ISSN 2227-6246 (Print)
ISSN 2663-6956 (Online)
COLLECTION OF RESEARCH PAPERS
"PROBLEMS OF MODERN PSYCHOLOGY"
DOI: https://doi.org/10.32626/2227-6246.2024-63
2024. ISSUE 63

UDC 159.955+159.956 УДК 159.955+159.956

Problems with the Requirement to "to Prove" as a Training of Creative Mathematical Thinking

Задачі з вимогою «довести» як тренінг творчого математичного мислення

Moiseienko Lidiia
Dr. in Psychology, Professor,
Ivano-Frankivsk National Technical University of Oil and Gas,
Ivano-Frankivsk, Ukraine
ORCID ID: http://orcid.org/0000-0001-9288-7355
Researcher ID: X-7506-2018
E-mail: Lmoyseyenko@i.ua

Мойсеєнко Лідія
доктор психологічних наук, професор,
Івано-Франківський національний технічний університет нафти і газу,
м. Івано-Франківськ, Україна

Shehda Liubov
Ph.D. in Physics and Mathematics, Assistant Professor,
Ivano-Frankivsk National Technical University of Oil and Gas,
Ivano-Frankivsk, Ukraine
ORCID ID: http://orcid.org/0000-0003-4721-7832
Researcher ID: Z-2516-2019
E-mail: shegda.luba@gmail.com

Шегда Любов
кандидат фізико-математичних наук, доцент,
Івано-Франківський національний технічний університет нафти і газу,
м. Івано-Франківськ, Україна

Address for correspondence, e-mail: kpnu_lab_ps@ukr.net
Copyright: © Moiseienko Lidiia, & Shehda Liubov

The article is licensed under **CC BY-NC 4.0 International (https://creativecommons.org/licenses/by-nc/4.0/)**

© Moiseienko Lidiia, & Shehda Liubov
DOI (article): https://doi.org/10.32626/2227-6246.2024-63.199-218

Авторський внесок: Мойсеєнко Лідія – 60%, Шегда Любов – 40%
The author's contribution: Moiseienko Lidiia – 60%, Shehda Liubov – 40%

ABSTRACT

According to the results of the analysis of research on creative mathematical thinking, it was established that the exploratory mathematical process is the process of setting and solving a mathematical problem. It has been found that questions activate the mental actions of both those who ask them and those who answer them.

It is emphasized that the process of solving a creative problem is determined by the performance of algorithmic and heuristic techniques that can and should be developed. They should contribute to the production of various ideas, their qualitative examination and selection.

The aim of the article research. *To analyze the potential possibilities of the experience of solving problems with the requirement to "to prove" with the active use of cause and effect questions to activate creative mathematical thinking. For this purpose,* ***the method*** *of analyzing students' searching actions while solving creative mathematical problems was used.*

The results of the research. *It was established that the application of a wide range of questions in the process of solving creative mathematical proof problems activates thinking components: understanding the problem, predicting solution ideas, approbation of thinking results, which become more complementary.*

It was found that special questions ensured the actualization of existing knowledge, experience, and skills, the flow of associations, imagination, which contributes to deepening the understanding of the meaning of a mathematical problem.

It was established that predictive thinking actions, which under the influence of special questions generate a high-quality leading idea for solving the problem, contribute to filling it with content due to the active mental modification of structural elements, the emergence of thought new formations.

The possibility of forming the skills of a critical attitude to various kinds of thinking discoveries with the help of their approbation with special questions has been proved. This becomes the basis for building a clear logical chain of reasoning from what is given to what needs to be proved in proof tasks.

Conclusions. *Solving mathematical proof problems with the active use of cause and effect questions activates creative mathematical thinking.*

© Moiseienko Lidiia, & Shehda Liubov

Key words: *creative mathematical thinking, mathematical proof problems, thinking training.*

Introduction

The relevance of the research results described in this article is proved by the need to introduce innovative methods into the educational process. Such implementation should be based on the formation of creative thinking of those who study and those who teach. This fully concerns the formation of creative mathematical thinking during the educational process, because mathematical thinking is the basis of any natural and scientific thinking.

Today, mathematics, not having direct connections with physics, chemistry, biology, economics, technology, is used with equal success in all these fields of knowledge. And, regardless of the fact that mathematics discovers nature with the help of its abstractions: numbers, quantities, functions, geometric shapes, etc. (Jonsson, Mossegård, Lithner, & Wirebring, 2022), today it has become not only tools for quantitative calculations, but also it has become a research method. Therefore, the **relevance of research on the activation of creative mathematical thinking** is provided by the combination of general features of intellectual creativity with the specifics of mathematical activity.

Search mathematical process is the process of setting and solving a mathematical problem. In such a thought process, there are no predetermined, fully defined guidelines that would unambiguously and directly determine the search activity (Моляко, 2007; Mahwah, 2015; Hidayah, Sa'dijah, Subanji, & Sudirman, 2020; Jäder, Lithner, & Sidenvall, 2020).

It is obvious that various thinking components function in such a search space. Scientists single out the process of understanding the problem, the process of forming a hypothesis for its solution, the process of approbation of such a hypothesis and study their procedural-dynamic and personal aspects (Моляко, 2007; Moiseienko, & Shegda, 2023).

Problem solving itself requires the activation of various information, the ability to apply it and to do beyond experience

© Moiseienko Lidiia, & Shehda Liubov

(Moreno-Armella, Hegedus, & Kaput, 2008). To solve a mathematical problem means to find a certain sequence of general provisions of mathematics (definitions, axioms, theorems, rules, laws, formulas, etc.), applying which to the condition of the problem or to its consequences (intermediate results of the solution), you can get what is required in the task. At the same time, in the process of solving the problem, it is the sequence of thinking steps that is important, and not the determination of the finished result – the answer (even if it is correct) (Tohir, Maswar, Atikurrahman, Saiful, & Pradita, 2020). That is why our research on creative mathematical thinking is based on the analysis of **the search** for solutions to creative mathematical problems.

The process of solving a creative problem is determined by the performance of algorithmic and heuristic techniques (Firmasari, Sulaiman, Hartono, & Noto, 2019; Borodina, 2020). At the same time, as is known, algorithms are a system of operations that provides the solution of a specific, known class of problems (Hidayah, Sa'dijah, Subanji, & Sudirman, 2020; Nurkaeti et al., 2020). The task of techniques that activate creative thinking is to "neutralize" the negative impact of algorithmic actions known to the subject, to transform them into auxiliary constructions that can be used both in their finished form and after certain adaptation (Desti, 2020; Ibrahim & Widodo, 2020; Hilmi, & Usdiyana, 2020; Selvy, Ikhsan, Johar, & Saminan, 2020). That is, being clearly regulated, algorithmic actions should not be at the forefront of the search process, but should always be "at hand".

We are talking about such means of influence on thinking activity, which are aimed at gaining experience in analyzing new situations. The main purpose of such tools is to promote the production of various ideas, their qualitative examination and selection. We set ourselves **the task** of finding and implementing the necessary means of influencing students' exploratory mathematical activity in order to activate and optimize it. In this article, we provide an analysis of one of the aspects of such influence.

© Moiseienko Lidiia, & Shehda Liubov
DOI (article): https://doi.org/10.32626/2227-6246.2024-63.199-218

In psychology, it has been found that questions activate the thinking actions of both those who ask them and those who answer them. We have conducted a number of studies, the results of which convincingly testify to the significant benefit for students of the introduction of the method of mutual questions and answers when the lecture material is presented. This method involves students asking thoughtful questions, which are then answered in turn. It turned out that they remember and understand the lecture material much easier than those students who were not covered by such "questioning" training.

Having evidence of the influence of a series of special questions on the process of understanding information, especially on the process of proving theorems and formulas, we became interested in their influence on the process of solving mathematical problems with the requirement to "to prove". In this article, we try to find out the psychological potential of the search process aimed at solving problems with the requirement to "to prove".

The aim of the article

To analyze the potential possibilities of active implementation of problems with the requirement to "to prove" in order to activate creative mathematical thinking.

The problems of the study are as follows: to analyze the possibility of active application of problems with the requirement to "to prove" as training for creative mathematical thinking, to prove the effectiveness of the positive impact of such training on the components of creative mathematical thinking.

Methods of the research

The research method is the analysis of students' search actions during their solving of creative mathematical problems of different classes. That is, we developed 25 mathematical problems with the requirement "to prove" and 5 problems with different tasks: calculate, build, find, research. We conducted an experimental study of creative mathematical thinking of students of Ivano-Frankivsk National Technical University of

© Moiseienko Lidiia, & Shehda Liubov

Oil and Gas. 50 students took part in the experiment: 25 students in the experimental group and 25 students in the control group.

In the experimental group, the study was conducted in three stages. The first 10 tasks (first stage) with the requirement to "to prove" were performed collectively, under the guidance of the experimenter. Such resolution was accompanied by numerous causal questions. The remaining 10 problems with the requirement "to prove" were performed by each student individually (second stage). The student was given the opportunity to work independently, there were no direct instructions on how to solve the problem. The performance of tasks was not limited to a certain time. At the third stage, students of the experimental group solved 5 control problems. Students of the control group solved 5 control problems.

Records, drawings, replicas and questions of students were analyzed.

Results and their discussion

We adhere to the fact that the creative mathematical process is three-stage: studying the condition, forming the idea of a solution, checking the found solution, which is based on three processes: understanding the problem of forecasting and approbation of mathematical results. When solving creative problems, these processes take place in parallel, complementing each other. Based on the results of the other two, the third process produces its intellectual assets, which in turn feed the previous two.

In the tasks of proving it is necessary to make sure of the validity of a certain confirmation. In general, to prove any statement means to show that it is a logical consequence of a system of statements already proven and accepted in mathematics. When studying the processes of proving mathematical facts, it is worth considering that mathematics operates with formalized objects of reality with the help of its symbols and on the basis of the laws of formal logic, while using unconscious thought acts.

© Moiseienko Lidiia, & Shehda Liubov
DOI (article): https://doi.org/10.32626/2227-6246.2024-63.199-218

At the first (collective) stage of the research, when the problems were solved under the guidance of the experimenter, the solution process was accompanied by the formulation of cause and effect questions regarding the essence of the problems. It was found that the majority of students were used to asking primitive questions that require a slight memory strain when answering them, activation of superficial information, or that which is most obviously related to the information that caused the question. There are almost no predictive questions: "What will happen if ...?", questions aimed at clarifying cause-and-effect relationships: "How does ... affect ...?" etc. Therefore, at this stage of the research, we formed the habit of students asking more complex, deeper questions. For this, a set of cliché questions was developed and offered to students, which they could use in collective and individual work.

Specially created conditions forced students to study the contents of the problems more deeply, to put forward hypotheses more actively, and to test mathematical results in more detail. The lack of natural activity was compensated by the influence of the experimenter.

When starting to get acquainted with the task, students first try to *understand* its content. In problems with the requirement "to prove", the main problem is to find out the logical chain of mathematical statements that lead from one part of the condition to another. That is, in the *process of understanding* proof problems, the starting and ending points of the search process are first outlined. In the future, the subject tries to identify the signs and properties of the "final" information in the "source", or in other words, in what is given, useful from a subjective point of view signs of what needs to be proven are sought. On the other hand, the meaning of what needs to be proved through the prism of the features of the objects given in the condition is clarified. That is, there is a need to activate the cognitive component of creative thinking.

Activating influences on the cognitive component were carried out through questions that ensured the actualization of

© Moiseienko Lidiia, & Shehda Liubov
DOI (article): https://doi.org/10.32626/2227-6246.2024-63.199-218

existing knowledge and skills: "What is meant by …?", "How can you apply … in a specific task?", "How is … related to…?", "How can … be used for …? Such questions contribute to the identification of the «dominant» components of the task.

Along with this, questions like: "What will happen if …?", "What is similar to …?", "Why is it important …?", "How are … and … similar?" contribute to the formation of internal freedom of choice of structural objects, their properties for search actions, activation of the flow of associations related to structural components, experience, fantasy. All these are important components of creative thinking.

Further, the deepening of the understanding of the condition of the problem is accompanied by the appearance of geometric constructions – illustrations. We recorded that in case of possible "materialization" of the sought objects in the form of specific figures, graphs, diagrams, blocks when solving proof problems, students try to fulfill them. And to facilitate this, the experimenter continued to ask questions: "What will happen if …?", "Explain how …?", "What do we already know about …?", "How does … affect .. ?", "Should we complete ... for use information about ...".

As a result of the descried actions, there is confidence in the understanding of the problem and thinking actions are directed towards finding a solution. Such confidence can be strengthened by questions such as: "What is the meaning of …?", "What is the difference between … and ...?".

After a detailed study of the conditions of the task (definition of the main task, establishment of connections between structural elements and those of their properties that were known), there is a specification and selection of such structural units and theoretical facts related to them, which become guidelines for the search process. For proof problems, they are mostly different theoretical facts related to the mathematical situation described in the problem.

As you know, the process of solving a new problem is based on putting forward and testing various hypotheses. We immediately note that there is no algorithm for solving proof problems. In proof tasks, more often than in tasks with other requirements, the first hypotheses were expressed at random and related to logical connections between the structural elements of the tasks. A large specific weight among the first hypotheses – the "learning hypothesis" does not in any way mean a waste of time and unproductive expenditure of intellectual efforts. Such hypotheses contributed to a deeper understanding of problems, a more complete clarification of the significance of the constituent elements of the problem, a more comprehensive study of their "possibilities".

That is, in the process of solving proof problems, understanding-prediction is initially more significant for the search process, and hypotheses about the solution (about building a chain of interconnections between structural elements) arise somewhat later.

The flow of predictions and hypotheses contributes to the emergence of a hypothesis about the solution path. We are talking about the dominant hypothesis, because when choosing it, there is a number of others, often quite distant from the problem situation, or very specific, which are true only for a certain part of the problem, with certain values of the symbols. In the future, the search process will be guided by this hypothesis (leading idea). Questions such as "What is the cause of … and why?", "What argument can be made against …?", "Compare … and … based on?" is contribute to this stage of resolution.

After all, the emerging leading idea is not clear and concrete yet. It needs development, detailing, but it is already an idea that determines the search strategy, which can give rise to the idea of a future solution. Such an idea causes further modification of structural elements. Those of their properties that contribute to its development and realization of the plan are taken into account.

By its essence, the content of the leading idea consists in the connection of the initial numerical ratios, the activation of asso-

© Moiseienko Lidiia, & Shehda Liubov

ciation, and as a result of the emergence of new structural units, which is a logical new formation from the elements of the tasks. It should be noted that structural new formations are the subject's own invention, this is what often helps him to solve the problem using a well-known technique - to transfer the problem from a new unknown, creative one to a known standard one. The content and complexity of new formations are related to the content and complexity of the task. In some cases, we are talking about joining, adding, replacing simple mathematical objects, in others – about a complex procedure.

Having mainly a positive meaning, nevertheless, such new formations sometimes became an obstacle. Such new formations are the result of the subject's search activity, the product of his mathematical creativity. He "gave birth" to them and it is often difficult for him to say goodbye to them. Therefore, we often observed that even when the possibility of a new formation to contribute to a positive result is exhausted, students connect them to the logical chain again and again in different angles, and this sometimes inhibited the search process. It is important to possess the skill of a critical attitude to various kinds of thought findings. This is helped by approbation of new formations with questions: "How can … be used for …?", "How is … related to …?", "Explain why …?", "Explain ...?".

After a certain time, on the basis of mental operations, the leading idea is filled with details, various functional relationships – a logical chain of reasoning is built. When solving proof problems, building a logical chain is a particularly important link in the formation of an idea. After all, the content of any proof is precisely in the sequence of logical steps. Attempting to use a certain element or theoretical statement without justifying the necessary logical steps, will not become an idea. Questions: "Which … is better and why?", "What is the difference between … and…?", "How does … affect …?" – are those questions that contribute to the construction of a clear logical chain of reasoning from what is given to what needs to be proven.

© Moiseienko Lidiia, & Shehda Liubov
DOI (article): https://doi.org/10.32626/2227-6246.2024-63.199-218

Having received the solution (final or intermediate), students check it with the content of the problem. It is also a productive process from the point of view of both understanding and forming the idea of a creative mathematical problem. At this stage, it is possible to evaluate the understanding of the problem as a complete system and the understanding of the content of the solution. The solution must not negate numerical and symbolic relationships, satisfy known mathematical facts that are relevant to the problem.

When solving mathematical problems, students often perform such operations that expand the set of solutions. You can find this out by checking. Many mathematical objects, such as functions and algebraic expressions, make sense under certain conditions. Having tested the solution with questions: "What will happen if ...?", "Why is it important ...?", "What can (should) be the solutions to the problem?", "Do you agree with the statement that ...?" directs the approbation of the obtained result and ensures the conviction of the correctness of the logical steps taken and the clarification of how general they are (whether a partial case leading to other consequences has not been omitted).

If a logical chain of consecutive thinking steps is found between what is given and what needs to be proved, the condition of the problem becomes a complete system, all structural elements of which are closely connected by logical connections, while no element "falls out", every element becomes necessary. The properties of the constituents that are involved in the proof process come to the fore. That is, understanding in this case takes the form of understanding-unification and becomes more complete and deeper. There is a subjective confidence that the statement has been proven.

At the second stage of the research, students independently solved problems with the requirement "to prove". Students were asked to write down a variety of questions in parallel with the search for a solution, which can be used to accompany the search process.

© Moiseienko Lidiia, & Shehda Liubov

It should be noted that the questions have become more difficult. Finding out the consequences and nature of the influence of the creative training organized by us on the exploratory mathematical process, we analyzed the process of independent solving of ten proof problems by the students of the experimental group. The attention of the experimenters was directed to the content, quality and place of the questions formulated by the students.

It can be stated that the process of solving the proof problems of the students who participated in the training is based on the facts that came to the fore after the students reformulated the conditions of the problem in their "own" language. The activity of such reformulation is caused by the questions formulated by the experimenter or the solver himself. As a result, students get new facts for analysis, use new concepts and, as a result, put forward new hypotheses. After that, a new analysis of the condition of the problem (analysis from a different angle), a new comparison of the known and the unknown, is actively started. That is, under the influence of such forced reformulations, more and more new data are included in the thinking process, leading to a deeper understanding of the task.

The mental search of some students turns into purposeful prediction of the solution. Intermediate results were examined in more detail, and therefore rational thinking findings were much or less likely to be rejected. The stage of understanding the solution, its justification becomes more significant in the structure of thinking procedures. The need to comprehensively test the found solution becomes a personal property of the thinking process of any student.

Approbation of the task by various questions helps to overcome inertia and stereotyping to a great extent. Students stop relying on close analogues, their imagination and mathematical forecasting become more active. Therefore, bolder hypotheses appear, hypotheses that involve knowledge from different sections of mathematics, often even knowledge from other branches of science.

© Moiseienko Lidiia, & Shehda Liubov

In order to find out the nature of the influence of the creative training organized by us on the exploratory mathematical process, we compared the process of solving five control problems with different requirements by students participating in the training and students of the control group.

The conditions for the students' performance of the control task were independent search activity at all stages of the solution, arbitrary completion time, termination of work at the student's request. We analyzed some quantitative indicators presented in the following table. Quantitative indicators were the solution time, the number of solutions, the average number of questions formulated by students while solving control problems. These data are presented in the following table.

Table 1

Quantitative indicators of the process of solving control problems by students of the experimental and control groups

	Benchmark				
	Average solution time (in minutes)	**Effectiveness of the solution (in %)**			**The average number of questions formulated by students while solving control problems**
		resolved	**not resolved**	**incorrectly resolved**	
Experimental group (25 students)	13.4	88.3	5.5	6.2	5.8
Control group (25 students)	21.3	48.1	25.4	26.5	1.2

Of course, the question on the obtaining a solution is the most general criterion for the quality of the search process in general and all its components (understanding, forming a solution hypothesis, testing the hypothesis). It can be seen from the table that when solving the control problems after the training,

© Moiseienko Lidiia, & Shehda Liubov

a much larger part of the students succeeds, while the average time spent on the solution decreases significantly. A significant decrease in incorrect solutions, a decrease in the number of unsolved problems indicates a deeper understanding of the problem, activation of search actions, greater perfection and significance of approbation actions.

The experience of formulating questions, which they acquired during the training course, contributes to the formulation of deeper questions and the search for the same answers.

A more detailed analysis of quantitative indicators gives the right to claim that after passing the training, the qualitative signs of the mathematical search process also change. The process of detailed examination of the content of the task of such students involves the ability to ask questions and seek answers to them. The experience of formulating questions, which they acquired during the training course, contributes to the formulation of deeper questions and the search for the same answers.

To analyze the possibility of active application of problems with the requirement "to prove" in the conditions of "interrogative" training for creative mathematical thinking. To prove the effectiveness of the positive impact of such training on the components of creative mathematical thinking.

An increase in the number of intermediate solutions is the result of an increase in the number of intermediate solution hypotheses, i.e. an increase in the number of certain angles (leading ideas), under the influence of which the problem situation was studied. That is, not every emerging primary concept of the solution was filled with mathematical content. It was often immediately rejected after being tested by various hypotheses and replaced by others.

Optimizing the process of understanding among the subjects of the experimental group was expressed in the correct allocation of the meaning of the task, in establishing the correct ratio of individual data of the task among themselves, in the qualitative selection of subjective standards necessary for the correct

© Moiseienko Lidiia, & Shehda Liubov

understanding of the task. Students of this group operated on their knowledge at a higher level, in contrast to the subjects of the control group, who significantly more often demonstrated ineffective, meaningless use of their mathematical knowledge, which led to a significantly higher number of errors.

When forming a solution hypothesis, the subjects of the experimental group demonstrated a greater variability of hypotheses, a much deeper content of these hypotheses. The formed ability to work on new material became a precautionary measure against making wrong decisions. The proportion of mental operations aimed at checking the obtained mathematical results significantly increased among the students who underwent the training. Knowledge is formed at a sufficient level of inspection quality and the skill to implement it necessarily in the search process is developed. The skill of comprehensively justifying the decisions made during solving mathematical problems is formed. This skill becomes its important component of students' mathematical thinking.

Conclusions

The implementation of training education contributed to positive changes in the creative mathematical thinking of students, which in general terms is expressed in the formation of the ability to comprehensively justify decisions made during solving mathematical problems, reducing the amount of time spent on solving problems, increasing the effectiveness of search actions, reducing the number of errors, reducing the number of refusals to solve the proposed problems.

The used methods of activation and optimization of creative mathematical thinking are adequate to the task of activation of the processes of understanding, formation of hypothesis of solution and approbation of mathematical results. They form systems of techniques for organizing students' thinking, which contribute to improving the study of the content of mathematical tasks, the production of a wide variety of hypotheses, forecasts, and

© Moiseienko Lidiia, & Shehda Liubov

the ability to deeply and comprehensively check mathematical results when solving creative mathematical problems.

The use of modified training contributed to changes in the basic components of the search process (cognitive, operational, personal-regulatory): the inadequate use of existing knowledge, skills, and abilities, which were the causes of identified errors in search mathematical thinking, is eliminated; the use of mental operations became more optimal and effective; the share of using remote analogues has increased, the use of reconstructive thinking actions has increased; the subjective confidence of students in their own intellectual abilities has increased, positive motivation and interest in mathematical activities are formed.

The perspective of further research of this problem is the study of the impact of the experience of solving problems in the conditions of "questioning" training on the personal aspect of students – on the search activity of students with different thinking styles.

Literature

Мойсеєнко Л.А. Психологія творчого математичного мислення. Івано-Франківськ : Факел, 2003. 481 с.

Моляко В.А. Творческая конструктология (пролегомены). Київ : Освита Украины, 2007. 388 с. Borodina T. Psychological and pedagogical conditions for developing students' creative thinking in teaching a foreign language in higher school. Talent Development & Excellence, 2020. Vol. 25, No 1, p. 12. http://www.iratde.com

Desti R.M., Pertiwi C.M., Sumarmo U., Hidayat W. Improving student's mathematical creative thinking and habits of mind using a problem-solving approach based on cognitive thinking stage. *Journal of Physics: Conference Series*, 2020. Vol. 1657 (1), pp. 1–8, 10.1088/1742-6596/1657/1/012042

Ibrahim I., Widodo S.A. Advocacy approach with open-ended problems to mathematical creative thinking ability. *Infinity Journal*, 2020. Vol. 9, No 1, pp. 93–102 DO*I:10.22460/infinity.v9i1*.

*Fi*rmasari S., Sulaiman H., Hartono W., Noto M.S. Rigorous mathematical thinking based on gender in the real analysis course. *Journal of Physics: Conference Series.* 2019. 11574 (042106). https://doi.org/10.1088/1742-6596/1157/4/042106.

© Moiseienko Lidiia, & Shehda Liubov

Hidayah I.N., Sa'dijah C., Subanji S., Sudirman S. Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*, 2020. Vol. 11, No 3, p. 347–362.
https://doi.org/10.22342/JME.11.3.11869.347-362.

Hilmi Y., Usdiyana D. An analysis of tenth grade students' mathematical creative thinking ability in vector. *Journal of Physics: Conference Series*, 2020. 1521, 032036. https://doi.org/10.1088/1742-6596/1521/3/032036.

Jablonka E. Critical thinking in mathematics education. Encyclopedia of mathematics education. Springer International Publishing, Cham, 2020. pp. 159–163.

Jäder J., Lithner J., Sidenvall J. Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*, 2020. Vol. 51, No 7. P. 1120–1136.
https://doi.org/10.1080/0020739X.2019.1656826

Jonsson B., Mossegård J., Lithner J., Karlsson Wirebring L. Creative Mathematical Reasoning: Does Need for Cognition Matter? Frontiers in Psychology, 2022, 6207. https://doi.org/10.3389/fpsyg.2021.797807

Moiseienko Lidiia, Shehda Liubov. Psychology of the Interaction of Understanding and Forecasting Processes in Creative Mathematical Thinking. *Збірник наукових праць «Проблеми сучасної психології»*. 2023. Вип. 59. С. 118–134. DOI (article): https://doi.org/10.32626/2227-6246.2023-59.118-134

Nurkaeti N., Turmudi Karso, Pratiwi V., Aryanto S., Gumala Y. Enhancement of mathematical creative thinking ability through open-ended approach based on metacognitive. *Journal of Physics: Conference Series*, 2020, Vol. 1521, 032030.

Selvy Y., Ikhsan M., Johar R., Saminan S. Improving students' mathematical creative thinking and motivation through GeoGebra assisted problem based learning. *Journal of Physics: Conference Series*, 2020, Vol. 1460, No 1, pp. 1–8. DOI 10.1088/1742-6596/1460/1/012004

Syarifuddin S., Nusantara T., Qohar A., Muksar M. Students' Thinking Processes Connecting Quantities in Solving Covariation Mathematical Problems in High School Students of Indonesia. Participatory Educational Research, 2020, Vol. 7, No 3. P. 59–78. https://doi.org/10.17275/per.20.35.7.3

Suherman S., Vidákovich T., Komarudin K. STEM-E: Fostering mathematical creative thinking ability in the 21st century. *Journal of Physics: Conference Series*, 2021. Vol. 1882, No 1, Article 012164

Tohir M., Maswar M., Atikurrahman M., Saiful S., Pradita D.A.R. Prospective Teachers' Expectations of Students' Mathematical Thinking Pro-

© Moiseienko Lidiia, & Shehda Liubov

cesses in Solving Problems. *European Journal of Educational Research*, 2020. Vol. 9, No 4, P. 1735–1748. URL: https://doi.org/10.12973/EU-JER.9.4.1735.

Wong T.Y. Is conditional reasoning related to mathematical problem solving? Developmental Science, 2018. Vol. 21, No 5, e12644. URL: https://doi.org/10.1111/desc.12644.

References

Moiseienko, L.A. (2003). *Psykholohiia tvorchoho matematychnoho myslennia [Psychology of creative mathematical thinking]*. Ivano-Frankivsk: Fakel [in Ukrainian].

Moliako, V.A. (2007). *Tvorcheskaia konstruktolohyia (prolehomeny) [Creative constructology]*. Kyiv: Osvita Ukrainy [in Russian].

Borodina, T. (2020). Psychological and pedagogical conditions for developing students' creative thinking in teaching a foreign language in higher school. *Talent Development & Excellence,* 25 (1), 12. Retrieved from http://www.iratde.com.

Desti, R.M., Pertiwi, C.M., Sumarmo, & U., Hidayat, W. (2020). Improving student's mathematical creative thinking and habits of mind using a problem-solving approach based on cognitive thinking stage. *Journal of Physics: Conference Series*, 1657 (1), 1–8. DOI 10.1088/1742-6596/1657/1/012042.

Ibrahim, I., & Widodo, S.A. (2020). Advocacy approach with open-ended problems to mathematical creative thinking ability. *Infinity Journal*, 9 (1), 93–102 DOI:10.22460/infinity.v9i1.

Firmasari, S., Sulaiman, H., Hartono, W., & Noto, M.S. (2019). Rigorous mathematical thinking based on gender in the real analysis course. *Journal of Physics: Conference Series,* 11574 (042106). Retrieved from https://doi.org/10.1088/1742-6596/1157/4/042106.

Hidayah, I.N., Sa'dijah, C., Subanji, S., & Sudirman, S. (2020). Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*, 11(3), 347–362. Retrieved from https://doi.org/10.22342/JME.11.3.11869.347-362.

Hilmi, Y., & Usdiyana, D. (2020). An analysis of tenth grade students' mathematical creative thinking ability in vector. *Journal of Physics: Conference Series*, 1521, 032036. Retrieved from https://doi.org/10.1088/1742-6596/1521/3/032036.

Jablonka, E (2020). Critical thinking in mathematics education. *Encyclopedia of mathematics education.* Springer International Publishing, Cham, (pp. 159–163).

Jäder, J., Lithner, J., & Sidenvall, J. (2020). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathe-*

© Moiseienko Lidiia, & Shehda Liubov

matical Education in Science and Technology, 51(7), 1120–1136. Retrieved from https://doi.org/10.1080/0020739X.2019.1656826.

Jonsson, B., Mossegård, J., Lithner, J., & Karlsson Wirebring, L. (2022). Creative Mathematical Reasoning: Does Need for Cognition Matter? *Frontiers in Psychology*, 6207. Retrieved from https://doi.org/10.3389/fpsyg.2021.797807.

Moiseienko, L., & Shehda, L. (2023). Psychology of the Interaction of Understanding and Forecasting Processes in Creative Mathematical Thinking. *Zbirnyk naukovykh prats "Problemy suchasnoi psykholohii" – Collection of research papers "Problems of modern psychology"*, 59, 118–134. DOI (article): https://doi.org/10.32626/2227-6246.2023-59.118-134

Nurkaeti, N., Turmudi, K., Pratiwi, V., Aryanto, S., & Gumala, Y. (2020). Enhancement of mathematical creative thinking ability through open-ended approach based on metacognitive. *Journal of Physics: Conference Series*, 1521, Article 032030.

Selvy, Y., Ikhsan, M., Johar, R., & Saminan, S. (2020). Improving students' mathematical creative thinking and motivation through GeoGebra assisted problem based learning. *Journal of Physics: Conference Series*, 1460 (1), 1–8. DOI 10.1088/1742-6596/1460/1/012004

Syarifuddin, S., Nusantara, T., Qohar, A., & Muksar, M. (2020). Students' Thinking Processes Connecting Quantities in Solving Covariation Mathematical Problems in High School Students of Indonesia. *Participatory Educational Research*, 7(3), 59–78. Retrieved from https://doi.org/10.17275/per.20.35.7.3.

Suherman, S., Vidákovich, T., & Komarudin, K. (2021). STEM-E: Fostering mathematical creative thinking ability in the 21st century. *Journal of Physics: Conference Series*, 1882 (1), Article 012164

Tohir, M., Maswar, M., Atikurrahman, M., Saiful, S., & Pradita, D. A. R. (2020). Prospective Teachers' Expectations of Students' Mathematical Thinking Processes in Solving Problems. *European Journal of Educational Research*, 9(4), 1735–1748. Retrieved from https://doi.org/10.12973/EU-JER.9.4.1735.

Wong, T.Y. (2018). Is conditional reasoning related to mathematical problem solving? *Developmental Science*, 21(5), e12644. Retrieved from https://doi.org/10.1111/desc.12644.

Мойсеєнко Лідія, Шегда Любов. Задачі з вимогою «довести» як тренінг творчого математичного мислення.

За результатами аналізу досліджень творчого математичного мислення констатовано, що пошуковий математичний процес – це процес постановки та розв'язання математичної задачі. З'ясовано, що

© Moiseienko Lidiia, & Shehda Liubov

запитання активізують мисленнєві дії і тих, хто їх ставить, і тих, хто на них відповідає.

Підкреслено, що процес розв'язання творчої задачі визначається продуктивністю функціонування алгоритмічних та евристичних прийомів, які можна і потрібно формувати. Вони повинні сприяти продукуванню різноманітних ідей, їх якісному обстеженню і селекціонуванню.

***Мета дослідження:** проаналізувати потенційні можливості досвіду розв'язування задач із вимогою «довести» з активним застосуванням причинно-наслідкових запитань для активізації творчого математичного мислення. Для цього було використано **метод** аналізу пошукових дій студентів упродовж розв'язування творчих математичних задач.*

***Результати дослідження.** Встановлено, що застосування широкого спектру запитань у процесі розв'язування творчих математичних задач на доведення активізує мисленнєві складові: розуміння задачі, прогнозування ідей розв'язування, апробація мисленнєвих результатів, які стають більш взаємодоповнювальними.*

З'ясовано, що спеціальні запитання забезпечували актуалізацію наявних знань, досвіду та навичок, потік асоціацій, фантазії, що сприяє поглибленню розуміння смислу математичної задачі.

Констатовано, що прогнозувальні мисленнєві дії, які під впливом спеціальних запитань породжують якісну провідну ідею розв'язування задачі, сприяють наповненню її змістом через активне мисленнєве видозмінення структурних елементів, виникнення мисленнєвих новоутворень.

Доведено можливість формування навиків критичного ставлення до різного роду мисленнєвих знахідок за допомогою їх апробації спеціальними запитаннями. Це стає основою побудови чіткого логічного ланцюга міркувань від того, що дано, до того, що необхідно довести у задачах на доведення.

***Висновки.** Розв'язування математичних задач на доведення з активним застосуванням причинно-наслідкових запитань активізує творче математичне мислення.*

***Ключові слова:** творче математичне мислення, математичні задачі на доведення, мисленнєвий тренінг.*

Original manuscript received 27.12.2023
Revised manuscript accepted 05.04.2024

© Moiseienko Lidiia, & Shehda Liubov

DOI (article): https://doi.org/10.32626/2227-6246.2024-63.199-218