

Psychological Aspects of Activating the Approbation Process of Creative Mathematical Thinking

Психологічні аспекти активізації апробаційного процесу творчого математичного мислення

Moiseienko Lidiia

Dr. in Psychology, Professor,
Ivano-Frankivsk National Technical University of Oil and Gas,
Ivano-Frankivsk, Ukraine
orcid.org/0000-0001-9288-7355
Researcher ID: X-7506-2018
E-mail: Lmoiseyenko@i.ua

Мойсеєнко Лідія

Доктор психологічних наук, професор,
Івано-Франківський національний технічний університет нафти і газу,
м. Івано-Франківськ, Україна

Shehda Liubov

Ph.D. in Physics and Mathematics, Assistant Professor,
Ivano-Frankivsk National Technical University of Oil and Gas,
Ivano-Frankivsk, Ukraine
orcid.org/0000-0003-4721-7832
Researcher ID: Z-2516-2019
E-mail: shegda.luba@gmail.com

Шегда Любов

Кандидат фізико-математичних наук, доцент,
Івано-Франківський національний технічний університет нафти і газу,
м. Івано-Франківськ, Україна

Address for correspondence, e-mail: kpnu_lab_ps@ukr.net
Copyright: © Moiseienko Lidiia, & Shehda Liubov



The article is licensed under **CC BY-NC 4.0 International**
(<https://creativecommons.org/licenses/by-nc/4.0/>)

© Moiseienko Lidiia, & Shehda Liubov

DOI (article): <https://doi.org/10.32626/2227-6246.2024-64.182-200>

Авторський внесок: Мойсеєнко Лідія - 50%, Шехда Любов – 50%

The author's contribution: Moiseienko Lidiia - 50%,. Shehda Liubov - 50%

ABSTRACT

The purpose of the article is to find out the potential influence of cause and effect questions on the approbation process of creative mathematical thinking in order to activate it.

Methods of the research. According to the results of the analysis of research on creative mathematical thinking, it was established that the exploratory mathematical process is the process of setting and solving a mathematical problem. It was found that the approbation of thinking results accompanies the solution of the problem at all its stages. It is emphasized that the process of solving a creative problem is determined by the performance of algorithmic and heuristic methods, which can and should be formed. They should contribute to the production of various ideas, their qualitative examination and selection.

The results of the research. It was established that the psychological essence of the approbation actions of mathematical thinking is the comparative interaction of the knowledge obtained during solving the problem with the existing subjective knowledge system.

The use of a wide range of cause and effect questions when solving creative mathematical problems is established to activate the components of the search process: understanding the problem, predicting solution ideas, approbation of thinking results, turns them into complementary ones.

The approbation actions are found to have different content and psychological significance at different stages of solving mathematical problems.

It was found that special questions ensured the actualization of existing knowledge, experience, and skills, the flow of associations, imagination, which contributes to deepening the understanding of the meaning of a mathematical problem and actualizes approbation actions.

It has been established that there is a significant activation of approbation actions when forming a solution to a mathematical problem. Under the influence of special questions, the structural elements of the task are modified, which requires the study of their properties and functional capabilities.

It is proven that the acquired experience of accompanying the search process with questions turns the approbation of the solution project into a subjectively meaningful process aimed at comparing the parameters of the solution hypothesis with the parameters required in the mathematical task.

It was found out that in such conditions, knowledge is formed about a sufficient level of inspection quality and the skill to necessarily introduce it into the search process is developed.

Conclusions. *Activating a wider range of thinking actions, the acquired skill of forming numerous questions changes the depth and quality of approbation actions that take place throughout the entire solution of mathematical problems, forms the ability to analyze and control one's own thinking steps, contributes to the formation of subjective confidence in the correctness of the solution found.*

Key words: *creative mathematical thinking, process of approbation of mathematical results, cause and effect questions.*

Introduction

Psychological research of creative mathematical processes is the research of processes related to solving non-standard mathematical problems, the birth of mathematical discoveries, and the creation of new mathematical theories. Although this topic of research is not new and contains a lot of work (Kozłowski J, Chamberlin, & Mann, 2019), there is still much that is unclear, incomprehensible to psychologists in the process of creative mathematical thinking.

Despite the fact that mathematics reveals nature with the help of its abstractions: numbers, quantities, functions, geometric figures, etc., today it has become not only a tool for quantitative calculations, as it was at its inception, but also a research method. Scientists note a specific property of mathematical results: they can be used in many phenomena or processes, the physical nature of which is fundamentally different from each other (Firmasari, Sulaiman, Hartono, & Noto, 2019; Jablonka, 2020). That is why an important task of psychology is the formation of creative thinking of an individual (and mathematical creative thinking in particular), because it contributes to the ability to find a solution in those cases for which there are still no developed rules of action (Molyako, Gulko, & Vaganova, and others 2021). Therefore, the combination of general features of intellectual creativity with the specifics of mathematical acti-

vity in creative mathematical thinking ensures *the relevance* of the study of its various aspects.

First of all, it should be emphasized that researchers of the search thinking process interpret it as a problem-solving process (Hidayah, Sa'dijah, Subanji, & Sudirmans, 2020; Jäder, Lithner, & Sidenvall, 2020). They emphasize that solving problems itself requires the activation of various information, the ability to apply it to create new beyond the experience of solving similar problems (Tohir, Maswar, Atikurrahman, Saiful, & Pradita, 2020; Jonsson, Mossegård, Lithner, & Karlsson Wirebring, 2022). That is why our research on creative mathematical thinking is based on the analysis of the search for solutions to **creative mathematical problems**.

In general terms, the condition of a mathematical problem is a set of mathematical facts and objects that sometimes have no obvious connection with each other. And despite the fact that the set directly presented in the condition of the problem is small, it can (and sometimes must) be supplemented by certain known statements, mathematical results (axioms, definitions, theorems, etc.). The boundaries of such a set of facts are not clear, because it is not known in advance what knowledge will be needed to solve the problem (Syarifuddin, Nusantara, Qohar, Muksar, 2020). To solve the problem, it is necessary to build your set of facts into a certain structure. At the same time, the solution is the construction of such a structure that contains component problems, reliable mathematical statements and the desired result. (Hilmi, & Usdiyana, 2020; Jäder, Lithner, & Sidenvall, 2020).

Various thinking components function in such a search space. Scientists distinguish the process of understanding the problem, the process of forming a hypothesis for its solution, and the process of approbation of such a hypothesis (Molyako, 2021, Moiseyenko, & Shegda, 2021). This article will analyze *issues related to the testing of various mathematical results* that arise in the process of solving problems, including the formed hypothesis of solving a creative mathematical problem.

Analysis of issues related to the psychology of approbation processes in the literature are not often found (Jonsson, Mossegård, Lithner, & Karlsson Wirebring, 2022). Information on this problem can be found in scientific literature on other issues, where the approbation process is one of its aspects. It is often considered to be a less creative, even routine, stage in exploratory thinking. There is no special method for researching the psychological aspects of checking the found solution.

However, it is worth noting that approbation, as a thinking process, accompanies the entire solution of the problem (from the study of the condition of the problem to the construction of the solution) and includes the approbation of the thinking result (intermediate or final). That is, approbation is a means of convincing the reliability of a certain statement, assumption, etc. Since assumptions arise at different stages of solving mathematical problems, there is a need for approbation actions throughout the entire solving process: in the process of understanding the problem, in the process of forming its solution, in the process of checking the formed solution. At the same time, approbation of the obtained result completes the performance of any type of activity related to the creation of something new. For mathematical problems, checking the formed hypothesis of the solution is both the comparison of the hypothesis with the condition and requirement of the problem, and the study of the obtained result, which is related to the specifics of mathematical activity. Therefore, the psychological result of such actions is subjective confidence in the correctness of the mathematical result and subjective knowledge about the conditions under which the obtained mathematical result satisfies the problem completely.

Scientists believe that testing hypotheses is a complex, multi-step process. It involves, according to them, the selection of information by asking questions and the use of this information by formulating conclusions from the answers to the questions (Jonsson, Mossegård, Lithner, & Karlsson Wirebring, 2022). Some scientists, investigating the approbation of the

formed hypothesis of solving, consider the mental experiment as its essence, the basis of which is knowledge about ways to test the hypothesis (Syarifuddin, Nusantara, Qohar, & Muksar, 2020).

However, thinking hypotheses are certain assumptions with varying degrees of justification that arise at different stages of the search process. They must correspond to the actual material of the problem, on the basis of which and for the explanation of which they are proposed; comply with the laws established in one or another science (in this case, mathematical). The level of such correspondence can be detected by hypothesis testing. Therefore, in our opinion, it is worth analyzing the approbation actions that took place throughout the entire search process aimed at solving the problem.

On the other hand, the process of solving a creative problem is determined by the performance of algorithmic and heuristic techniques. Algorithms are a system of well-known mental operations that provides the solution of a specific, subject's known, class of problems. However, such techniques can have a negative effect (for example, they are used where it is not rational). The task of techniques that activate creative thinking is to "neutralize" the negative impact of algorithmic actions known to the subject, to transform them into auxiliary constructions that can be used both in their finished form and after certain adaptation.

The main purpose of such tools is to promote the production of various ideas, their qualitative examination and selection, to promote in-depth analysis of new situations. We paid attention to psychological studies in which the activating significance of questions formed in the process of solving problems was established (Borodina, 2020). Researchers confirm that questions activate the thinking actions of both those who ask them and those who answer them. However, despite this, they state that the ability to ask questions is a difficult skill that should be taught to pupils, students, etc. Having evidence of the influence of a series of special questions on the thinking process, we became interested in their influence on the approbation process when

© Moiseienko Lidiia, & Shehda Liubov

DOI (article): <https://doi.org/10.32626/2227-6246.2024-64.182-200>

solving mathematical problems. In this article, we try *to find out the psychological impact of causal questions on the approbation process* of solving creative mathematical problems.

The purpose of the article is to find out the potential influence of cause and effect questions on the approbation process of creative mathematical thinking in order to activate it.

The task of the article is to analyze the process of approbation as an end-to-end process of solving creative mathematical problems in conditions of active application of cause and effect questions.

Research methods and techniques

The research method is the analysis of students' search actions during their solving of creative mathematical problems, focusing on the process of approbation. That is, we developed 40 mathematical problems and conducted an experimental study of creative mathematical thinking of students of the Ivano-Frankivsk National Technical University of Oil and Gas. 50 students took part in the experiment: 25 students in the experimental group and 25 students in the control group. Each student solved 8 mathematical problems of different classes. In the experimental group, the study was conducted in three stages. Students performed the first 10 tasks (first stage) collectively, under the guidance of the experimenter. Such resolution was accompanied by numerous cause-and-effect questions formulated by the experimenter and research participants. The remaining 10 were performed by each student individually (second stage). The student was given the opportunity to work independently, in dialogue with the experimenter. There were no direct instructions on how to solve it, but the solution was accompanied by numerous questions. Such questions were formulated by the experimenter and students.

At the third stage, students of the experimental and control groups solved 5 control problems without any restrictions or requirements. Students' notes, drawings, replicas and questions were analyzed.

Results and their discussion

We will consider the psychological essence of approbation actions of mathematical thinking as the *comparative interaction* of the acquired knowledge with the existing subjective knowledge system. At the same time, the basis of approbation actions is the analysis of the applied logical steps and their consequences for comparing the obtained result with the condition and requirement of the task.

Approbation actions in the mathematical thinking of students are observed even at the stage of studying the condition, that is, in the process of understanding a creative mathematical problem. At this stage, the essence of structural elements, their properties, and relationships were clarified. This is done through putting forward and testing hypotheses about them, i.e., approbative actions become a tool for understanding the problem. Since understanding is often based on comparison, and the standard of comparison is chosen from many possible ones, the comparison operation turns into the process of approbation of the selected element. The quality of such an approbation determines to a certain extent the quality of the understanding of the task, and the quality of the approbation itself is determined by the state of understanding.

When studying the condition of the task under the guidance of the experimenter, the solution process was accompanied by cause-and-effect questions that the experimenter formulated himself and encouraged the students to do that. For this, a set of cliché questions was developed and offered to students, which they could use in collective and individual work. In terms of content, the following questions related to cause-and-effect relationships: "How does ... affect ...?", "How can ... be used for ...?", "What is meant by ...? etc.

When working on a task, students' thinking is first directed to the study of its structural elements (the beginning of the process of understanding the task). Questions "How are ... and ... similar?", "What is the difference between ... and ...?" are

© Moiseienko Lidiia, & Shehda Liubov

DOI (article): <https://doi.org/10.32626/2227-6246.2024-64.182-200>

deepen that analysis, contributing to the discovery of various properties. This is how the need to more actively use mental approbation actions is formed.

Students combined the components of problems with certain of their properties and checked the usefulness of such a combination for finding a solution. At the same time, the questions “Explain why ...?”, “Explain how ...?”, “Why is it important ...?”, “What argument can be given against ...?” contributing to a deeper understanding of the task, activate approbation as a component of the thinking process. On the other hand, “interrogative training”, contributing to the improvement of orientation in the context of a mathematical problem, activates a wider range of thinking actions, including approbation actions.

The solution of the problem is based on the subjective selection of certain elements from those that exist and certain actions on them. If this process is accompanied by the questions «What do we already know about ...?», «What does ... look like?», then such a set can be expanded at the expense of students' knowledge, that is, include structural components that are not represented by the condition of the problem, but are known in mathematics, and their properties. Of course, such an expansion requires approbation of the associations that have arisen: approbation of the structural elements of the problem, which are clearly not given by its condition, and the identification of relationships between them in the fullest possible scope. Such a need contributes to the activation of mental approbation actions. That is, «interrogative training», increasing the perception of the context of a new task, actualizes approbation actions.

In future, the thinking steps are directed to the search for a solution, more precisely, to the formation of hypotheses regarding the solution. The task situation changes, each next step has to be performed in conditions that differ from the previous ones. The importance of structural elements and their properties is being reassessed. The task situation (the situation obtained after all transformations of the source information) is characterized

by the extent and specific form of participation of structural elements. This can be actively facilitated by questions such as «How can you apply ... in a specific task?», «Which ... is better and why?», «What will happen if ...?». The search for answers to such questions requires activation of approbation actions that help clarify hidden connections between structural elements and their consequences, promote activation of unconscious thought acts. And although structural elements can still have not only explicit, but also implicit, hidden meanings, students still managed to form certain hypotheses regarding the solution.

That is, the solution to the problem, which is hypothetical in origin, needs numerous approbation actions before it turns into a subjectively meaningful thought product, which will be declared a solution. It is worth noting that the formal, low-quality approbation of intermediate links of thinking leads to the impossibility of forming a full solution hypothesis from an intermediate idea. Accompanying the process of solving with questions (for example, «What will happen if ...?», «Which ... is better and why?») helps to analyze and control one's own thinking steps through a detailed examination of the consequences to which these steps lead. This proves the importance of approbation actions in the formation and filling with content of the primary concept of the solution, and hence the need to activate them throughout the entire process of solving a creative mathematical problem.

The execution of any type of activity related to the creation of something new is completed by approbation of the obtained result. This fully applies to the process of solving creative mathematical problems. Having completed the formation of the solution hypothesis, the subject, as a rule, tests it: checks for compliance to the condition, finds out to what extent it satisfies the requirements of the task. That is, it requires subjective conviction in conformity with the condition and requirement of the task. Such subjective conviction occurs as a result of approbation actions aimed at comparing the parameters of the solution hypothesis with the parameters required in the mathematical

task. Now the approbation of the results turns into a subjectively significant process, the effectiveness of which can be strengthened by questions like «What's the point of ...?», «What are the strengths and weaknesses of ...?», «Do you agree with the statement that ...?».

Finding out the consequences and nature of the influence of the creative training organized by us on the approbation process, we analyzed the process of independent solving of ten problems by the students of the experimental group. The attention of the experimenters was directed to the content, quality and place of the questions formulated by the students.

It can be stated that the process of solving mathematical problems of the students participating in the training was based on the facts that came to the fore after the students reformulated the conditions of the problem in «their» language. The questions that were formulated by the experimenter or the solver himself cause the activity of approbation actions. As a result, already at the early stages of solving, students actively research and select the structural elements of the problem, include new data that lead to a deeper understanding of the problem. A mental search turns into a purposeful prediction of a solution. Intermediate results were examined in more detail, and therefore rational thinking findings were much less likely to be rejected. The stage of understanding the solution, its justification becomes more significant in the structure of thinking procedures. The need to comprehensively test the found solution becomes a personal property of the thinking process of any student.

Approbation of the task with various questions helps to overcome inertia and stereotyping to a great extent. Students stop relying on close analogues, their imagination and mathematical forecasting become more active. Therefore, bolder hypotheses appear, hypotheses that involve knowledge from various branches of mathematics, often even knowledge from other branches of science.

In order to find out the nature of the influence of the creative training organized by us on the search mathematical pro-

cess, we compared the process of solving five control problems by students who participated in the training and students of the control group. In general, we observed that when solving the control problems after the training, a much larger part of the students succeeds. At the same time, the number of incorrect solutions significantly decreases

It should be noted a positive change in the quality of the search process among students who have undergone training. The acquired experience contributes to the formulation of deeper questions about the content of the task and the search for answers to them. An example can be the task: «There are 40 cars and motorcycles in the parking lot». They have a total of 40 steering wheels and 100 wheels. How many cars, how many motorcycles in the parking lot?». This task caused the students of the experimental group questions (What is the shape of a car steering wheel? Does the number of steering wheels of a car relate to the number of wheels?), which we did not record in the students of the control group.

The process of understanding among the subjects of the experimental group was based on the correct allocation of the meaning of the task, on establishing the correct ratio of individual data of the task, on the qualitative selection of subjective standards. Students of this group operated on their knowledge at a higher level, in contrast to the subjects of the control group, who significantly more often demonstrated ineffective, meaningless use of their mathematical knowledge, which led to a significantly higher number of errors.

For example, when solving the following problem: «Prove that the equation $3x^2 - 4y^2 = 13$ does not have integer solutions.», the students of the experimental group came to the conclusion that the expressions $3x^2$, $4y^2$ cannot be both even or odd at the same time, because their sum – odd number 13. In contrast, the students of the control group were empirically convinced that the square of an odd number $3^2 = 9$, $7^2 = 49$ is an odd number: and

the square of an even number: $2^2 = 4$, $4^2 = 16$ is an even number and did not draw a proper conclusion for further considerations.

Approbation actions contribute to the formation content of the hypothesis of the solution, because with the help of approbation, the expediency of using a certain structural element of the problem is clarified on the basis of the identified properties and actualized specific mathematical facts, newly formed structural elements. When forming a solution hypothesis, the subjects of the experimental group demonstrated a greater variability of hypotheses, a much deeper content of these hypotheses. For example, in relation to the previous problem, they rearrange the elements in the equation $3(x_0^2 - 1) - 4y_0^2 = 10$ and put forward a hypothesis about the impossibility of dividing the left expression by 10. For this, the constituent elements are examined for parity: $3(x_0^2 - 1)$ - an even number, $x_0^2 - 1$ - an even number, x_0^2 - an odd number, x_0 - an odd number; write an odd number in the general form: $x_0 = 2k - 1$, (k is an integer): get an equation in the form $4(3(k^2 - k) - y_0^2) = 10$, which can have whole roots only when the left side of the equation is divisible by 10, which is not possible. This gives grounds for asserting: the equation $3x^2 - 4y^2 = 13$ does not have integer roots. Instead, students in the control group continued to explore the numerical values of x and y .

Approbation of the obtained result completes the implementation of any type of activity related to the creation of something new. The new system of knowledge, which was formed as a result of the formation of the solution hypothesis, becomes an integral part of the psychological mechanism, which directs the subject's further activity to the approbation of the quality of the thinking results of the search actions of the previous stages. The main feature of the research activity of the students of the experimental group was the timely detection (even the desire to detect) thinking traps and timely correction of the search actions in order to avoid unfounded conclusions. Instead, participants in the control group were often led to false conclusions due to inertness in the references.

© Moiseienko Lidiia, & Shehda Liubov

DOI (article): <https://doi.org/10.32626/2227-6246.2024-64.182-200>

On the other hand, at the stage of checking mathematical results, students achieve subjective confidence in the correctness of the solution. Such a subjective conviction occurs as a result of comparing the parameters of the solution hypothesis with the parameters required in the mathematical problem. Therefore, we observed an increase in the frequency and significance of approbation actions among students of the experimental group when solving control problems. At the same time, the quality of approbation actions has changed even more: the students of the experimental group immediately reject irrelevant references and conclusions that have arisen from them.

However, it is important that such confidence does not come prematurely and does not become distorted, turning into self-confidence, which we often observed in the control group. In contrast, students from the experimental group, having the experience of testing the obtained mathematical result with cause and effect questions, were more considerate of the results of their mathematical activity.

The formed ability to work on new material became a precautionary measure against making wrong decisions.

The proportion of thinking operations aimed at checking and researching the obtained mathematical results significantly increased among the students who completed the training. Knowledge about a sufficient level of inspection quality is formed and the skill to necessarily implement it in the search process is developed. Such a skill becomes an important component of students' mathematical thinking. This can be illustrated by our example of a proof problem. Some students, even after finding the solution, continued to investigate the obtained result: they considered cases where one root is whole and the other is fractional. Most often, students managed to find out that such a situation gives a positive result and is a partial case of the general result.

Conclusions

The implementation of training education contributed to positive changes in students' creative mathematical thinking: the perception of the context of a new task deepens with the help of the formed skill to comprehensively examine the structural components of tasks; the effectiveness of search actions increases, the number of errors decreases due to the formed subjective need to justify the decisions made during solving mathematical problems; approbation actions are becoming more relevant.

Activating a wider range of thinking actions, the acquired skill of forming numerous questions changes the depth and quality of approbation actions that take place throughout the entire solution of mathematical problems (when studying the condition of the problem, when searching for a solution, when checking of the found solution), forms the ability to analyze and control one's own thinking steps.

The content of approbation actions depends on the stages of solving the problem and can be both a formal routine stage of the search process and a creative process that continues or supplements the search process, turning into a component of the main task, which can change the understanding of the problem, change the already formed hypothesis of the solution connection, form a scale of its evaluation.

The psychological essence of the approbation actions of mathematical thinking is the comparative interaction of the acquired knowledge with the existing subjective knowledge system. The psychological result of the process of approbation of any hypothesis is the moment of subjective confidence in its correctness (or incorrectness).

The use of inquiry training contributed to changes in the basic components of the search process: the inadequate use of existing knowledge, skills, and abilities, which were the causes of identified errors in search mathematical thinking, is eliminated; the use of mental operations became more optimal and effective;

the subjective confidence of students in their own intellectual capabilities increased.

The perspective of further research of this problem is to study the impact of the experience of solving problems in the conditions of «questioning» training on the personal aspect of students – on the search activity of students with different thinking styles.

Literature

- Функціонування творчого мислення в інформаційно-віртуальному просторі суб'єкта: монографія / В.О. Моляко, Ю.А. Гулько, Н.А. Ваганова [та ін.] ; за ред. В.О. Моляко. Київ ; Львів : Видавець Вікторія Кундельська, 2021. 194с. ISBN 978-617-8116-05-7.
- Firmasari S., Sulaiman H., Hartono W., Noto M.S. Rigorous mathematical thinking based on gender in the real analysis course. *Journal of Physics: Conference Series*. 2019. P. 11574 (042106). URL: <https://doi.org/10.1088/1742-6596/1157/4/042106>.
- Hidayah I.N., Sadijah C., Subanji S., Sudirman, S. Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*. 2020. Vol. 11, No. 3. P. 347–362. URL: <https://doi.org/10.22342/JME.11.3.11869.347-362>.
- Hilmi Y., Usdiyana D. An analysis of tenth grade students' mathematical creative thinking ability in vector. *Journal of Physics: Conference Series*. 2020. P. 1521, 032036. URL: <https://doi.org/10.1088/1742-6596/1521/3/032036>.
- Ibrahim I., Widodo S.A. Advocacy approach with open-ended problems to mathematical creative thinking ability. *Infinity Journal*. 2020. Vol. 9, No. 1. P. 93–102 DOI:10.22460/infinity.v9i1.
- Jablonka E. Critical thinking in mathematics education. *Encyclopedia of mathematics education*. 2020. Springer International Publishing, Cham, pp. 159–163.
- Jäder J., Lithner J., Sidenvall, J. Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*. 2020. Vol. 51, No. 7. P. 1120–1136.
- Jonsson B., Mossegård J., Lithner J., Karlsson Wirebring L. Creative Mathematical Reasoning: Does Need for Cognition Matter? *Frontiers in Psychology*. 2022. P. 6207. URL: <https://doi.org/10.3389/fpsyg.2021.797807>.
- Kozłowski J.S., Chamberlin S.A., Mann E. Factors that influence mathematical creativity. *The Mathematics Enthusiast*. 2019. Vol.16, No. 1. P. 505–540. DOI 10.54870/1551-3440.1471.

- Moiseienko Lidiia, Shehda Liubov. Psychology of the interaction of understanding and forecasting processes in creative mathematical thinking. *Збірник наукових праць «Проблеми сучасної психології»*. 2023. Вип. 54. P. 118–135. URL: <https://doi.org/10.32626/2227-6246.2023-59.9-28>.
- Syarifuddin S., Nusantara T., Qohar A., Muksar M. Students' Thinking Processes Connecting Quantities in Solving Covariation Mathematical Problems in High School Students of Indonesia. *Participatory Educational Research*. 2020. Vol. 7, No. 3. P. 59–78. URL: <https://doi.org/10.17275/per.20.35.7.3>.
- Tohir M., Maswar M., Atikurrahman M., Saiful S., Pradita D.A.R. Prospective Teachers' Expectations of Students' Mathematical Thinking Processes in Solving Problems. *European Journal of Educational Research*. 2020. Vol. 9, No. 4. P. 1735–1748. URL: <https://doi.org/10.12973/EU-JER.9.4.1735>.

References

- Firmasari, S., Sulaiman, H., Hartono, W., & Noto, M. S. (2019). Rigorous mathematical thinking based on gender in the real analysis course. *Journal of Physics: Conference Series*, 11574 (042106). Retrieved from <https://doi.org/10.1088/1742-6596/1157/4/042106>.
- Hidayah, I.N., Sadijah, C., Subanji, S., & Sudirman, S. (2020). Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*, 11(3), 347–362. Retrieved from <https://doi.org/10.22342/JME.11.3.11869.347-362>.
- Hilmi, Y., & Usdiyana, D. (2020). An analysis of tenth grade students' mathematical creative thinking ability in vector. *Journal of Physics: Conference Series*, 1521, 032036. Retrieved from <https://doi.org/10.1088/1742-6596/1521/3/032036>.
- Ibrahim, I. & Widodo, S.A. (2020). Advocacy approach with open-ended problems to mathematical creative thinking ability. *Infinity Journal*, 9 (1), 93–102 DOI:10.22460/infinity.v9i1.
- Jablonka, E. (2020). Critical thinking in mathematics education. *Encyclopedia of mathematics education*. Springer International Publishing, Cham. (pp. 159–163).
- Jäder, J., Lithner, J., & Sidenvall, J. (2020). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*, 51(7), 1120–1136. Retrieved from <https://doi.org/10.1080/0020739X.2019.1656826>.
- Jonsson, B., Mossegård, J., Lithner, J., & Karlsson Wirebring, L. (2022). Creative Mathematical Reasoning: Does Need for Cognition Mat-

- ter? *Frontiers in Psychology*, 6207. Retrieved from <https://doi.org/10.3389/fpsyg.2021.797807>.
- Kozłowski, J.S., Chamberlin, S.A., & Mann E. (2019). Factors that influence mathematical creativity. *The Mathematics Enthusiast*, 16 (1), 505–540. DOI 10.54870/1551-3440.1471.
- Moiseienko, L., & Shehda, L. (2023). Psychology of the interaction of understanding and forecasting processes in creative mathematical thinking. *Zbirnyk naukovykh prats «Problemy suchasnoi psykholohii» – Collection of research papers “Problems of modern Psychology”*, 54, 118–135. Retrieved from doi: <https://doi.org/10.32626/2227-6246.2023-59.9-28>.
- Moliako, V.O., Hulko, Yu.A., Vahanova, N.A. et.al. (2021). *Funktsionuvannia tvorchoho myslennia v informatsiino-virtualnomu prostori subiekta: monohrafiia [Functioning of creative thinking in the informational and virtual space of the subject]* / V.O. Moliako (Ed.). Kyiv ; Lviv : Vydavets Viktoriia Kundelska. ISBN 978-617-8116-05-7 [in Ukrainian].
- Syarifuddin, S., Nusantara, T., Qohar, A., & Muksar, M. (2020). Students' Thinking Processes Connecting Quantities in Solving Covariation Mathematical Problems in High School Students of Indonesia. *Participatory Educational Research*, 7(3), 59–78. Retrieved from <https://doi.org/10.17275/per.20.35.7.3>.
- Tohir, M., Maswar, M., Atikurrahman, M., Saiful, S., & Pradita, D.A.R. (2020). Prospective Teachers' Expectations of Students' Mathematical Thinking Processes in Solving Problems. *European Journal of Educational Research*, 9(4), 1735–1748. Retrieved from <https://doi.org/10.12973/EU-JER.9.4.1735>.

Мойсеєнко Лідія, Шегда Любов. Психологічні аспекти активізації апробаційного процесу творчого математичного мислення.

Мета статті полягає у з'ясуванні потенційних можливосту впливу причинно-наслідкових запитань на апробаційний процес творчого математичного мислення з метою його активізації.

Методи дослідження. За результатами аналізу досліджень творчого математичного мислення констатовано, що пошуковий математичний процес – це процес постановки та розв'язання математичної задачі. З'ясовано, що апробація мисленневих результатів як складовий мисленневий процес, супроводжує розв'язування задачі на всіх його етапах.

З'ясовано, що спеціальні запитання забезпечували актуалізацію наявних знань, досвіду, та навичок, потік асоціацій, фантазії, що сприяє

поглибленню розуміння смислу математичної задачі, прогнозування розв'язку й актуалізує апробаційні дії.

Результати дослідження. Підкреслено, що процес розв'язання творчої задачі визначається продуктивністю функціонування алгоритмічних та евристичних прийомів, які можна і потрібно формувати.

Встановлено, що застосування широкого спектру причинно-наслідкових запитань при розв'язуванні творчих математичних задач активізує складові пошукового процесу: розуміння задачі, прогнозування ідей розв'язування, апробацію мисленнєвих результатів, перетворює їх у взаємодоповняльні.

З'ясовано, що спеціальні запитання забезпечували актуалізацію наявних знань, досвіду та навичок, потік асоціацій, фантазії, що сприяє поглибленню розуміння смислу математичної задачі, прогнозування розв'язку й актуалізує апробаційні дії.

Доведено, що набутий досвід супроводжувати пошуковий процес запитаннями, перетворює апробацію проєкту розв'язку у суб'єктивно значущий процес.

Висновок. Активізуючи ширший спектр мисленнєвих дій, здобута навичка формувати численні запитання змінює глибину і якість апробаційних дій, що мають місце впродовж всього розв'язування математичних задач, формує вміння аналізувати і контролювати власні мисленнєві кроки, сприяє формуванню суб'єктивної впевненості у правильності знайденого розв'язку.

Ключові слова: творче математичне мислення, процес апробації математичних результатів, причинно-наслідкові запитання.

Original manuscript received 21.06.2024

Revised manuscript accepted 07.11.2024