SOUND AND DISSIPATIVE RELAXATION IN SUPERFLUID ^{3}He-^{4}He MIXTURES

K. Nemchenko, S. Rogova

Karazin Kharkiv National University, Svobody Sq. 4, Kharkiv, 61077
e-mail: svetlanochka85@mail.ru

A system of hydrodynamic equations is considered to describe temperature and concentration relaxation in superfluid ^{3}He-^{4}He mixtures. Temperature and concentration relaxation is found to be determined both by the second sound mechanism and by the dissipative diffusive process. An analytical solution of this system is found. The obtained solution is compared with the results of experiment, where the kinetics of temperature change and concentration change in the superfluid ^{3}He-^{4}He mixture, when the thermal flow was switched on or switched off, was measured.

Introduction

Relaxation and concentration processes in superfluid ^{3}He-^{4}He mixtures are very unusual [1]. When heat flux presents, simultaneous appearing of temperature and concentration gradients in this system means that thermal conductivity, thermal diffusion and mass diffusion are interconnected. Theoretical investigations of kinetic and relaxation processes in superfluid mixtures [2-3] showed that in this system temperature and concentration relaxation is found to be determined both by the second sound mechanism and by the dissipative diffusive process.

A similar situation when heat is carried not only by thermal conductivity mechanism exists in all physical media. In order to solve the problem of such heat distribution, one should start with the complete system of hydrodynamic equations.

The System of Hydrodynamic Equations and Its Solution

We solve the system of hydrodynamic equations of the form:

$$\begin{cases} \dfrac{\partial}{\partial t}\rho(x,\mathrm{t}) + \rho_0 \dfrac{\partial}{\partial x}u(x,t) = 0, \\ \dfrac{\partial}{\partial t}u(x,t) + \dfrac{1}{\rho_0}\left(\dfrac{\partial p}{\partial T}\right)_\rho \dfrac{\partial}{\partial x}T(x,t) + \dfrac{1}{\rho_0}\left(\dfrac{\partial p}{\partial \rho}\right)_T \dfrac{\partial}{\partial x}\rho(x,t) = 0, \\ \dfrac{\partial}{\partial t}T(x,t) + \dfrac{T_0}{C_V}\left(\dfrac{\partial p}{\partial T}\right)_\rho \dfrac{\partial}{\partial x}u(x,t) = \chi \dfrac{\partial^2}{\partial x^2}T(x,t), \end{cases} \tag{1}$$

with the initial conditions:

$$\begin{cases} \rho(x,\mathrm{t}=0) = 0, \\ u(x,t=0) = 0, \\ T(x,t=0) = \delta(x), \end{cases} \tag{2}$$

where *ρ(x,t)* is the density deviation from the equilibrium value ρ_0, *u(x,t)* – is velocity, *T(x,t)* – temperature deviation from the equilibrium value T_0, *p* is pressure, $\chi = \kappa / C_V$ – temperature conductivity, κ is the thermal conductivity, C_V is the density of thermal capacity at constant volume.

We investigate the solution of these equations in the infinite area assuming that the deviations of macroscopic variables tend to zero on infinity. Then we can define Fourier transform for $\rho(x,t)$, $u(x,t)$, $T(x,t)$, and for Fourier transformations of these variables one then obtains the system of equations:

$$\begin{cases} \frac{\partial}{\partial t}\rho_q(t) = -i\rho q u_q, \\ \frac{\partial}{\partial t}u_q(t) = -i\alpha_1 q\rho_q - i\beta_1 qT_q, \\ \frac{\partial}{\partial t}T_q(t) = -i\mu_1 qu_q - \chi q^2 T_q, \end{cases} \quad (3)$$

where

$$\alpha_1 = \frac{1}{\rho_0}\left(\frac{\partial p}{\partial \rho}\right)_T, \ \beta_1 = \frac{1}{\rho_0}\left(\frac{\partial p}{\partial T}\right)_\rho, \quad \mu_1 = \frac{T_0}{C_V}\left(\frac{\partial p}{\partial T}\right)_\rho . \quad (4)$$

Let us consider three-dimensional space of vectors with coordinates corresponding to the hydrodynamic variables:

$$\mathbf{a}(t) = \begin{pmatrix} \rho_q(t) \\ u_q(t) \\ T_q(t) \end{pmatrix}. \quad (5)$$

Then the system (3) can be rewritten as the differential equation:

$$\frac{\partial}{\partial t}\mathbf{a}(t) = M_q \mathbf{a}(t) \quad (6)$$

with the initial condition:

$$\mathbf{a}(t=0) = \begin{pmatrix} \rho_q(0) \\ u_q(0) \\ T_q(0) \end{pmatrix} \equiv \mathbf{a}_0 . \quad (7)$$

Matrix of the system (6) has the form:

$$M_q = \begin{pmatrix} 0 & -iq\rho & 0 \\ -i\alpha_1 q & 0 & -i\beta_1 q \\ 0 & -i\mu_1 q & -\chi q^2 \end{pmatrix}. \quad (8)$$

To find eigenvalues λ_i $(i=1, 2, 3)$ of this matrix we note that hydrodynamic description can be used only for phenomena slowly changing in space. This means that in Fourier transform only terms with small parameter q are significant. Hence, we can consider eigenvalues only in case of $q \to 0$. In this case values λ_i can be written as follows: $\lambda_i = a_i q + b_i q^2 + O(q^3)$. One then obtains three eigenvalues of the form:

$$\begin{aligned} \lambda_1 &= -ic_S q - \Gamma_S q^2, \\ \lambda_2 &= -ic_S q - \Gamma_S q^2, \\ \lambda_3 &= -\chi q^2, \end{aligned} \quad (9)$$

where $c_s = \left(\frac{C_p}{C_V}\left(\frac{\partial p}{\partial \rho}\right)_T\right)^{\frac{1}{2}}$ is the sound velocity, $\Gamma_s = \frac{\kappa}{2}\left(\frac{1}{C_V} - \frac{1}{C_p}\right)$ is damping of sound.

Since matrix M_q is non-Hermitian both left and right eigenvectors exist, and they can be derived from equalities: $M_q \boldsymbol{X}_i = \lambda_i \boldsymbol{X}_i$, $\boldsymbol{Y}_i M_q = \lambda_i \boldsymbol{Y}_i$, $(i=1,2,3)$. And when $q \to 0$ are given by:

$$X_{1,2} = \left(\frac{C_v}{2C_p}\right)^{1/2} \begin{pmatrix} \rho \\ \pm c_s \\ \frac{T\left(\partial p/\partial T\right)_\rho}{\rho C_v} \end{pmatrix},$$
$$X_3 = \left(\frac{C_p - C_v}{C_p}\right)^{1/2} \begin{pmatrix} \rho \\ 0 \\ -\frac{\rho\left(\partial p/\partial \rho\right)_T}{\left(\partial p/\partial T\right)_\rho} \end{pmatrix}, \quad (10)$$

$$Y_{1,2}=\left(\pm\frac{c_s}{(\partial p/\partial\rho)_T}\ ,\ 0\ ,\ \frac{(\partial p/\partial T)_\rho}{(\partial p/\partial\rho)_T}\right),$$
$$Y_3=\left(\frac{1}{\rho}\ ,\ 0\ ,\ \frac{\rho C_v}{T(\partial p/\partial T)_\rho}\right). \qquad (11)$$

The found eigenvalues and respective eigenvectors of matrix M_q allow the system (3) to be solved.

The Problem of Temperature Relaxation in the Infinite Media

We solve the system of hydrodynamic equations of the form

We develop ***a*** upon eigenvectors: $\boldsymbol{a}=\alpha_1\boldsymbol{X_1}+\alpha_2\boldsymbol{X_2}+\alpha_3\boldsymbol{X_3}$, where $\alpha_i=(\boldsymbol{Y_i}, \boldsymbol{a})$, $i=1,2,3$. Then the solution of system (3) has the form:

$$\begin{aligned}
\rho_q(t) &= \alpha_1 e^{-\lambda_1 t}x_1^{(1)} + \alpha_2 e^{-\lambda_2 t}x_1^{(2)} + \alpha_1 e^{-\lambda_3 t}x_1^{(3)},\\
u_q(t) &= \alpha_1 e^{-\lambda_1 t}x_2^{(1)} + \alpha_2 e^{-\lambda_2 t}x_2^{(2)} + \alpha_1 e^{-\lambda_3 t}x_2^{(3)},\\
T_q(t) &= \alpha_1 e^{-\lambda_1 t}x_3^{(1)} + \alpha_2 e^{-\lambda_2 t}x_3^{(2)} + \alpha_1 e^{-\lambda_3 t}x_3^{(3)}.
\end{aligned} \qquad (12)$$

After inverse Fourier transform we obtain the solution for the instant sources of the form (12):

$$\begin{aligned}
T(x,t) &= \frac{C_V}{C_p}G_\chi(x,t) + \frac{1}{2}\frac{C_p - C_V}{C_p}\left(G_S^{(+)}(x,t) + G_S^{(-)}(x,t)\right),\\
\rho(x,t) &= \rho_0\frac{C_p - C_V}{C_p}\frac{1}{\mu_1}\left[\frac{1}{2}\left(G_S^{(+)}(x,t) + G_S^{(-)}(x,t)\right) - G_\chi(x,t)\right],\\
u(x,t) &= \frac{C_p - C_V}{C_p}\frac{c_s}{\mu_1}\left(G_S^{(+)}(x,t) - G_S^{(-)}(x,t)\right).
\end{aligned} \qquad (13)$$

Here Green's functions are introduced, that correspond to the hydrodynamic modes (10) and (11). Function

$$G_\chi(x,t) = \frac{1}{\sqrt{4\pi\chi t}}\exp\left(-\frac{x^2}{4\chi t}\right), \qquad (14)$$

describes relaxation caused by dissipative mechanism of thermal conductivity, and functions

$$G_S^{(\pm)}(x,t) = \frac{1}{\sqrt{4\pi\Gamma_s t}}\exp\left(-\frac{(x \pm c_s t)^2}{4\Gamma_s t}\right) \qquad (15)$$

describe relaxation caused by sound propagation. Examples of calculations on formula (13) are given in Fig.1.

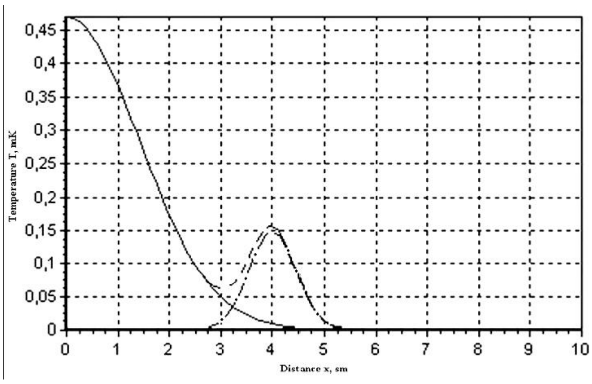

Fig.1.Temperature distribution in infinite one-dimensional area from the point heat source.Dotted line is temperature dependence on distance. Solid line presents the dissipative mode. Stroke-dotted line presents the sound mode.

The Problem of Switching on a Heat Flux on a Segment

We solve system (1) on a segment $0 \le x \le l$, with $0 \le t \le \infty$ and with the boundary conditions:

$$T'_x(x=0,t) = -q_0 / \kappa \,,$$

$$T(x=l,t) = 0. \qquad (16)$$

$$u(x=l,t) = u(x=0,t) = 0\,,$$

where q_0 is the heat flux that switches at $t=0$.

Using method of source images we obtain expressions that express solution on a segment via solutions (13):

$$T_{omp}(x,t) = \int_0^t 2q_0 \sum_{k=0}^{\infty} \left((-1)^k \left(T(2kl + x,t) - T(2(k+1)l + x,t) \right) \right) dt$$

$$u_{omp}(x,t) = \int_0^t 2q_0 \sum_{k=0}^{\infty} \left((-1)^k \left(u(2kl + x,t) - u(2(k+1)l + x,t) \right) \right) dt \qquad (17)$$

$$\hat\rho(x,t) = \int_0^t 2q_0 \sum_{k=0}^{\infty} \left((-1)^k \left(\hat\rho(2kl + x,t) - \hat\rho(2(k+1)l + x,t) \right) \right) dt$$

$$\rho_{omp}(x,t) = \hat\rho(x,t) - \frac{1}{l}\int_0^l \hat\rho(x,t)\,dx$$

The examples of calculations according to Eq. (16) are given in Figs.2, 3.

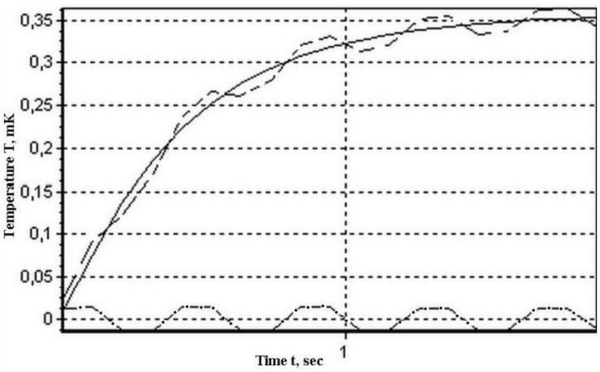

Fig.2. Temperature dependence on time(in case of the large sound mode and weak sound damping) in the middle of a segment at one end of which is constant heat source and constant temperature is supported at the other end. Dotted line presents temperature dependence on distance. Solid line presents the dissipative mode. Stroke-dotted line presents the sound mode.

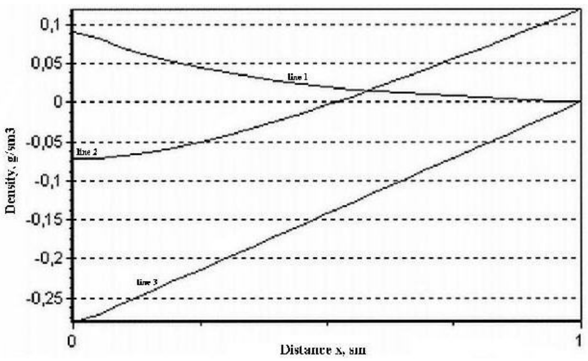

Fig.3. Density distribution at a segment at one end of which is constant heat source and constant temperature is supported at the other end. Line 1 presents the sound mode. Line 2 presents temperature dependence on distance. Line 3 presents the dissipative mode.

Description of the Experiment

Quantum superfluid ^{3}He-^{4}He mixtures have a great number of unique features. One of them is the fact that in these mixtures the sound mode and the dissipative mode are connected with the factor that has the meaning of thermal extension for usual substances.

$$\frac{C_p - C_V}{C_p} \to \frac{u_{2\varepsilon}^2}{u_2^2}. \qquad (18)$$

Here u_2^2 is the second sound velocity, $u_{2\varepsilon}^2 = \frac{\rho_s \bar{S}^2 T}{\rho_n \rho C_V}$, ρ_s, ρ_n, $\bar{S}$ are thermodynamic parameters of quantum mixtures [1].

When temperature is low enough so that thermal exitations of ^{4}He can be neglected the system of hydrodynamic equations for mixtures becomes analogous to the system (1) after respective variables substitution. Hence, solutions (13) and (17) that were obtained here for usual liquids can be used for quantum mixtures.

The main difference of quantum mixtures is that the factor (18) can take any values from much smaller till much larger than 1 [4-6]. This fact stimulated investigators to make a number of experiments related with observation of temperature relaxation in ^{3}He-^{4}He quantum mixtures [7-11]. In this manuscript those experimental data are investigated.

Experiments were made at constant temperature of the higher flange, herewith specific power from 0.5 till 25 mW/sm^2 was given to a heater. Temperature relaxation and concentration relaxation were measured when heat flux was switched on. Mixture with initial concentration 9.8% of ^{3}He was investigated at temperature range 150 – 400 mK with pressure 0.38 bar.

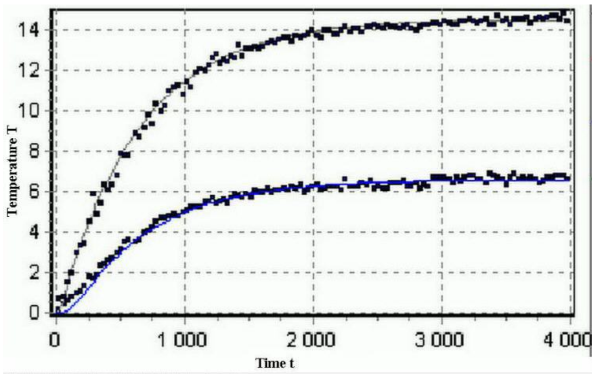

Fig.4. Temperature dependence on time. Solid lines are theoretical curves. Dots are experimental data.

From stationary temperature values T_1, T_2 we find thermal conductivity value $\kappa = q_0 \Delta l/(T_2 - T) = 2.1 \cdot 10^{-4}$ J/(cm K s), where Δl is distance between detectors. For given experimental conditions we know thermal capacity value $C_V = 0.26$ J /(g K) and density value $\rho_0 = 0.1451$ g/cm^3. Substituting this values into the solution we obtain dependency temperature on time, that is shown as solid lines in Fig.4. As we can see from Fig.4 theoretical calculations are in good agreement with experimental data which are indicated as dots in Fig.4.

References

1. I.M. Khalatnikov.V. A. Zharkov, J. Phys. 32, 1108 (1957).
2. L. Gorkov, L. Piaevskii, J. Phys. 33, 634 (1957).
3. A. Griffin, J. Phys. 47, 429 (1969).
4. I. Adamenko, V. Tsyganyuk, JLTP, 88, 1641 (1985).
5. K. Nemchenko, JLTP, 23, 799 (1997).
6. K. Nemchenko, JLTP, 24, 941 (1998).
7. A. Zadorozhko, T. Kalko, E. Rudavskii, V. Chagovets, G. Sheshin, JLTP, 30, 169 (2004).
8. A. Zadorozhko, T. Kalko, E. Rudavskii, V. Chagovets, G. Sheshin, JLTP, 29, 367 (2003).
9. A. Zadorozhko, T. Kalko, E. Rudavskii, V. Chagovets, G. Sheshin, JLTP, 29, 367 (2003).
10. A. Zadorozhko, T. Kalko,K. Nemchenko, E. Rudavskii, V. Chagovets, G. Sheshin, JLTP, 30, 1019 (2004).
11. A. Zadorozhko, T. Kalko, E. Rudavskii, V. Chagovets, G. Sheshin, JLTP, 31, 134-140 (2005).

ЗВУКОВА І ДИСИПАТИВНА РЕЛАКСАЦІЯ В НАДПЛИННИХ РОЗЧИНАХ ^{3}He-^{4}He

К. Немченко, С. Рогова

Харківський національний університет ім. В.Н.Каразіна,
пл. Свободи, 4, Харків, 61077
e-mail: svetlanochka85@mail.ru

Розглянуто систему рівнянь гідродинаміки для опису релаксації температури в надплинних розчинах ^{3}He-^{4}He. Релаксація температури і концентрації визначається як механізмом другого звуку, так і дисипативним дифузійним процесом. Знайдено аналітичний розв'язок цієї системи. Проведено порівняння отриманого розв'язку з експериментальними даними вимірювань кінетики зміни температури і концентрації надплинного розчину ^{3}He-^{4}He при включенні чи виключенні теплового потоку.