varphi), \qquad (1)$$

where $h_{nm}^{G} = \left\{\bar{C}_{nm}^{G}, \bar{S}_{nm}^{G}\right\}$, are the normalized harmonic coefficients of the EGM2008 geoid model, $h^T_{nm} = \left\{\bar{C}^T_{nm}, \bar{S}^T_{nm}\right\}$ are the normalized harmonic coefficients of the ETOPO1 topography model.

This approach allowed us to move from geospatial data to their spectral representation with the ability to compare relief and geoid components in the frequency domain. In this work, two independent spectral slices (*n*≤20 and *n*≤70) are deliberately used as complementary levels of description of characteristic features: the first to assess the global component and general dynamics; the second to verify the stability of spatial patterns when transitioning to regional detail. The schedule was implemented using custom-developed software that calculated the harmonic coefficients $\bar{C}^G_{nm}, \bar{S}^G_{nm}$ and $\bar{C}^T_{nm}, \bar{S}^T_{nm}$ using the least-squares method with attached Legendre polynomials.

Subsequently, the transfer function (n was calculated using the formula [Tserklevich et al., 2022]:

$$\lambda_n = \frac{\sum_{m=0}^{n}\left(\bar{C}^G_{nm}\bar{S}^T_{nm} + \bar{S}^G_{nm}\bar{S}^T_{nm}\right)}{\left[\sum_{m=0}^{n}\left(\left(\bar{S}^T_{nm}\right)^2 + \left(\bar{S}^T_{nm}\right)^2\right)\right]}, \qquad (2)$$

λ_n — a transfer function that shows how much the topography affects the geoid for a given degree *n*. It is calculated separately for each degree, in our case, from *n*=1 to *n*=70. If the value of λ_n is close to 1, the relief of the lithosphere surface almost completely explains the geoid, and if λ_n is close to 0, the relief does not affect the geoid.

Note that λ_n — the transfer function (or admittance) is mainly used in automatic control theory, statistical communication theory, and digital signal processing, and is a differential operator that expresses the relationship between the input and output signals of a linear stationary system.

Thus, having determined from calculations based on spherical harmonics $\bar{C}^G_{nm}, \bar{S}^G_{nm}$, $\bar{C}^T_{nm}, \bar{S}^T_{nm}$ the transfer function λ_n, we can calculate the harmonic coefficients $\bar{C}^{G_{\text{cor}}}_{nm}, \bar{S}^{G_{\text{cor}}}_{nm}$ of the «residual/synthetic» geoid, corrected for the influence of topography, using the following formulas:

$$\bar{C}^{G_{\text{cor}}}_{nm} = \bar{C}^G_{nm} - \lambda_n \bar{C}^T_{nm}, \qquad (3)$$

$$\bar{S}^{G_{\text{cor}}}_{nm} = \bar{S}^G_{nm} - \lambda_n \bar{S}^T_{nm}. \qquad (4)$$

Thus, at this stage, the corresponding fraction of the topographic influence is removed from each harmonic coefficient $\bar{C}^G_{nm}, \bar{S}^G_{nm}$ of the EGM2008 geoid model. To do this, we use the transfer function λ_n, which takes into account how much the topography affects the shape of the geoid for each degree *n*. In other words, we «clean» the modern geoid by removing everything that can be explained by the relief of the lithosphere's surface. As a result, we obtain the corrected coefficients $\bar{C}^{G_{\text{cor}}}_{nm}$ and $\bar{S}^{G_{\text{cor}}}_{nm}$ of the «residual/synthetic» geoid, which reflect the influence of the planet's internal mass structures, i.e., deep sources (such as mantle convection or uneven density distribution). Such a geoid can be conditionally called «internal» because it is free from the influence of the relief of the continental and oceanic surface of the Earth.

The next step is a harmonic decomposition of paleotopography with coefficients $\bar{C}^{PT}_{nm}, \bar{S}^{PT}_{nm}$. These harmonic coefficients of paleotopography are obtained from PaleoDEM models for each geological epoch (e.g., 100 million years (Ma), 200 Ma, ... 500 Ma), which must be compatible with the previous coefficients in terms of degree and order.

The final stage is the reconstruction (restoration of the geoid) of the paleogeoid using spherical harmonics $\bar{C}^{PG}_{nm}, \bar{S}^{PG}_{nm}$, which were determined by the formulas:

$$\bar{C}^{PG}_{nm} = \bar{C}^{G_{\text{cor}}}_{nm} - \lambda_n \bar{C}^{PT}_{nm}, \qquad (5)$$

$$\bar{S}^{PG}_{nm} = \bar{S}^{G_{\text{cor}}}_{nm} - \lambda_n \bar{S}^{PT}_{nm}. \qquad (6)$$

This stage allows us to calculate the harmonic coefficients of the paleogeoid for a specific geological epoch based on data on paleotopography and the cleaned (corrected) geoid. The essence of the calculations is that we take the part of the harmonic coefficients of the geoid that does not depend on the modern relief (i.e., $\bar{C}^{G_{\text{cor}}}_{nm}$ and $\bar{S}^{G_{\text{cor}}}_{nm}$) and add to them the restored part of the coefficients $\lambda_n \bar{C}^{PT}_{nm}$ and $\lambda_n \bar{S}^{PT}_{nm}$, but already for a specific geological period.

Thus, the obtained coefficients $\overline{C}_{nm}^{PG}$ and $\overline{S}_{nm}^{PG}$ allow us to reconstruct the geoid for the corresponding epoch (for example, 200 Ma) and construct a paleogeoid map. This algorithm also allows us to trace the evolution of the geoid in geological time and estimate the manifestations of TPW by analyzing second-order harmonics.

Given that, for example, large clusters of crustal blocks or mantle upwellings can shift the axis of maximum moment of inertia, and thus alter the coefficients $\overline{C}_{21}^{PG}$ and $\overline{S}_{21}^{PG}$. These coefficients are usually dimensionless and reflect the global redistribution of Earth's masses in a particular geological epoch. Using the coefficients $\overline{C}_{21}^{PG}$ and $\overline{S}_{21}^{PG}$, it is possible to calculate the position of the Earth's pole in different geological eras.

The angular displacement of the pole's movement (or amplitude) is determined by the formula

$$A=\sqrt{\left(C_{21}^{PG}\right)^2+\left(S_{21}^{PG}\right)^2}\,. \qquad (7)$$

This is the value of the pole's deviation from its current position — the actual length of the displacement vector in spherical coordinates. The amplitude can be converted to latitude using normalization:

$$\varphi=\arcsin(A/K). \qquad (8)$$

Here, K is the normalization coefficient, which is set depending on the selected model or scale.

The geographic longitude of the pole is calculated using the formula

$$\lambda=\arctan 2\left(S_{21}^{PG},C_{21}^{PG}\right). \qquad (9)$$

This is the azimuth angle that indicates the direction of the pole's displacement relative to the equator. The arctan2 function automatically determines the correct quadrant of the coordinates.

Thus, the coefficients of spherical harmonics C_{21}^{PG} and S_{21}^{PG} allow us to restore the position of the Earth's rotation pole in different geological eras.

Results and discussion. Figs 1—4 show selected variants of map schemes of the geoid (paleogeoid) and the contribution of topography to the geoid height in the Surfer software using harmonic coefficients up to the 20th and 70th degrees and orders. It is important to note that, according to the calculations, the heights of the geoid and paleogeoids vary within a range of approximately –100 to +80 meters. At the same time, the contribution of topography to geoid heights varies from –26 to +26 meters, while the contribution of paleotopography to paleogeoid heights is significantly smaller, ranging from –16 to +18 meters. In addition, it can be noted that the degree (20th or 70th) of decomposition by spherical functions does not significantly affect the shape of the geoid image or the contribution of topography to geoid heights on a global scale. However, when comparing the obtained images of paleogeoids with the modern geoid, changes in their topography become more noticeable, although the main features of the geoid topology are preserved. Therefore, it can be argued that the main contribution to geoid heights is due to the influence of the heterogeneous structure of the mantle. In the case of paleogeoid reconstruction with the restriction of using spherical coefficients up to $n\approx 20$, corresponding to spatial scales of more than 1000 km, the reflection of deep mantle structures that cause a long-wave dynamic geoid is characteristic, while high degrees $n\approx 70$ of harmonic coefficients introduce insignificant short-wave changes and, accordingly, have less influence on the formation of the geoid.

Most leading geophysicists believe that mantle convection is the primary driver of tectonic and geodynamic processes and leads to the formation of long-wavelength mass anomalies within the Earth. Density distribution anomalies detected at various depths and along horizontal extensions alter the planet's geoid and inertia tensor. These parameters are not isolated but interrelated factors in a single process — the long-term redistribution of mass—which shapes the Earth's actual shape (the geoid) and determines the position of the pole over geological time.

Fig. 5 shows a diagram of the change in

the position of the North Pole based on paleogeoid modeling at 100-million-year intervals. The maximum deviation from the current pole of rotation reaches ~2600 m.

There are three fundamentally different approaches to estimating the displacement (drift) of the Earth's rotational pole, which yield results of different orders of magnitude [Gubbins, Herrero-Bervera, 2007].

Astronomical and geodetic observations (short time scale). Optical astrometric and modern geodetic measurements show that the average displacement of the Earth's rotational pole between 1900 and 1992 is ~3.5 milliarcseconds per year, or ~107 mm/year (107 km/million years) [Gubbins, Herrero-Bervera, 2007; Ghelichkhan et al., 2025]. However, this pole migration is largely due to postglacial isostatic uplift — the response of the lithosphere and mantle to the melting of the ice sheet — rather than deep mantle convection. Therefore, a direct extrapolation of this rate to the Phanerozoic is incorrect.

Paleomagnetic estimates (geological time scale, millions to hundreds of millions of years). The TPW track reflects an alternation of prolonged pauses (~50 million years) and relatively faster episodes of displacement; the typical TPW velocity in the interval 130—70 million years ago is approximately 30 km/million years [Evans, 2003; Gubbins, Herrero-Bervera, 2007]. Over the last ~8—59 million years, the pole has hardly shifted at all, with TPW points lying within 3.3°—5.9° of the present-day pole. Thus, over the Phanerozoic (540 million years), the total angular

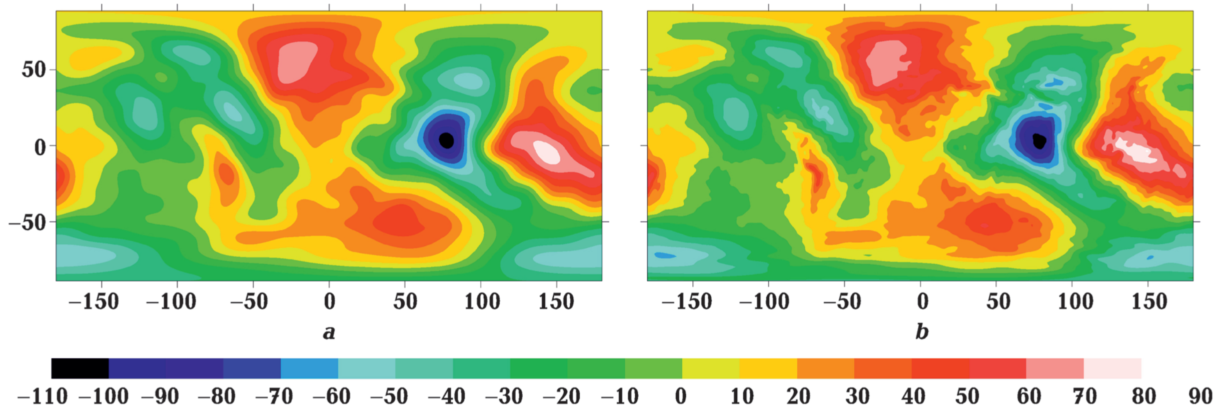


Fig. 1. Visualization of EGM2008 geoid heights in meters: *a* — harmonics $\bar{C}^G_{nm}, \bar{S}^G_{nm}$ up to the 20th degree were used; *b* — harmonics $\bar{C}^G_{nm}, \bar{S}^G_{nm}$ up to the 70th degree were used.

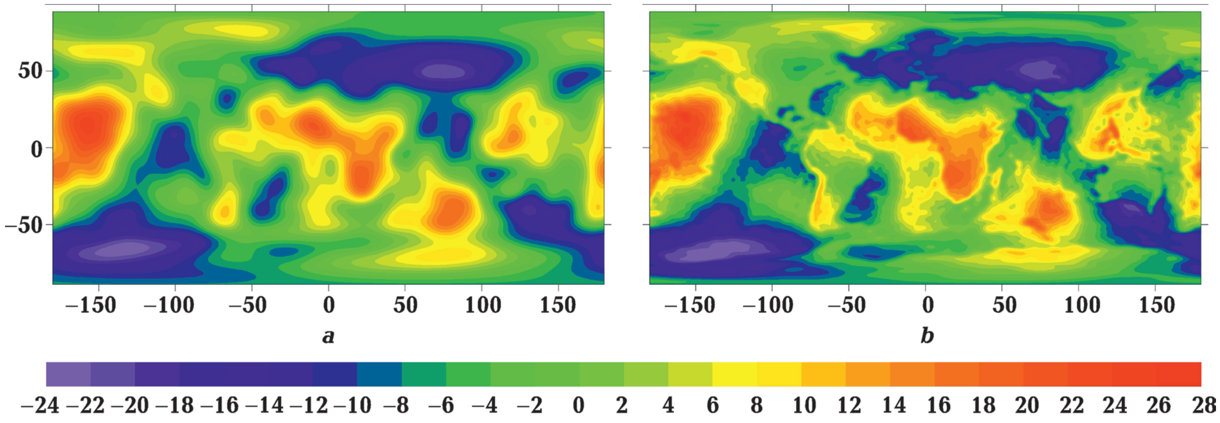
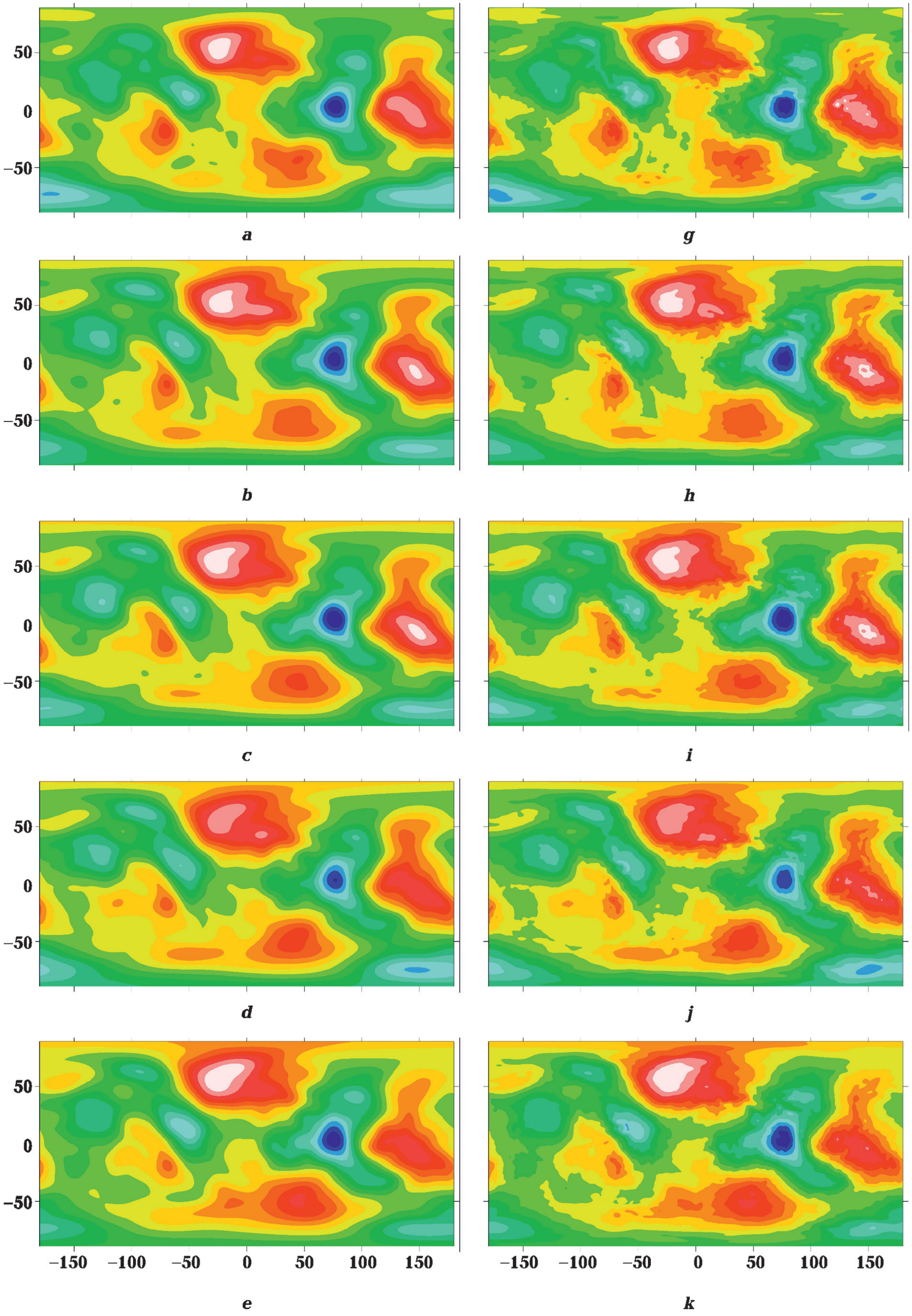


Fig. 2. Visualization of the topographic contribution to the EGM2008 geoid heights in meters: *a* — harmonics $\lambda_n \bar{C}^T_{nm}$, $\lambda_n \bar{S}^T_{nm}$ up to the 20th degree used; *b* — harmonics $\lambda_n \bar{C}^T_{nm}$, $\lambda_n \bar{S}^T_{nm}$ up to the 70th degree were used.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displacement is indeed tens of degrees, i.e., thousands of kilometers.

The approach proposed in this study (analysis of spherical harmonics and the paleogeoid) differs fundamentally from the paleomagnetic approach: we do not measure the angular displacement of plates relative to the hot-spot system, but rather reconstruct changes in the C_{21} and S_{21} coefficients in the spherical function decomposition—that is, the component of mass redistribution encoded in the long-wavelength ($n \leq 20$) geoid. By its nature, this approach acts as a bandpass filter: it isolates stable, depth-driven mantle density anomalies and excludes short-wavelength effects associated with local topography and isostasy [Dovbnych, 2007; Tserklevich et al., 2022].

But a question may arise: why is the result given in hundreds of meters rather than degrees? The obtained value of ~2,600 m is not an estimate of the total angular displacement of the pole during the Phanerozoic. It reflects the effective change in the spatial position of the axis of maximum moment of inertia, which is preserved in the long-wavelength component of the paleogeoid under our filtering method. This is the lower limit, not the full amplitude of TPW, since the transfer function λ_n is constructed based on the modern «relief—geoid» correlation and, according to the accepted assumption, is preserved over time only for low orders. Mantle viscosity controls the rate at which hydrostatic compression adjusts to the new position of the axis of rotation; this steady-state response develops over characteristic timescales of ~10^4 years in the upper mantle [Terkot, Schubert, 2002]. When analyzing Phanerozoic time intervals (~10^8 years), short-term changes in the geoid are not preserved in PaleoDEM reconstructions. PaleoDEM models have a spatial resolution of 1°×1°, and their accuracy decreases significantly for epochs older than 100 million years, where the uncertainty of paleotopographic reconstructions increases. However, the paleomagnetic TPW estimate captures the signal from all tectonic plates in the hotspot reference frame; in contrast, our «geodetic» approach does not depend on paleomagnetic reconstructions and is an independent tool that captures only the TPW component reflected in the long-wavelength geoid in the WGS 84 coordinate system.

One of the key elements of adaptation to an adequate perception of the given algorithm and the results obtained from reconstructed geoids is their reliability and relevance assessment. Since no direct measurements allow for the construction of a paleogeoid over geological time, the validation criterion can be based on two principles: the physical correspondence of the reconstructed spectrum to the modern geoid spectrum at the same degrees, and internal consistency across geological epochs. In particular, for low degrees

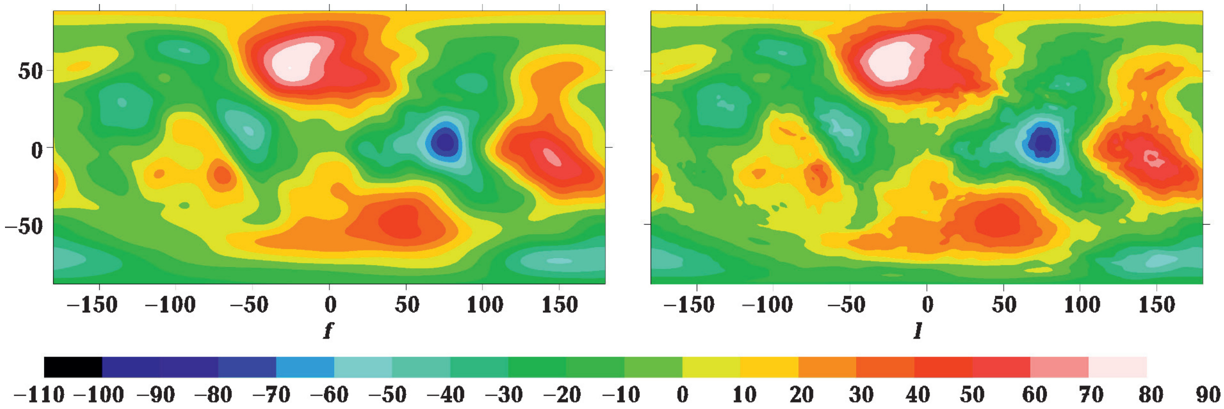
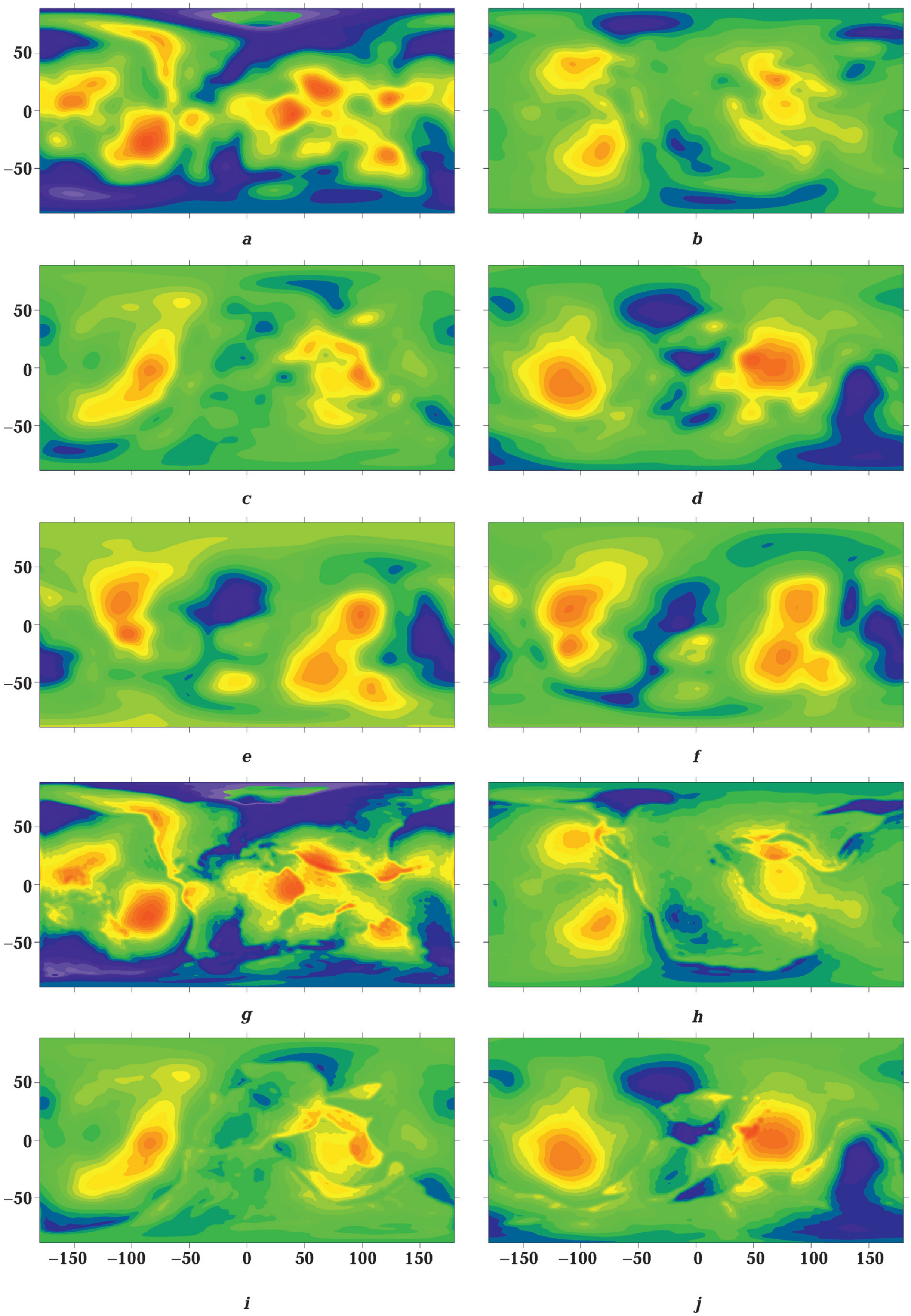


Fig. 3. Visualization of paleo-geoid heights in meters. Harmonic components used $\bar{C}_{nm}^{PG}$ and $\bar{S}_{nm}^{PG}$ up to the 20th degree: *a* — 100 Ma, *b* — 200 Ma, *c* — 300 Ma, *d* — 400 Ma, *e* — 500 Ma, *f* — 540 Ma. Harmonics $\bar{C}_{nm}^{PG}$ and $\bar{S}_{nm}^{PG}$ up to the 70th degree were used: *g* — 100 Ma, *h* — 200 Ma, *i* — 300 Ma, *j* — 400 Ma, *k* — 500 Ma, *l* — 540 Ma.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$n \leq 20$, the reconstructed geoid should show slow changes in amplitudes, corresponding to periods of mantle convection reorganization. For degrees $n > 20$, amplitudes may vary more significantly, as this range corresponds to medium-scale structures sensitive to topography reconstructions.

During the study, geodynamic modeling of geoid changes in the geological past was carried out, which made it possible to assess the relationship between lithospheric processes and the Earth's gravitational field. The spectral analysis method we used, based on the transfer function, confirmed that the paleogeoid had different parameters in past geological epochs compared to the modern model. However, the question remains what semantic meaning the obtained paleogeoids convey and how they can be used to interpret geodynamic processes. In our opinion, these digital elevation data for paleogeoids can be used to track changes in polar motion, which are reflected in the patterns of lithospheric plate movement. The TPW can be modeled geodynamically, since the spin (rotational) axis follows the axis of maximum inertia, which can be determined from the second-order geoid in a spherical function expansion. The proposed modeling technique can be used to predict changes in the Earth's gravitational field, which is important for global geodynamics. The results of the study can also be applied to the study of sea-level changes, isostatic processes, and the stability of Earth's rotation. On the other hand, our assumption of an unchanging statistical correlation between geoid heights and topography in the present era can be questioned — can such a relationship be extrapolated to past geological epochs?

The proposed approach assumes that the spectral transfer function λ_n, which describes the relationship between topography and the geoid, retains its form over geological time, at least for low- and medium-degree spherical harmonics. This assumption requires separate justification, given the known temporal variability of the rheological properties of the mantle and lithosphere.

Modern geodynamic models show that the long-wave gravitational response of the Earth ($n \leq 20$) is formed mainly by deep-mantle heterogeneities with characteristic time scales of hundreds of millions of years [Burke et al., 2008; Simmons et al., 2015]. Structures such as large low-velocity zones (LLSVPs) in the lower mantle or cold remnants of subduction plates have high thermal and dynamic inertia and do not undergo rapid restructuring even when the configuration of lithospheric plates changes [Burke, Torsvik, 2004; Torsvik et al., 2010]. At this large scale, the transfer func-

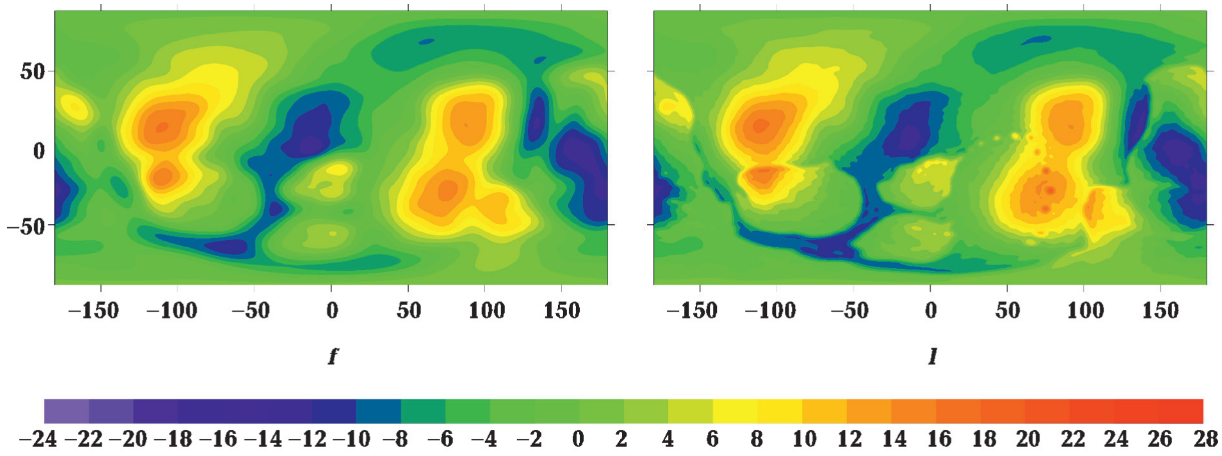

Fig. 4. Visualization of the contribution of topography to the heights of the paleogeoid in meters. Harmonic components used $\lambda_n \overline{C}_{nm}^{PT}$ and $\lambda_n \overline{S}_{nm}^{PT}$ up to the 20th degree: *a* — 100 Ma, *b* — 200 Ma, *c* — 300 Ma, *d* — 400 Ma, *e* — 500 Ma, *f* — 540 Ma. Harmonics $\lambda_n \overline{C}_{nm}^{PT}$ and $\lambda_n \overline{S}_{nm}^{PT}$ up to the 70th degree were used: *g* — 100 Ma, *h* — 200 Ma, *i* — 300 Ma, *j* — 400 Ma, *k* — 500 Ma, *l* — 540 Ma.

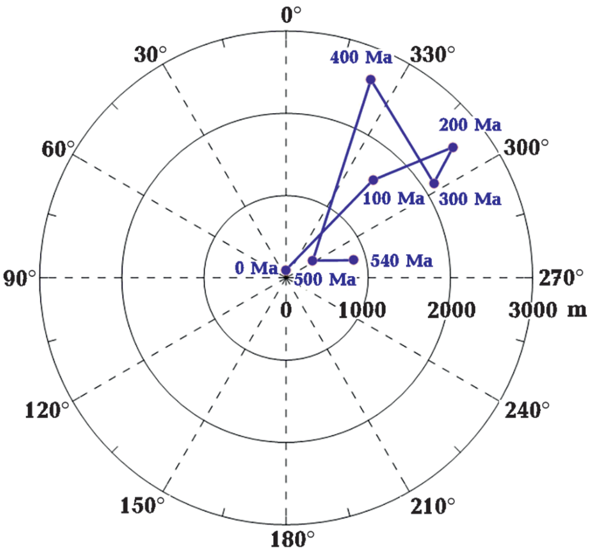

Fig. 5. Graphs of changes in the position of the North Pole based on modeling results.

tion λ_n reflects not so much the instantaneous properties of the lithosphere as the integral gravitational response of the lithosphere-mantle system, taking into account viscoelastic relaxation. Numerical models of mantle convection with temperature-dependent viscosity show that changes in absolute viscosity values mainly affect the time components of the relaxation of inhomogeneities, while the spectral form of the gravitational response at long wavelengths remains relatively stable [Van der Voo et al., 1999; Aryasova, Khazan, 2018; Zaccagnino, Doglioni, 2022].

Thus, the assumption of quasi-stationarity (n is justified for low and medium degrees of spherical decomposition, which determine the evolution of the inertia tensor and the position of the Earth's axis of rotation. For higher degrees ($n>40\div50$), which are sensitive to short-wave lithospheric structures and local isostatic effects, this assumption is likely to be violated, which justifies limiting the analysis of the geoid to predominantly long-wave components.

It should also be emphasized that, in the proposed algorithm, λ_n is not interpreted as a universal physical constant but rather as an effective spectral operator that accumulates the actions of isostatic mechanisms, deep support of topography, and mantle dynamics. In this sense, its use is a first approximation adequate for the analysis of global geodynamic trends, but it can be refined in future studies by introducing time- or region-dependent transfer functions.

In conclusion, we should also note that, until recently, there were not enough well-founded paleogeographic reconstructions that provided information on the paleorelief with satisfactory resolution. Today, we were able to use digital models of the relief of PaleoDEM [Scotese, Wright, 2018] lithosphere paleo-reconstructions, and, based on the studies conducted, we were convinced that the movement of lithospheric plates only partially changes the heights of the geoid. This so-called dynamic geoid relief is transitional, changing both in space and time in response to major mantle flows. Thus, the results confirm that the geoid is a sensitive indicator of tectonic and mantle processes, and its changes in the past can be used to reconstruct the geodynamic history of the Earth. Thus, the end result of this process is the creation of a stable time series of geoid models that allows tracking the evolution of global gravitational anomalies and reconstructing dynamic processes in the Earth's mantle, including the displacement of the pole of inertia and the reorganization of lithospheric plates.

Conclusions. The geoid in the geological past cannot be reconstructed using direct measurement methods and therefore requires coordination between different types of models — topographic reconstructions, tomographic models, modern gravity models, and data on the movement of lithospheric plates. The spectral approach allows the complex morphology of paleotopography to be converted into a mathematically controlled algorithm, in which not only physically significant components are separated, but also those directly related to dynamic processes in the mantle. In this sense, spherical harmonics are a universal mathematical tool that ensures uniformity of spatial scales, suitability for statistical operations, and compatibility with modern models of the gravitational field and topography, as well as with the dynamic topography reproduced in paleo-reconstruction models. TPW is also a key element in analyzing Earth's geodynamic evolution, and reconstructing it helps us indirectly understand the processes that have taken place in the planet's deep layers over geological time.

It has been shown that the main contribution to the formation of both the modern geoid and reconstructed paleogeoids is determined by long-wave mass heterogeneities in the mantle, while the contribution of surface topography is secondary and limited to the middle degrees of spherical harmonic decomposition. This is consistent with current ideas about the dominant role of mantle convection in the formation of the global shape of the Earth and its inertia tensor.

Reconstructed paleogeoids for different stages of the Phanerozoic demonstrate the relative stability of long-wave structures with simultaneous variability in the amplitudes and orientation of low-order harmonics. An analysis of the second-order coefficients $\overline{C}_{nm}^{PG}$ and $\overline{S}_{nm}^{PG}$ showed that changes in the mass distribution within the mantle lead to repeated systematic shifts in the axis of maximum moment of inertia, which is interpreted as manifestations of TPW. The estimated amplitudes of pole displacement do not exceed a few kilometers, which is consistent with current geodynamic constraints on the shortest of the Earth's axes of inertia for the Phanerozoic Earth.

The proposed approach does not directly depend on paleomagnetic reconstructions and is based solely on an analysis of mass distribution and the influence of plate tectonic dynamics on the gravitational field, making it an independent tool for studying the long-term evolution of the planet's moment of inertia. The results indicate that the geoid can be considered as an integral indicator of deep geodynamic processes and used to quantitatively assess their impact on the stability of the Earth's rotation.

Reconstructions of the paleogeoid are of an integrated nature and are not intended to interpret local or short-wavelength features. Further development of the method may involve accounting for temporal variability in the mantle's rheological properties and integrating it with seismic tomography.

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Геодинамічне моделювання змін геоїда та істинної міграції полюса в геологічному минулому з використанням сферичного гармонічного аналізу

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Представлено результати реконструкції палеогеоїда для різних геологічних епох фанерозою (0—540 млн років) з використанням сферичного гармонічного аналізу. Методика базується на припущенні про відносну стабільність зв'язку між топографією та геоїдом у геологічному часі для низьких і середніх ступенів сферичних гармонік. Підставою для такого припущення є те, що більша частина топографії геоїда (близько 9/10) має походити від неоднорідностей розподілу підкорової густини, які є більш стабільними в геологічному часі. Знаючи сучасний зв'язок топографії і геоїда і маючи реконструкції палеотопографії, можна оцінити давню форму геоїда через процедуру вилучення внеску сучасної топографії літосфери у висоти геоїда і відновлення геоїда (палеогеоїда) на основі палеореконструкцій топографії в минулі геологічні періоди. Використано сучасні моделі геоїда EGM2008, топографії ETOPO1 та палеотопографічні реконструкції PaleoDEM. Запропоновано алгоритм, що включає визначення передавальної функції між топографією та геоїдом, вилучення впливу сучасної топографії та відновлення палеогеоїда на основі реконструйованої палеотопографії. Результати моделювання демонструють, що основний внесок у формування глобального геоїда (близько 90 %) обумовлений неоднорідностями розподілу густини в мантії до глибин ~1000 км, тоді як вклад поверхневої топографії є другорядним. Аналіз гармонічних коефіцієнтів другого ступеня дав змогу реконструювати траєкторію міграції полюса обертання Землі з максимальним відхиленням близько 2600 м протягом фанерозою. Отримані результати узгоджуються із сучасними уявленнями про домінуючу роль мантійної конвекції у формуванні глобальної фігури Землі та підтверджують зв'язок між тектонічними процесами, перерозподілом мас і змінами тензора інерції планети.

Ключові слова: глобальна геодинаміка, мантійна конвекція, тектоніка плит, геоїд, палеогеоїд, сферичні гармоніки, істинна міграція полюса, динамічна топографія, фанерозой.