DOI: http://dx.doi.org/10.15673/2072-9812.1/2015.50224
Proc. Intern. Geom. Center 2015 **8**(1) 100-105 *dω*

Chaos-Geometric approach to analysis of chaotic attractor dynamics for the one-ring fibre laser

G.P. Prepelitsa

Abstract Earlier we have developed new chaos-geometric approach to modelling and analysis of nonlinear processes dynamics of the complex systems systems. It combines together application of the advanced mutual information approach, correlation integral analysis, Lyapunov exponent's analysis etc. Here we present the results of its application to studying low-and high-D attractor dynamics of the one-ring fibre laser

Keywords geometry of chaos, non-linear analysis, laser system

Mathematics Subject Classification: (2000) 55R01-55B13

1. Introduction

Earlier [1]–[8] we have developed a new, chaos-geometrical combined approach to treating of chaotic low- and high-D attractor dynamics of complex dynamical systems and forecasting its temporal evolution. Here we use this approach to carry out an analysis of nonlinear processes dynamics in the one-ring fibre laser. Such systems has a great practicalk interest and is used in different technical applications. Our approach combines together application of a few techniques, namely, an advanced mutual information approach, correlation integral analysis, Lyapunov exponent's analysis etc. Let us remind that during the last two decades, many studies in various fields of science have appeared, in which chaos theory was applied to a great number of dynamical systems, including those are originated from nature [5]–[16]. The outcomes of such studies are very encouraging, as they reported very good predictions using such an approach for different systems.

2. Chaos-geometrical approach to the one-ring fibre laser attractor dynamics

In this work we study low-and high-D ttractor dynamics of the the one-ring fibre laser. To speak more exactly, we make a detailes analysis of the output voltage temporal variations series with two controlling parameters (the modulation frequency f and dc bias voltage of the electro-optical modulator) and as an analysis technique use the non-linear prediction approache and chaos theory method (in versions) [1]–[8]. The output voltage temporal variations series for the the one-ring fibre laser are described and listed in [9].

The fundamental aspects of our chaos-geometric approach version have been in details presented earlier. So, below, we will give only ashort description of the fundamental sapects, following to our papers [1]–[8]. As usually, one should formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is a start time, Δt is time step, and n is number of the measurements. In a general case, $s(n)$ is any time series (f.e. atmospheric pollutants concentration). As processes resulting in a chaotic behaviour are fundamentally multivariate, one needs to reconstruct phase space using as well as possible information contained in $s(n)$. Such reconstruction results in set of d-dimensional vectors $\mathbf{y}(n)$ replacing scalar measurements. The main idea is that direct use of lagged variables $s(n + \tau)$, where τ is some integer to be defined, results in a coordinate system where a structure of orbits in phase space can be captured. Using a collection of time lags to create a vector in d dimensions, $\mathbf{y}(n) = [s(n), s(n + \tau), s(n + 2\tau), .., s(n + (d-1)\tau)]$, the required coordinates are provided. In a nonlinear system, $s(n + j\tau)$ are some unknown nonlinear combination of the actual physical variables. The dimension d is the embedding dimension, d_E.

Let us remind that following to [1]–[8], the choice of proper time lag is important for the subsequent reconstruction of phase space. If τ is chosen too small, then the coordinates $s(n + j\tau)$, $s(n + (j+1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. If τ is too large, then $s(n+j\tau)$, $s(n+(j+1)\tau)$ are completely independent of each other in a statistical sense. If τ is too small or too large, then the correlation dimension of attractor can be under-or overestimated. One needs to choose some intermediate position between above cases. First approach is to compute the linear autocorrelation function $C_L(\delta)$ and to look for that time lag where $C_L(\delta)$ first passes through 0. This gives a good hint of choice for τ at that $s(n + j\tau)$ and $s(n + (j + 1)\tau)$ are linearly independent. It's better to use approach with a nonlinear concept of independence, e.g. an average mutual information. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to 0 if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$. Earlier it was suggested, as a prescription, that it is necessary to choose that τ where the first minimum of $I(\tau)$ occurs.

In [5]–[6] it has been stated that an aim of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. If the time series is characterized by an attractor, then correlation integral $C(r)$ is related to a radius r as $d = \lim_{r \to 0, N \to \infty} \frac{\log C(r)}{\log r}$, where d is correlation exponent.

3. Conclusions

As input data we have used detailed numerical data for time series of the output voltage temporal variations series in depedence of with two controlling parameters: the modulation frequency f and dc bias voltage of the electro-optical modulator [9]. In depending upon f and dc bias voltage V values there are realized 1-period (f = 75MHz, V = 10V and f = 60MHz, V = 4V), 2-period (f = 68 MHz, V = 10V or f = 60MHz, V = 6V), chaotic (f = 64MHz, V = 10 V and f = 60MHz, V = 10V) regimes in dynamics of the system. We are interested by a chaotic regime, when there is realized chaotic attractor.

It table 1 we list the values of the autocorrelation function C_L and the first minimum of mutual information I_{min1} for the output voltage amplitude of the one-ring fiber laser system. Itis analyszed the time series of the output voltage amplitude when the controlling parameter, namely frequency f of the electro-optical modulator is changing (correspondingly, the dc bias voltage parameter is constant). We call this regime as the chaos 1 one.

Table 1. Time lags (hours) subject to different values of C_L, and first minima of average mutual information,I_{min1}, (time series of output voltage amplitude of the one-ring fiber laser system).

	Series 1	Series 2	Series 3	Series 4
C_L=0.1	78	96	124	138
C_L=0.5	11	13	9	22
I_{min1}	14	16	18	230

The values, where the autocorrelation function first crosses 0.1, can be chosen as τ, but in [10]–[12] it's showed that an attractor cannot be adequately reconstructed for very large values of τ. So, before making up final decision we calculate the dimension of attractor for all values in Table 1. The large values of τ result in impossibility to determine both the correlation exponents and attractor dimensions using Grassberger-Procaccia method [12]. Here the outcome is explained not only inappropriate values of τ but also shortcomings of correlation dimension method. If algorithm [4] is used, then a percentages of false nearest neighbours are comparatively large in a case of large τ. If time lags determined by average mutual information are used, then algorithm of false nearest neighbours provides $d_E = 6$.

Table 2 shows the correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), and average limit of predictability ($Pr_{max}, hours$) for time series of the output voltage amplitude in dependence of changing the frequency f (dc bias voltage of the electro-optical modulator is fixed; chaos 1 regime) and changing dc bias voltage of the electro-optical modulator (frequency f is constant; chaos 2 regime).

Table 2. The Time lag (τ), correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L) for time series of the output voltage amplitude in dependence of changing the frequency f (dc bias voltage of the electro-optical modulator is fixed; chaos 1 regime) and changing dc bias voltage of the electro-optical modulator (frequency f is constant; chaos 2 regime).

	Chaos 1	Chaos 2
τ	6	6
(d_2)	3.0	3.1
(d_E)	4	4
d_L	2.85	2.88

Further the numerical calculation give the following positive values results for two Lyapunov's exponents (LE) λ_i, namely, one LE pair for chaos 1 regime: 0.168 and 0.0212 and for chaos 2 regime: 0.172 and 0.0215). It is obvious that the positive confirm a chaotic feature of the system dynamics. Besides, we have found that the time series of the output voltage amplitude in the chaos regimes have acceptable predictability than other time series, for example, in the hyperchaos one.

So, in this paper we first have considered an application of an advanced chaos-geometrical approach (combinatin of the advanced mutual information approach, correlation integral analysis, Lyapunov exponent's analysis etc) to numerical modelling and analysis of an attractor dynamics of the one-ring fibre laser phase space.

References

1. Glushkov A.V., Kuzakon' V.M., Khetselius O.Yu., Prepelitsa G.P. and Svinarenko A.A., Geometry of Chaos: Theoretical basis's of a consistent combined approach to treating chaotic dynamical systems and their parameters determination// Proc. Int. Geom. Centre.-2013.-Vol.6.-N1.-P6-12.
2. Glushkov A.V., Khetselius O.Yu., Florko T.A., Prepelitsa G.P., Chaos-Geometric approach to analysis of quantum-generator systems// Proc. Int. Geom. Centre.- 2014.-Vol.7,N4.-P.77-82.
3. Bunyakova Yu.Ya., Glushkov A.V.,Fedchuk A.P., Serbov N.G., Svinarenko A.A., Tsenenko I.A., Sensing non-linear chaotic features in dynamics of system of couled autogenerators: standard multifractal analysis// Sensor Electr. and Microsyst. Techn.-2007.-N1.-P.14-17.
4. Glushkov A.V., Khokhlov V.N., Loboda N.S., Bunyakova Yu.Ya., Short-range forecast of atmospheric pollutants using non-linear prediction method// Atmospheric Environment (Elsevier).-2008.-Vol.42.-P. 7284–7292.
5. Glushkov A.V., Svinarenko A.A., Buyadzhi V.V., Zaichko P.A., Ternovsky V.B., Chaos-geometric attractor and quantum neural networks approach to simulation chaotic evolutionary dynamics during perception process// Advances in Neural Networks, Fuzzy Systems and Artificial Intelligence, Series: Recent Advances in Computer Engineering (Gdansk,EU, World Sci.).-2014.-Vol.21.-P.143–150.
6. Glushkov A.V., Prepelitsa G.P., Lepikh Ya.I., Buyadzhi V.V., Ternovsky V.B., Zaichko P.A., Chaotic dynamics of non-linear processes in atomic and molecular systems in electromagnetic field and semiconductor and fiber laser devices: new approaches, uniformity and charm of chaos// Sensor Electronics and Microsystems Techn.-2014.-Vol.11,N4.-P.43–57.
7. Glushkov A.V., Khetselius O.Yu., Bunyakova Yu.Ya., Prepelitsa G.P., Solyanikova E.P., Serga E.N., Non-linear prediction method in short-range forecast of atmospheric pollutants: low-dimensional chaos// Dynamical Systems – Theory and Applications. – Lodz: Lodz Univ. Press (Poland). –2011.- LIF111 (6p.).
8. Glushkov A.V., Bunyakova Yu.Ya., Zaichko P.A., Geometry of Chaos: Consistent combined approach to treating chaotic dynamics atmospheric pollutants and its forecasting// Proc. of Int. Geometry Center.-2013.-Vol.6,N3.-P.6-14.
9. Feng C., Yu-Ling F., Zhi-Hai Y., Jian F., Yuan-Chao S., Yu-Zhu Z.: Experimental investigation on chaos generation in erbium-doped fiber single-ring lasers//Chin. Phys. B.-2013.-Vol. 21.-P.100504.
10. Kennel M., Brown R., Abarbanel H., Determining embedding dimension for phase-space reconstruction using a geometrical construction//Phys Rev A.-1992.-Vol.45.-P.3403–3411.
11. Packard N., Crutchfield J., Farmer J., Shaw R., Geometry from a time series//Phys Rev Lett.-1988.-Vol.45.-P.712–716.
12. Grassberger P., Procaccia I., Measuring the strangeness of strange attractors//Physica D.-1983.-Vol.9.-P.189–208.
13. Fraser A., Swinney H., Independent coordinates for strange attractors from mutual information// Phys Rev A.-1986.-Vol.33.-P.1134–1140.
14. Takens F (1981) Detecting strange attractors in turbulence. In: Rand DA, Young LS (eds) Dynamical systems and turbulence, Warwick 1980. (Lecture notes in mathematics No 898). Springer, Berlin Heidelberg New York, pp 366–381
15. Mane R (1981) On the dimensions of the compact invariant sets of certain non-linear maps. In: Rand DA, Young LS (eds) Dynamical systems and turbulence, Warwick 1980. (Lecture notes in mathematics No 898). Springer, Berlin Heidelberg N.-Y., p. 230–242

16. Sano M, Sawada Y (1985) Measurement of the Lyapunov spectrum from a chaotic time series//Phys Rev.Lett.-1995.-Vol.55.-P.1082–1085

G.P. Prepelitsa
Information Technologies Department
State Environmental University, Odessa, Ukraine
E-mail: quantpre@mail.ru