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Psychology of the Interaction of Understanding and Forecasting Processes in Creative Mathematical Thinking

Психологія взаємодії процесів розуміння і прогнозування в творчому математичному мисленні

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ABSTRACT

Based on the results of the analysis of research on mathematical thinking, its creative nature has been ascertained.

The results of research on creative mathematical thinking were analyzed and the expediency of studying the psychological essence of the interaction of thought processes of understanding and forecasting when solving creative mathematical problems was ascertained.

***The aim of the article** is to find out the psychological essence of the interaction of thought processes of understanding and forecasting in creative mathematical thinking.*

*To study the interaction of the processes of understanding and forecasting in mathematical thinking, **the method** of analyzing students' search actions during solving creative mathematical problems of different classes was used.*

***The results of the research**. It was established that creative mathematical thinking is a complete system of interrelated actions, with the help of which the thinking mathematical result is achieved.*

It was established that the processes of understanding mathematical problems and predicting thinking results function throughout the entire process of solving mathematical problems.

It was found that the content of search actions aimed at understanding the problem and predicting thinking results depend on the stages of solving the problem (study of the condition, search for a solution, verification of the found solution), in which their procedural and dynamic side is not only manifested, but is also being formed. At the same time, the process of understanding a creative mathematical problem and the process of forecasting are complementary.

It is established that the understanding of the condition of the problem forms the content of forecasting actions, and the process of forecasting cont-

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ributes to the formation of understanding of the mathematical problem. It was established that in the search mathematical process it is not possible to record such a state of understanding of the problem that would ensure the emergence of a hypothesis regarding the solution.

It has been found that forecasting, which takes place throughout the entire search process, can generate a solution hypothesis at different stages of the solution, with different states of understanding of the mathematical problem.

The hypothesis of solving the problem is an indicator of the state of understanding of the problem, and its approbation contributes to deepening the understanding of the essence of the problem itself.

At the same time, the content of the hypothesis, its approval determines the state of understanding of the problem.

Conclusion. *The process of the subject's understanding of a creative mathematical problem and the process of prediction take place throughout all stages of the solution process and are mutually complementary.*

Key words: *creative mathematical thinking, understanding of a mathematical problem, prediction of thinking results.*

Introduction

The study of creative thinking process, including mathematical, is an important task of psychology. This creates the basis for the formation of a personality capable of solving creative non-standard tasks. That is, he/she will be able to find a solution in those cases for which there haven't been developed rules of action yet, will be able to turn knowledge into tools for active actions.

Along with this, today mathematics has become not only a tool for quantitative calculations, as it was at its inception, but also a method of research. Mathematics, as a science that does not have direct connections with physics, chemistry, biology, economics, technology, is used with equal success in all these fields of knowledge (Jonsson, Mossegård, Lithner & Wirebring, 2022).

Mathematics reveals nature with the help of its own abstractions: numbers, magnitude, functions, geometric figures, etc. Scientists (Moiseienko, 2003; Firmasari, Sulaiman, Hartono & Noto, 2019) state that mathematical thinking has its speci-

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fic manifestations related to: the use of mathematical symbols; dominance of the logical method of proof; the presence of an algorithm for solving many problems; simultaneous functioning of axiomatic and constructive methods of constructing mathematical theories, etc. At the same time, the mathematical result has the property that it can be used not only in the study of a certain phenomenon or process, but can also be used in many others, the physical nature of which is fundamentally different from those previously considered (Wong, 2018). Therefore, ***the relevance*** of the study of various aspects of creative mathematical thinking is provided by the combination of general features of intellectual creativity with the specifics of mathematical activity.

Researchers of the exploratory thinking process interpret it as a problem-solving process (Moliako, 2007; Mahwah, NJ, 2015; Hidayah, Sa'dijah, Subanji & Sudirman, 2020; Jäder, Lithner, Sidenvall, 2020). They emphasize that solving tasks itself requires the activation of various information, the ability to apply it to create beyond experience (Moreno-Armella, Hegedus & Kaput, 2008). To solve a mathematical problem means to find a certain sequence of general provisions of mathematics (definitions, axioms, theorems, rules, laws, formulas, etc.), applying which to the condition of the problem or to its consequences (intermediate results of the solution), you can get what is required in the task. At the same time, in the process of solving the problem, the sequence of thinking steps is important, and not the determination of the finished result – the answer (even if it is correct) (Tohir, Maswar, Atikurrahman, Saiful & Pradita, 2020). That is why our research on creative mathematical thinking is ***based on the analysis of the search for solutions*** to creative mathematical problems.

In general terms, the condition of a mathematical problem is a set of mathematical facts and objects that sometimes have no obvious connection with each other. And despite the fact that the set directly presented in the condition of the problem is small,

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it can (and sometimes must) be supplemented by certain known statements, mathematical results (axioms, definitions, theorems, etc). The boundaries of such a set of facts are not clear, because it is not known in advance what knowledge will be needed to solve the problem (Francisco, 2013; Mielicki & Wiley, 2016; Syarifuddin, Nusantara, Qohar & Muksar, 2020). To solve the problem, it is necessary to build your set of facts into a certain structure. At the same time, the solution is the construction of such a structure that gives a specific result (Ortiz Enrique, 2016; Adu-Gyamfi, Bossé & Chandler, 2017& Yunita, Maharani & Sulaiman, 2019).

It is obvious that different thinking components function in such a search's space. Scientists single out the process of understanding the problem, the process of forming a hypothesis for its solution, the process of approbation that hypothesis and study their procedural-dynamic and personal aspects of such components (Moliako, 2007; Moiseienko & Shehda, 2021). The process-dynamic aspect acts as a holistic form of synthesis of various mental phenomena of the subject and is a characteristic of creative thinking (and mathematical thinking in particular). It covers all stages of solving a creative problem (studying the condition, finding a solution, checking the solution found). The content of search actions depends on the stages of solving the problem, in which its procedural and dynamic side is not only manifested, but also formed.

It is generally recognized that understanding the condition of the problem is the initial moment of the stage of finding a solution, and the central moment in solving the problem is finding ways (methods, principles) or the main way of solving it. That is, researchers consider that a prerequisite for the success of any thinking process aimed at solving the task is understanding it (Moliako, 2007; Kovalenko, 2015; Moiseienko & Shehda, 2021). On the basis of a certain state of a hypothesis regarding the solution, which is "… an anticipatory synthesis or idea of a possible solution, based on a preliminary analysis of the condition of the problem" (Kostiuk, 1989: 228). At the same time, scientists are unanimous about the process of forming a solution as a process of proposing and testing hypotheses ***(forecasting)***.

So, the questions arise: What state of understanding of the problem ensures the emergence of a solution hypothesis? Due to what does this condition deepen? How does the state of understanding of the problem relate to the quality of the hypothesis about the solution? etc. Therefore, in order to develop a holistic idea of the procedural and dynamic nature of the interaction of the processes of understanding and forecasting when solving creative tasks, it is advisable to conduct an end-to-end comparative analysis of both processes.

The purpose of the article is to find out the psychological essence between the interaction of the processes of understanding and forecasting in creative mathematical thinking.

The task of the article is to analyze the processes of understanding and forecasting as end-to-end processes solving creative mathematical problems; to find out the essence of the interaction of understanding and forecasting in the thinking process.

Research methods and techniques. The method of research is the analysis of students' search actions during the solution of creative mathematical problems of different classes. That is, we have developed 30 mathematical problems. All problems have been divided by the nature of the requirement for the problem into 4 classes of problems: problems for finding an unknown quantity, problems for proof, problems for construction and heuristic problems.

We have conducted an experimental study of creative mathematical thinking of students of Ivano-Frankivsk National Technical University of Oil and Gas. 100 students have taken part in the experiment. Each student has solved 10 different math problems from different classes.

The tasks have been performed by each student in the presence of the experimenter. The work has been carried out individually. The student has been given the opportunity to work independently, there have been no direct instructions on how to solve the problem. The tasks have not been limited in time.

Records, drawings, replicas and questions of students were analyzed.

Results and discussions

The condition of a mathematical problem is a set of facts and constituent objects that sometimes have no obvious connection with each other. When reading the problem for the first time, students tried to recognize the meaning of words and symbols: they selected from the context of the problem known terms, symbols, numbers that have certain qualitative characteristics ($\sum_{n=1}^{\infty} a_n$ – series; $\frac{\partial z}{\partial x}$ – partial derivative). Through recognition, a certain understanding of the problem comes and it becomes clear whether they have encountered similar problems before, or whether the algorithm for its solution is known. The consequence of such thinking actions can be a hypothesis (true or false) about the application of a known solving algorithm. That is, even after the selection of certain structural elements of the problem, without their further study, finding a solution can start.

Understanding of the problem is deepened by recalling known axioms, definitions, theorems, formulas, etc. (Mumford & Gustafson 1988; Francisco, 2013). However, understanding is not reduced to the reproduction of what was previously known, but is a process of further enrichment of knowledge through the discovery of new connections between things. Knowledge, previous experience, subjective systems of meanings and vocabulary ensure accurate recognition of the functional capabilities of its constituent elements. Knowledge is selected, and the task itself is reformulated, or, more precisely, a certain ratio of its conditions and requirements changes based on a single system of concepts.

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That is, the first thinking steps aimed at studying the condition of the problem are steps aimed at understanding the problem, and the meaning acquired by the structural elements of the problem is the basis for starting to put forward hypotheses about the solution. At the same time, at the beginning of the thinking activity aimed at the task, the structural element acquires the "shade" that can be useful in the solution. Thinking constructions are built, they connect the new object with the subject's mathematical knowledge.

In the act of understanding the mathematical task, the process of building such thinking structures is quite multifaceted. It can be the joining of several elements that will further function as a single whole (for example, a transition from the equation $4x^4 + 4x^2 + 1 = 0$ to equation. $x^2 + 2t + 1 = 0$ where $(2x)^2 = t$ composition of several elements (for example, functions $y=sinx$ i $y = |x|$ consist of $y = |\sin x|$); replacement of several elements (for example, the diameter of a circle is replaced by the hypotenuse of an inscribed right triangle); logical transformation (for example, a whole expression including various functions is taken as a single variable). In such new formations a certain state of understanding of the task is formed. But not only like that, at this stage it is already possible to follow the emergence of hypotheses regarding the future solution.

Further thinking actions are aimed at recoding the problem into "one's own" language. "One's" vision of the condition of the problem consists in providing a familiar mathematical interpretation of a specific situation, in graphic illustrations of the task, or in a textual description of the graphic illustration that is given by condition of the problem.

Experimental tasks were proposed in textual or symbolic form, while some of them were supplemented with graphs and drawings. We observed at the initial stages of thinking activity students' desire to match the text with the available pictures or supplement the tasks with "illustrations". Graphical or schematic interpretation of the text and textual description of graphic

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or schematic information is also a translation of the content of the task into "one's own" language, which often plays a key role in understanding the task. Accompanying the text task with "provocative" pictures often led to a misunderstanding of the task, if the discrepancy between the text and the picture was not detected in time.

It should be especially emphasized that such search actions at the same time determine a certain direction of the search for a solution, that is, an active process of predicting the solution of the problem begins.

On the other hand, the meaning of a mathematical problem, and therefore the state of its understanding, is determined by the interpretation of structural elements by using some hypothesis. That is, the understanding of the problem moves to a higher level, which is determined by the content of such a hypothesis. We observed the emergence of many alternative and non-alternative hypotheses, which led to different understandings of the same problem, which, in turn, contributed to the formation of different directions in the search for a solution.

So, in the process of understanding a mathematical problem, the first hypothesis regarding the solution is put forward. Further thinking actions are guided by this hypothesis. Its verification is carried out under the influence of the content of the structural components of the problem. Without meeting the conditions, but having significantly explored the objects specified in the problem, it is rejected and replaced by another one. The verification of the hypothesis, its agreement with the condition of the problem, leads to a new content of the understanding of the problem. That is, the process of understanding the problem continuously transitioned into the process of building a solution project.

All this can be summarized as follows. In the process of understanding a creative mathematical problem, students first identify the structural elements of the problem (numbers, symbols, operations, geometric shapes, etc.) and recognize their purpose. After that, they find connections between them both by studying and testing a number of hypotheses about these connections (at the stage of studying the condition) and by putting forward hypotheses about ways to solve them (at the stage of forming a solution project). Hypotheses aimed at unifying the disparate elements of the problem condition (in the process of understanding) are gradually intertwined with hypotheses regarding the content of the solution. At the same time, the proposed hypotheses regarding the content of the solution contribute to the process of understanding the problem, because they illuminate certain aspects of the structural objects that contain the creative mathematical problem. That is, the process of understanding the problem runs parallel to the search for its solution. As a result, the subject receives a new vision of the problem as a whole, a new more perfect model of the problem.

In the subsequent search activity, the structural elements of the problem are examined in detail, and a greater number of their properties are revealed. It is about identifying some subjective significance of structural elements and their properties. For mathematics, which operates with symbols and numbers, it is important that the semantic and formal content of the task is reconciled. It is necessary to make a decision about the need for more or less extensive information about those objects that are expressed by certain symbols, or to establish admissible values for symbols. For example, when establishing a set of points described by the inequality $\frac{x}{a}+\frac{y}{b}\leq 1$, it is necessary to know where this set of points is located: on a plane (ellipse) or in space (cylindrical surface). Such information is often not directly specified by the condition of the problem. Then the hypothesis about this information can be tested by an intermediate solution. That is, the understanding of the problem is superimposed on a certain prediction regarding its solution.

In any case, the problem arises in the form of a more or less complete system of mathematical objects – a certain model of the problem is completed and a certain direction of finding its solution is highlighted. Now, instead of separate structural elements, complexes of mathematical objects with their own mathematical properties begin to operate. In some cases, this happens on the basis of the connection of real quantitative ratios, which will further function with the formed symbolic expressions. However, the task still contains a number of blanks (there are "extra" or "missing" elements). A complete understanding of the task has not come yet, which often resulted in a solution (intermediate or final).

Note that the described actions aimed at understanding a mathematical problem can be attributed both to the stage of finding a solution and to the stage of checking the solution (intermediate or final). So it can be argued that ***the process of understanding is "embedded" in the processes of the forecasting solutions and approvals of thinking results.***

The resulting solution (as a rule, an intermediate one) can be checked by a condition. It was the intermediate solutions that were the clearest indicator of students' understanding of the mathematical problem. After all, if the subject gets the impression that there is a correspondence between the condition of the problem and the found new formation, it is declared the solution of the problem. At the same time, the content of this new formation provides information about a certain state of understanding of the problem.

The ineffectiveness of the intermediate solution was revealed at the stage of its approval. The content of mental actions of the approbation process of creative mathematical thinking in comparing the result with the condition and requirement of the task; in reviewing the chain of thinking steps that led to the result; in its approval in various conditions that do not go beyond the conditions of the task and are allowed by it. Such actions, forming subjective confidence in the correctness (or incorrectness) of the found solution, at the same time, are aimed at deepening the understanding of the essence of the problem itself. On

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the other hand, approbation actions confirm or deny the validity of the working hypothesis regarding the search for a solution.

Students often after finding a solution and explaining it to the experimenter refused it, having realized its inconsistency. However, mainly the explanations indicate the level of understanding of the problem, and the quality of the functioning hypothesis of the solution.

Conclusions

The creative mathematical process is a process that harmoniously combines the general features of intellectual creativity with the specifics of mathematical activity, which is associated with the presence of numerical and symbolic elements, the operation of formalized objects often with the help of spatial imagination, the existence of algorithms for solving certain types of problems.

The content of search actions depends on the stages of solving the problem (study of the condition, search for a solution, verification of the found solution), in which its procedural and dynamic side is not only manifested, but also formed.

The process of the subject's understanding of a creative mathematical problem and the process of prediction take place at all stages of the solution process and are mutually complementary. That is why, in the search mathematical process, it is not possible to record such a state of understanding of the problem, which would ensure the emergence of a hypothesis regarding the solution. The prediction that takes place throughout the search process can generate a solution hypothesis at different stages of the solution. Moreover, it can be argued that it is formed from a barely noticeable preference of the direction of thinking actions to the actual hypothesis of the solution in the course of deepening the understanding of the problem. Moreover, it can be argued that it is formed from a barely noticeable preference of the direction of thinking actions to the actual hypothesis of the solution in the course of deepening the understanding of the problem. At the same time, the content of the hypothesis, its approval determines the state of understanding of the problem.

The perspective for further studies on this problem is the study of the personal aspect of the interaction processes of understanding and forecasting, its influence on the course and mutual coordination of all the component processes of creative mathematical thinking.

Literature

Коваленко А.Б. Проблема розуміння в працях українських психологів. *Теоретичні і прикладні проблеми психології*, 2015. № 1(36). С. 190–197.

Костюк Г.С. Навчально-виховний процес і психологічний розвиток особистості. Київ : Радянська школа, 1989. 108 с.

Мойсеєнко Л.А. Психологія творчого математичного мислення. Івано-Франківськ: Факел, 2003. 481 с.

Моляко В.А. Творческая конструктология (пролегомены). Київ: Освита Украины, 2007. 388 с.

Adu-Gyamfi K., Bossé M.J., Chandler K. Student connections between algebraic and graphical polynomial representations in the context of a polynomial relation. *International Journal of Science and Mathematics Education*, 2017. Vol. 15, No 5. https://doi.org/10.1007/s10763-016-9730-1.

Firmasari S., Sulaiman H., Hartono W., Noto M.S. (2019). Rigorous mathematical thinking based on gender in the real analysis course. *Journal of Physics: Conference Series,* 2019. 11574 (042106). https://doi.org/10.1088/1742-6596/1157/4/042106.

Francisco J.M. The mathematical beliefs and behavior of high school students: Insights from a longitudinal study. *The Journal of Mathematical Behavior*, 2013. Vol. 32, No 3. P. 481–493. https://doi.org/10.1016/j.jmathb.2013.02.012.

Hidayah I.N., Sa'dijah C., Subanji S., Sudirman S. Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*, 2020. Vol. 11, No 3. P. 347–362. https://doi.org/10.22342/JME.11.3.11869.347-362.

Jäder J., Lithner J., Sidenvall J. Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*, 2020. Vol. 51, No 7. P. 1120–1136. https://doi.org/10.1080/0020739X.2019.1656826.

Jonsson B., Mossegård J., Lithner J., Karlsson Wirebring L. Creative Mathematical Reasoning: Does Need for Cognition Matter? *Frontiers in Psychology,* 2022. 6207. https://doi.org/10.3389/fpsyg.2021.797807.

Timo Reuter, Wolfgang Schnotz, Renate Rasch. Drawings and Tables as Cognitive Tools for Solving Non-Routine Word Problems in Primary School. *American Journal of Educational Research*, 2015. Vol. 3, No. 11. P. 1387–1397. doi: 10.12691/education-3-11-7.

Mielicki M.K., Wiley J. Alternative representations in algebraic problem solving: When are graphs better than equations? *The Journal of Problem Solving,* 2016. Vol. 9, No 1. P. 3–12. https://doi.org/10.7771/1932-6246.1181.

Moiceienko Lidiia, Shehda Liubov. Thinking styles of Understanding Creative Mathematical Problems in the Process of Solving Them. *Збірник наукових праць «Проблеми сучасної психології»*. 2021. Вип. 51. С. 142–164. DOI (article): https://doi.org/10.32626/2227-6246.2021-51.142-164

Syarifuddin S., Nusantara T., Qohar A., Muksar M. Students' Thinking Processes Connecting Quantities in Solving Covariation Mathematical Problems in High School Students of Indonesia. *Participatory Educational Research*, 2020. Vol. 7, No 3. P. 59–78. https://doi.org/10.17275/per.20.35.7.3

Tohir M., Maswar M., Atikurrahman M., Saiful S., Pradita D.A.R. Prospective Teachers' Expectations of Students' Mathematical Thinking Processes in Solving Problems. *European Journal of Educational Research*, 2020. Vol. 9, No 4. P. 1735–1748. https://doi.org/10.12973/EU-JER.9.4.1735.

Wong T.T.Y. Is conditional reasoning related to mathematical problem solving? *Developmental Science*, 2018. Vol. 21, No 5. P. 12644. https://doi.org/10.1111/desc.12644.

Yunita D.R., Maharani A., Sulaiman H. Identifying of rigorous mathematical thinking on olympic students in solving non-routine problems on geometry topics. *In 3rd Asian Education Symposium (AES 2018). Atlantis Press.* 2019, April. https://doi.org/10.2991/aes-18.2019.111.

References

Adu-Gyamfi, K., Bossé, M. J., & Chandler, K. (2017). Student connections between algebraic and graphical polynomial representations in the context of a polynomial relation. *International Journal of Science and Mathematics Education*, 15(5). https://doi.org/10.1007/s10763-016-9730-1.

Firmasari, S., Sulaiman, H., Hartono, W., & Noto, M. S. (2019). Rigorous mathematical thinking based on gender in the real analysis course.

Journal of Physics: Conference Series, 11574 (042106). https://doi.org/10.1088/1742-6596/1157/4/042106.

Francisco, J. M. (2013). The mathematical beliefs and behavior of high school students: Insights from a longitudinal study. *The Journal of Mathematical Behavior,* 32(3), 481–493. https://doi.org/10.1016/j.jmathb.2013.02.012.

Hidayah, I. N., Sa'dijah, C., Subanji, S., & Sudirman, S. (2020). Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*, 11(3), 347–362.

Jäder, J., Lithner, J., & Sidenvall, J. (2020). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*, 51(7), 1120–1136.

Jonsson, B., Mossegård, J., Lithner, J., & Karlsson Wirebring, L. (2022). Creative Mathematical Reasoning: Does Need for Cognition Matter? *Frontiers in Psychology*, 6207. https://doi.org/10.3389/fpsyg.2021.797807.

Kostiuk, H.S. (1989). *Navchalno-vykhovnyi protses i psykholohichnyi rozvytok osobystosti[Educational process and psychological development of personality]*. Kyiv: Radianska shkola [in Ukrainian].

Kovalenko A.B. (2015) Problema rozuminnia v pratsiakh ukrainskykh psykholohiv [The problem of understanding in the works of Ukrainian psychologists]. *Teoretychni i prykladni problemy psykholohii – Theoretical and applied problems of psychology*, 1(36), 190–197 [in Ukrainian].

Mielicki, M. K., & Wiley, J. (2016). Alternative representations in algebraic problem solving: When are graphs better than equations? *The Journal of Problem Solving*, 9 (1), 3–12. https://doi.org/10.7771/1932-6246.1181.

Moiseienko L.A. (2003). *Psykholohiia tvorchoho matematychnoho myslennia [Psychology of creative mathematical thinking]*. Ivano-Frankivsk: Fakel [in Ukrainian].

Moiseienko Lidiia, Shehda Liubov (2021). Thinking styles of Understanding Creative Mathematical Problems in the Process of Solving Them. Zbirnyk naukovykh prats *«Problemy suchasnoi psykholohii» – Collection of research papers "Problems of modern psychology"*, 51, 142–164. DOI: https://doi.org/10.32626/2227-6246.2021-51.142-164.

Molyako V.A. (2007) *Tvorcheskaya krnstruktologiya (prolegomenyi) [Creative Constructology (prolegomen)]*. Kyiv: Osvita Ukrainy [in Ukrainian].

Syarifuddin, S., Nusantara, T., Qohar, A., & Muksar, M. (2020). Students' Thinking Processes Connecting Quantities in Solving Covariation Mathematical Problems in High School Students of Indonesia. *Partici-*

patory Educational Research, 7(3), 59–78. https://doi.org/10.17275/per.20.35.7.3

Timo Reuter, Wolfgang Schnotz, & Renate Rasch (2015). Drawings and Tables as Cognitive Tools for Solving Non-Routine Word Problems in Primary School. *American Journal of Educational Research*, 3 (11), 1387–1397. doi: 10.12691/education-3-11-7.

Tohir, M., Maswar, M., Atikurrahman, M., Saiful, S., & Pradita, D. A. R. (2020). Prospective Teachers' Expectations of Students' Mathematical Thinking Processes in Solving Problems. *European Journal of Educational Research*, 9 (4), 1735–1748. https://doi.org/10.12973/EU-JER.9.4.1735.

Wong, T. T. Y. (2018). Is conditional reasoning related to mathematical problem solving? *Developmental Science*, 21(5), 12644.

Yunita, D. R., Maharani, A., & Sulaiman, H. (2019, April). Identifying of rigorous mathematical thinking on olympic students in solving non-routine problems on geometry topics. *In 3rd Asian Education Symposium (AES 2018). Atlantis Press*. https://doi.org/10.2991/aes-18.2019.111.

Мойсеєнко Лідія, Шегда Любов. Психологія взаємодії процесів розуміння і прогнозування в творчому математичному мисленні.

За результатами аналізу досліджень математичного мислення констатовано його творчий характер. Наголошено, що результативність пошукових дій досягається завдяки взаємодії числової, символьної та просторової складової математичного мислення.

Проаналізовано результати досліджень творчого математичного мислення і констатовано доцільність вивчення психологічної сутності взаємодії мисленнєвих процесів розуміння і прогнозування при розв'язуванні творчих математичних задач.

Мета. *З'ясувати психологічну сутність взаємодії мисленнєвих процесів розуміння і прогнозування у творчому математичному мисленні.*

Для вивчення взаємодії процесів розуміння і прогнозування у математичному мисленні було використано ***метод*** *аналізу пошукових дій студентів упродовж розв'язування творчих математичних задач різних класів.*

Результати дослідження. *Констатовано, що творче математичне мислення – цілісна система взаємопов'язаних дій, за допомагаю яких досягається мисленнєвий математичний результат.*

Встановлено, що процеси розуміння математичних задач та прогнозування мисленнєвих результатів функціонують упродовж усього процесу розв'язування математичних задач.

З'ясовано, що зміст пошукових дій, спрямованих на розуміння задачі та прогнозування мисленнєвих результатів, залежать від етапів розв'язання задачі (вивчення умови, пошук розв'язку, перевірка знайденого розв'язку), в яких їх процесуально-динамічна сторона не лише проявляється, але й формується. При цьому процес розуміння творчої математичної задачі та процес прогнозування є взаємодоповнювальними.

Встановлено, що розуміння умови задачі формує зміст прогнозувальних дій, а процес прогнозування сприяє формуванню розуміння математичної задачі.

Констатовано, що у пошуковому математичному процесі не можна зафіксувати такого стану розуміння задачі, який забезпечив би виникнення гіпотези щодо розв'язку.

З'ясовано, що прогнозування, яке має місце впродовж усього пошукового процесу, може породити гіпотезу розв'язку на різних етапах розв'язування, при різних станах розуміння математичної задачі. Гіпотеза розв'язування задачі є індикатором стану розуміння задачі, а її апробація сприяє поглибленню розуміння сутності самої задачі. При цьому зміст гіпотези, її апробація визначає стан розуміння задачі.

***Висновки.** Процес розуміння суб'єктом творчої математичної задачі та процес прогнозування проходять упродовж усіх етапів процесу розв'язання і є взаємодоповнювальними.*

***Ключові слова:** творче математичне мислення, розуміння математичної задачі, прогнозування мисленнєвих результатів.*

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