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New chaos-geometric and information technology analysis of chaotic generation regime in a single-mode laser system with absorbing cell

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Abstract Here we present the results of application of a new chaos-geometric approach and some information technology algorithms to analysis of chaotic generation regime in a single-mode laser system with absorbing cell. Earlier developed chaos-geometric approach to modelling and analysis of nonlinear processes dynamics of the complex systems combines together application of the advanced mutual information approach, correlation integral analysis, Lyapunov exponent's analysis etc.

Keywords geometry of chaos, non-linear analysis, laser system

Mathematics Subject Classification: (2000) 55R01-55B13

1. Introduction

As it is known a chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena [1]–[10]. The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently irregular phenomena could be understood from a chaotic deterministic point of view but reported very good predictions using such an approach for different systems too.

Earlier [1]–[8] we have developed a new, chaos-geometrical and information technology combined approach to treating of chaotic low- and high-D attractor dynamics of complex dynamical systems and forecasting its temporal evolution. Here we use this approach to carry out an analysis of chaotic self-oscillations in a single-mode laser system with absorbing cell. Such systems have a great practical interest and are used in different technical applications. Our approach combines together application of a few techniques, namely, an advanced infomation technolgy, including a mutual information approach, correlation integral analysis, Lyapunov exponent's analysis etc.

2. Chaos-geometrical approach to complex self-oscillations in a single-mode laser system with absorbing cell

In this work we study complex self-oscillations in a single-mode laser system with absorbing cell. As an analysis technique use an advanced non-linear prediction approach, based on some chaos theory methods and information technolgy algorithms (in versions) [1]–[8]. We consider the output data of a theoretical model of a single-mode laser resonator in which the reinforcement is placed along with a nonlinear absorbing medium [9]. Each of the environments consists of identical two-level atoms. The gain and absorption lines are uniformly broadened and their centers align and coincide with one of the frequencies of the cavity. Such a model can describe the real system of five differential equations, which have been numerically solved within different approximations [9]–[10]. At last, let us note that the system studied is used for the experimental observation of a dynamical chaos.

The fundamental aspects of our chaos-geometric approach version have been in details presented earlier. So, here we are limited only by a short description of the key aspects, following to our papers [1]–[8]. As usually, one should formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is a start time, Δt is time step, and n is number of the measurements. In a general case, $s(n)$ is any time series (f.e. atmospheric pollutants concentration). As processes resulting in a chaotic behaviour are fundamentally multivariate, one needs to reconstruct phase space using as well as possible information contained in $s(n)$. Such reconstruction results in set of d-dimensional vectors $\mathbf{y}(n)$ replacing scalar measurements. The main idea is that direct use of lagged variables $s(n+\tau)$, where τ is some integer to be defined, results in a coordinate system where a structure of orbits in phase space can be captured. Using a collection of time lags to create a vector in d dimensions, $\mathbf{y}(n) = [s(n), s(n+\tau), s(n+2\tau), .., s(n+(d-1)\tau)]$, the required coordinates are provided. In a nonlinear system, $s(n+j\tau)$ are some unknown nonlinear combination of the actual physical variables. The dimension d is the embedding dimension, d_E.

Let us remind that following, for example, to [7]–[8], the choice of proper time lag is important for the subsequent reconstruction of phase space. If τ is chosen too small, then the coordinates $s(n + j\tau)$, $s(n + (j + 1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. If τ is too large, then $s(n + j\tau)$, $s(n + (j + 1)\tau)$ are completely independent of each other in a statistical sense. If τ is too small or too large, then the correlation dimension of attractor can be under-or overestimated. One needs to choose some intermediate position between above cases. First approach is to compute the linear autocorrelation function $C_L(\delta)$ and to look for that time lag where $C_L(\delta)$ first passes through 0. This gives a good hint of choice for τ at that $s(n + j\tau)$ and $s(n + (j + 1)\tau)$ are linearly independent.

It's better to use approach with a nonlinear concept of independence, e.g. an average mutual information [10]–[12]. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to 0 if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$. Earlier it was suggested, as a prescription, that it is necessary to choose that τ where the first minimum of $I(\tau)$ occurs.

Earlier (look [5]–[12]) it has been stated that an aim of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. If the time series is characterized by an attractor, then correlation integral $C(r)$ is related to a radius r as $d = \lim\limits_{r \to 0, N \to \infty} \frac{\log C(r)}{\log r}$, where d is correlation exponent [13].

3. Some results and Conclusions

As it has been noted above as data for analysis we use the output data of theoretical model of a single-mode laser resonator, more exactly, solutions of a system of five differential equations, which have been numerically solved within different approximations [9]–[10]. The cited system includes the equations for intensity, and simuteniously for absorbing medium. The functions to be determined are amplitude of the laser of the field, polarizations in the environment and difference between populations of the working levels in the two-level atomic ensemble.

The key physical parameters include longitudinal and transverse relaxation rates d_k , related to the half-width of the resonator $dw/2$, the ratio of the coefficients of saturation of the absorbing and amplifying media b. We are interested by a chaotic regime of the system, when there is realized a chaotic attractor. Indeed, according to [9]–[10], strange attractors occur as a result of the sequence of bifurcations of solutions of above cited dynamical equations system, the first of which is the Hopf bifurcation of stationary solutions with zero intensity of the laser field. Appearance of bifurcations is linked with the governing parameter $N = F[d_i, b_i]$ (i=1,2). Our analysis shows that the Hopf bifurcation occurs at moderate values N, if the relative width of the absorption line d_2 is quite small, and the relative width of the gain line d_1 is quite large. The numerical calculation showed that in order to get the chaotic lasing it is necessary the following: to saturate the absorber should be saturated stronger than the amplifier (b>1). At low b the limit cycles generated from the stationary solutions with the zeroth intensity is stable up to very large values of N.

It table 1 we list the values of the autocorrelation function C_L and the first minimum of mutual information I_{min1} for time series of the output function (amplitude, polarization)for the studied single-mode laser system with absorbing cell (four sets of data).

Table 1. Time lags subject to different values of C_L, and first minima of average mutual information,I_{min1}, (see text).

	Series 1	Series 2	Series 3	Series 4
C_L=0.1	42	53	68	96
C_L=0.5	7	8	10	16
I_{min1}	8	10	12	112

The values, where the autocorrelation function first crosses 0.1, can be chosen as τ, but in [10]–[13] it's showed that an attractor cannot be adequately reconstructed for very large values of τ. So, before making up final decision we calculate the dimension of attractor for all values in Table 1. The large values of τ result in impossibility to determine both the correlation exponents and attractor dimensions using Grassberger-Procaccia method [13]. Here the outcome is explained not only by inappropriate values of τ but by shortcomings of correlation dimension method too. If algorithm [4] is used, then a percentages of false nearest neighbours are comparatively large in a case of large τ. If time lags determined by average mutual information are used, then algorithm of false nearest neighbours provides $d_E = 8$.

Table 2 shows the time lags, correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L) for time series of the output function (amplitude, polarization)for the studied single-mode laser system with absorbing cell (two sets of data, accordingly, regimes: chaos 1 and chaos 2).

Table 2. The Time lag (τ), correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L) for time series of the the output function for the studied single-mode laser system with absorbing cell.

	Chaos 1	Chaos 2
τ	7	8
(d_2)	4.7	4.7
(d_E)	5	5
d_L	4.15	4.17

The further computing give the following values for two Lyapunov's exponents (LE) λ_i, namely, one LE pair for chaos 1 regime: 0.215 and 0.154 and for the chaos 2 regime: 0.218 and 0.152). Naturally, the positive values of the first two Lyapunov's exponents confirm a chaotic feature of the system dynamics [14]–[16]. So, in this paper we have presented results of computing and numerical analysis of the strange attractor dynamics of the single-mode laser system with absorbing cell with using an advanced chaos-geometrical and information technology approach (combinatin of the advanced mutual information approach, correlation integral analysis, Lyapunov exponent's analysis etc). The numerical data on the time lags, correlation dimension, embedding dimension (d_E), Kaplan-Yorke dimension (d_L) and the LE for time series of the output function for the studied single-mode laser system with absorbing cell are listed.

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