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MATHEMATICAL MODELS OF DECISION SUPPORT IN THE PROBLEMS OF LOGISTICS NETWORKS OPTIMIZATION

The **subject** of research in the article is the process of decision support in the problems of logistics networks optimization. The **goal** of the work is to develop a set of mathematical models of logistics network optimization problems to increase the efficiency of decision support systems by coordinating the interaction between automatic and interactive procedures of computer-aided design systems. The following **tasks** are solved in the article: review and analysis of the current state of the problem of decision support in the problems of logistics networks optimization; decomposition of the problem of decision support for the optimization of logistics networks; development of a mathematical model of the general problem of network optimization in terms of economy, efficiency, reliability and survivability; development of a set of technological mathematical models for the correct reduction of many effective options for building logistics networks for the final choice, taking into account difficult to formalize factors, knowledge and experience of the decision maker (DM). The following **methods** are used: systems theory, utility theory, optimization and operations research. **Results.** Analysis of the current state of the problem of logistics networks optimization has established the existence of the problem of correct reduction of a subset of effective options for their construction for ranking, taking into account difficult to formalize factors, as well as knowledge and experience of DM. The decomposition of the problem into tasks is performed: definition of the principles of network construction; network structure selection; determination of the topology of network elements; choice of network operation technology; determination of parameters of elements and communications (means of cargo delivery); multi criteria evaluation and selection of the best option for building a network. A mathematical model of the general problem of network optimization in terms of economy, efficiency, reliability and survivability is proposed. To coordinate the interaction between automatic and interactive network optimization procedures, it is proposed to use a combined method of ranking options, which allows you to identify and correctly reduce the subset of effective options for ranking DM. To implement the method, mathematical models of problems of the procedure of ranking options in the technologies of project decision support have been developed, which allow to combine the advantages of the technologies of the ordinalistic and cardinalistic approaches. **Conclusions.** The developed set of mathematical models expands the methodological bases of automation of processes of support of multi criteria decisions on optimization of logistic networks, allows to carry out correct reduction of set of effective options of their construction for the final choice taking into account factors, knowledge and experience of DM. The practical use of the proposed models and procedures will reduce the time and capacity complexity of decision support technologies, and through the use of the proposed selection procedures - to improve their quality across a variety of functional and cost indicators.

Keywords: logistics network; optimization; multi criteria evaluation; effective option; decision support.

Introduction

The efficiency of production and trade companies in modern conditions is largely determined by the quality of their logistics. Logistics processes cover all major stages of economic activity from raw materials to the supply of products and services to consumers [1]. One of the most important problems of logistics is the design of supply chain networks [2]. Promising supply chains must be sufficiently stable in the face of changes in the external environment. This fully applies to the chains of modern types of environmental and reversible logistics. They fully cover product life cycles, from the rational use of raw materials to waste disposal [3].

A systematic approach to the optimization of supply chain networks involves the joint solution of direct and reverse logistics problems [4-5], which have traditionally been considered conditionally independent. In this approach, logistics networks are considered as geographically distributed objects, optimization problems of which are combinatorial in nature and are solved on a variety of functional and cost indicators in terms of incomplete definition of goals and data [6]. This creates many new challenges to support decision-making on the optimization of logistics networks, which require formalization and development of effective methods for solving them.

Modern technologies of design (reengineering) of such objects provide joint solution of problems of their

structural, topological and parametric optimization. This involves the generation and analysis of powerful sets of possible options for building networks. The vast majority of acceptable options for building networks are inefficient, and the choice of the best option is made by a decision-maker (DM), who is able to choose from only a few options [7]. Due to the fact that in practice it is not possible to substantiate a single scalar criterion for assessing the effectiveness of options, DM evaluates them based on the analysis of a given set of conflicting local criteria [8–10]. The methods of individual or collective expert evaluation are used [11–13]. At their realization in designing technologies, there are problems of coordination of interaction of automatic and interactive procedures. One of them is the problem of correctly reducing the number of effective options for building networks for the final selection of the best among them, taking into account factors that are difficult to formalize, knowledge and experience of DM.

Analysis of the problem and methods of solving it

In modern logistics, there are macro- and micrologistic processes [4]. In the systems of macrologistics the processes of interaction of several independent objects of property, not connected with territorial distribution, are realized. Macro-logistics tasks are solved in the interests of international, transcontinental companies or intermediary organizations. The problems of

micrologistics are solved to organize the interaction of elements of one or more enterprises gathered in a group of common economic interests [14].

An example of the problem of optimizing global traffic can be the problem of structural and topological synthesis of the centralized transport and warehousing system of the regional level [15]. Directed search methods are successfully used to solve such problems. The problems of local transportation optimization are formulated as varieties of the salesman's problem, for the solution of which, depending on their dimension, accurate, heuristic and metaheuristic methods are used [16].

Conditions of competition lead to the need for rapid development of new products, which can lead to significant changes in flows in supply chains. With relatively small changes in flows, the adaptation of existing network options is carried out, which requires decision-making in conditions of uncertainty in demand [17]. With significant changes in demand, other characteristics of the network, environmental or functional constraints there is a need for its reengineering [5].

Regardless of the level of the network in the problem of decision support for its optimization, there are tasks [18]: goal setting; formation of a universal set of options S^U ; selection of a set of valid options $S^* \subseteq S^U$; selection of a subset of effective options $S^E \subset S^* \subset S^U$; ranking and choosing the best option $s^o \in S^E$.

Logistics networks are characterized by significant territorial dispersion. As a result, their structural, functional and cost characteristics are significantly dependent on the topology (location) of their elements (manufacturers, terminals, consumers). Based on this, formally, the option of building a logistics network should be presented in the form of a tuple, which reflects the set of its elements E , the relationships between the elements R and the location of its elements G [5]:

$$s = \langle E, R, G \rangle, \quad (1)$$

where $R = [r_{ij}]$, $i, j = \overline{1, n}$: $r_{ij} = 1$, if there is a direct connection between the i -th and j -th network elements, $r_{ij} = 0$ – in other case; $n = |E|$ – number of network elements.

Each of the options for building a network is evaluated by a set of local criteria $k_j(s)$, $j = \overline{1, m}$ that reflect their functional and cost indicators (delivery time, reliability, delivery costs, etc.). At the first stage of network optimization, based on its desired properties, subsets of elements E' , structures (connections between elements) R' and topologies (locations of terminals) G' on which it can be created are determined. They identify many possible options for building a network [6]:

$$S' \subseteq E' \times R' \times G'. \quad (2)$$

The established ecological, economic, other restrictions reduce subsets of possible elements, connections and topologies of a network to subsets of

admissible, which define set of admissible variants of its construction:

$$S^* \subseteq E^* \times R^* \times G^*, E^* \subseteq E', R^* \subseteq R', G^* \subseteq G'. \quad (3)$$

The essence of the problem is to remove from the universal set of options for building a network that does not satisfy the constraints:

$$k_j(s) \geq k_j^* \quad \forall k_j(s) \in Q(s), k_i(s) \leq k_i^* \quad \forall k_i(s) \in C(s), \quad (4)$$

where $Q(s)$, $C(s)$ are the sets of indicators of effects from the use of the network and the cost of its creation and operation.

The task of allocating a subset of effective options for building a network $S^E \subset S^*$ is to remove from the set of acceptable subsets of those that can be improved by at least one of the local criteria $k_j(s)$, $j = \overline{1, m}$ without compromising the quality of others. A variant of network construction will be called effective $s^E \in S^E$ if there is no variant on the set of admissible S^* for which the relations could be fulfilled [18]:

$$k_i(s) \geq k_i(s^E), \text{ if } k_i(s) \rightarrow \max, \quad (5)$$

$$k_i(s) \leq k_i(s^E), \text{ if } k_i(s) \rightarrow \min \quad (6)$$

and at least one of them was strict.

Subsets of effective variants $S^E \subset S^*$ by the classical methods of Carlin and Hermeyer are found by combining the solutions s_l^o of the corresponding sets of problems [19–20]:

$$s_l^o = \arg \max_{s \in S^*} \{ P(s) = \sum_{j=1}^m \lambda_j \xi_j(s) \}, \quad (7)$$

$$s_l^o = \arg \max_{s \in S^*} \{ P(s) = \min_j \lambda_j \xi_j(s) \} \quad (8)$$

for various sets of parameters (sets of weighting factors of local criteria)

$$A^l = \{ \lambda_j : \lambda_j > 0 \quad \forall j = \overline{1, m}, \sum_{j=1}^m \lambda_j = 1 \}, \quad (9)$$

where $\xi_j(s)$ – the value of the utility function or the normalized value of the j -th local criterion; λ_j – weighting factor of j -th criterion.

An evolutionary method based on a genetic algorithm with non-dominant sorting is used to determine the Pareto front on admissible sets of extra-large sizes [21]. A method for reducing the number of objective functions based on the principal components method is used to accelerate the rate of its convergence to the Pareto front [22].

Models and methods of utility theory are used to rank and select the best option for building a network from a set of effective $s^o \in S^E$. Moreover, in automated

technologies, these problems are solved by methods of both quantitative and qualitative evaluation [7–13].

Arranging small sets of effective options S^E is done by DM or experts. To date, a large number of methods of multicriteria analysis of solutions, including: AHP, MULTIMOORA, MAUT, TOPSIS, VIKOR, COPRAS, STEP, PROMETHEE, ELECTRE [11, 23]. Each of the methods uses its own technology of ordering, so even in problems with the same input data, different orders of alternatives can be established and different options can be chosen as the best. The simplest from the point of view of expertise can be considered the method of comparative identification, which allows on the basis of the established order of options to synthesize a function for their quantitative scalar estimation $P(s)$ [7, 18, and 20]. Using the obtained function, it is possible to organize powerful sets of effective options S^E and choose the best among them:

$$s^o = \arg \max_{s \in S^E} P(s). \quad (10)$$

In both cases, each of the options for building a network $s \in S^E$ is assigned the meaning of its value $P(s)$, which determines their order on a subset of effective $S^E \subseteq S^*$:

$$\begin{cases} \{\forall s, v \in S^* : s \succ v \leftrightarrow P(s) > P(v); \\ s \sim v \leftrightarrow P(s) \geq P(v); \\ s \sim v \leftrightarrow P(s) = P(v). \end{cases} \quad (11)$$

Additive function in the form of convolution of local criteria has become the most widespread for quantitative estimation of variants [7]:

$$P(s) = \sum_{j=1}^m \lambda_j \xi_j(s), \quad (12)$$

$$\xi_j(s) = \left[\frac{k_j(s) - k_j^-}{k_j^+ - k_j^-} \right]^{\mu_j}, \quad j = \overline{1, m}, \quad (13)$$

where λ_j – weighting factors that assess the mutual importance of local criteria $k_j(s)$, $j = \overline{1, m}$, $\lambda_j \geq 0$,

$\sum_{j=1}^m \lambda_j = 1$; $\xi_j(s)$ – the value of the utility function of the

j -th local criterion for the option s ; k_j^+ , k_j^- , $j = \overline{1, m}$ – the best and the worse values of the j -th local criterion on the set S^* ; μ_j – parameter that determines the specific type of function (13): linear, concave or convex.

To reconcile the interaction between automatic and expert procedures, a combined method of ranking options has been proposed, which combines the advantages of ordinalistic and cardinalistic ordering [18]. However, the method of selecting options for preliminary peer review remains open.

According to the results of the review of the current state of the problem of decision support in the optimization of logistics networks, it was found that [18]:

- most network optimization tasks are multi-criteria and combinatorial in nature, and the process of solving them involves the automatic generation and analysis of powerful sets of options for their construction;

- the final decision-making process is carried out using expert evaluation methods, in the process of which only a small number of options can be analyzed.

There is a need to correctly reduce the subsets of effective options for ranking at both stages of the examination, taking into account factors that are difficult to formalize, knowledge and experience of DM.

The aim is to develop mathematical models for technology to support decision-making in the problems of optimization of logistics networks based on the procedures of ordinalistic and cardinalistic ordering.

Results of the Study

The methodological complexity of the problem of optimization of logistics networks as large-scale objects does not allow creating a holistic formalized description and finding an effective option for their construction in one procedure. Let's divide the description of the network into hierarchical levels and aspects, and the optimization process – into groups of optimization procedures [24]. Then the selected procedures will allow you to obtain and convert descriptions of the tasks of the selected levels (aspects) and their aggregation to obtain the best option for building a network.

At the first stage, the problem of network optimization is presented as a *MetaTask*, which contains many local tasks. Such problems $Task_i^l$, $i = \overline{1, i_l}$, $l = \overline{1, n_l}$ (where i_l is the number of levels of decomposition, n_l - the number of problems at the l -th level of decomposition) belong to different levels of decomposition [6]:

$$MetaTask = \{Task_i^l\}, Task^l = \{Task_i^l\}. \quad (14)$$

The basic tasks of the lower level are: $Task_1^l$ – defining the principles of network construction; $Task_2^l$ – network structure selection; $Task_3^l$ – determining the topology of network elements; $Task_4^l$ – choice of operating technology; $Task_5^l$ – determination of parameters of elements and connections (means of cargo delivery); $Task_6^l$ – multicriteria evaluation and selection of the best network construction option.

Each of the selected tasks $Task_i^l$, $i = \overline{1, i_l}$ will be considered as a converter of its input data into In_i^l its output data Out_i^l :

$$Task_i^l : In_i^l \rightarrow Out_i^l, l = \overline{1, n_l}, i = \overline{1, i_l}. \quad (15)$$

Consider the problem of network optimization on four indicators: economy, efficiency, reliability and survivability. Indicators of efficiency and efficiency are taken into account when optimizing almost all logistics networks. Reliability and survivability indicators are important for military logistics and critical logistics systems [25].

To assess the cost-effectiveness of options, we use the criterion of the costs of creating and operating a network: $k_1(s) \rightarrow \min_{s \in S^*}$. To assess the efficiency of the network, we use the time of delivery of goods. Then the maximum efficiency will correspond to the minimum time for delivery of goods: $k_2(s) \rightarrow \min_{s \in S^*}$. As an indicator of network reliability we use the coefficient of its readiness: $k_3(s) \rightarrow \max_{s \in S^*}$. To assess the survivability, we use the value of the share of consumers who will receive cargo in case of single damage to its components: $k_4(s) \rightarrow \max_{s \in S^*}$.

Taking into account the introduced notations, the mathematical model of the multi-criteria problem of logistics network optimization can be presented as follows:

$$\left\{ \begin{array}{l} k_1(s) \rightarrow \min_{s \in S^*}; \quad k_1(s) \leq k_1^*; \\ k_2(s) \rightarrow \min_{s \in S^*}; \quad k_2(s) \leq k_2^*; \\ k_3(s) \rightarrow \max_{s \in S^*}; \quad k_3(s) \geq k_3^*; \\ k_4(s) \rightarrow \max_{s \in S^*}; \quad k_4(s) \geq k_4^*. \end{array} \right. \quad (16)$$

where k_1^* , k_2^* , k_3^* , k_4^* – maximum allowable values of cost, efficiency, reliability and survivability of network construction options.

By excluding some local criteria or constraints from model (16), it is possible to obtain models of different network optimization problems according to one, two or three local criteria.

Table 1. Power subsets of efficient options

$m / S^E $	10000	20000	30000	40000	50000
2	7	8	10	11	12
3	47	51	52	54	58
4	151	164	216	253	292
5	404	501	553	591	785
6	822	1309	1394	1587	1750
7	1762	2572	3010	3157	3468
$m / S^E $	60000	70000	80000	90000	100000
2	13	15	16	18	19
3	61	62	68	71	77
4	295	298	302	329	342
5	791	941	975	1023	1141
6	2116	2237	2298	2351	2450
7	4071	4741	4942	5235	5391

The solution of the network optimization problem will be carried out by a combined expert-machine method [18]. It provides for the implementation of such stages:

- allocation on a set of admissible variants of construction of a network S^* a set of effective options S^E ;
- allocation on a subset of effective S^E a set of options $S' \subseteq S^E$ for preliminary expert evaluation;
- parametric synthesis of the function of general utility of variants $P(s)$;
- ranking options using the general utility function;
- using estimates $P(s)$ of allocation on a subset S^E of a given number of the most effective options $S'' \subseteq S^E$;
- DM choice on a set of options $S'' \subseteq S^E$ of the best option $s^o \in S'' \subseteq S^E$ on a set of indicators $k_j(s)$, $j = \overline{1, m}$.

The total number of possible options for building a logistics network with a radial-node structure, provided that the nodes are located only near consumers is 2^n (where $n = |E|$ – number of network elements).

Given that, the methods of Carlin and Hermeyer (7) - (9) for a reasonable time allow us to select only the approximation of a subset of effective options S^E from the set of allowable S^* , we use the method of pairwise comparisons [18, 20]. The results of experimental studies using the method of pairwise comparisons for uniform distribution of network characteristics in the space of local criteria $k_j(s)$, $j = \overline{2, 7}$ showed that: the absolute size (capacity) of subsets of effective options $|S^E|$ depending on the number of local criteria m and the capacity of sets of acceptable options of network construction $|S^*|$ have a steady upward trend, and the relative size of subsets of effective options $|S^E|/|S^*|$ – downward (tables 1 and 2).

Table 2. Relative power subsets of efficient options

$m / S^E / S^* $	10000	20000	30000	40000	50000
2	0,0007	0,0004	0,0003	0,0003	0,0002
3	0,0047	0,0026	0,0017	0,0014	0,0012
4	0,0151	0,0082	0,0072	0,0063	0,0058
5	0,0404	0,0251	0,0184	0,0148	0,0157
6	0,0822	0,0655	0,0465	0,0397	0,0350
7	0,1762	0,1286	0,1003	0,0789	0,0694
$m / S^E / S^* $	60000	70000	80000	90000	100000
2	0,0002	0,0002	0,0002	0,0002	0,00019
3	0,0010	0,0009	0,0009	0,0008	0,00077
4	0,0049	0,0043	0,0038	0,0037	0,00342
5	0,0132	0,0134	0,0122	0,0114	0,01141
6	0,0353	0,0320	0,0287	0,0261	0,02450
7	0,0679	0,0677	0,0618	0,0582	0,05391

If possible, within the framework of the network design technology used, it is suggested not to select a subset S^E of the set of admissible variants S^* by solving a separate problem, but to form it already at the stage of variant $s \in S^*$ generation. This will significantly reduce the time and capacity complexity of decision-making tasks (table 1).

To automatically select the specified number of the most effective options $S^n \subseteq S^E$, it is necessary first to select the type of function of general utility $P(s)$ and to solve the problem of its parametric synthesis.

We will use to quantify the overall usefulness of options for building networks with a universal function [7, 18, and 20]:

$$P(s) = \sum_{i=1}^m \lambda_i \xi_i(s) + \sum_{i=1}^m \sum_{j=i}^m \lambda_{ij} \xi_i(s) \xi_j(s) + \sum_{i=1}^m \sum_{j=i}^m \sum_{l=j}^m \lambda_{ijl} \xi_i(s) \xi_j(s) \xi_l(s) + \dots, \quad (17)$$

where $\lambda_i, \lambda_{ij}, \lambda_{ijl}$ – weighting factors that assess the mutual importance of the criteria $k_i(s), k_j(s), k_l(s)$ and their products; $0 < \xi_i(s) < 1, i = \overline{1, m}$ – the value of the

utility criterion of the local criterion $k_i(s), i = \overline{1, m}$ for the option $s \in S^E$.

To be able to make S- and Z-like approximation of estimates of the values of local criteria, we use the best in terms of "accuracy-complexity" universal function-gluings [26–27]:

$$\xi(s) = \begin{cases} \bar{a}(b_1 + 1) \left(1 - \left(b_1 / \left(b_1 + \frac{\bar{k}(s)}{\bar{k}_a} \right) \right) \right), & 0 \leq \bar{k}(s) \leq \bar{k}_a; \\ \bar{a} + (1 - \bar{a})(b_2 + 1) \left(1 - \left(b_2 / \left(b_2 + \frac{\bar{k}(s) - \bar{k}_a}{1 - \bar{k}_a} \right) \right) \right), & \bar{k}_a < \bar{k}(s) \leq 1, \end{cases} \quad (18)$$

where $\xi(s) = \bar{k}(s); \bar{k}_a, \bar{a}$ – coordinates of the gluing point, $0 \leq \bar{k}_a \leq 1, 0 \leq \bar{a} \leq 1; b_1, b_2$ – parameters that determine the type of dependence on the first and second segments of the function.

To solve the problem of parametric synthesis of the general utility function, we use the method of comparative identification [7, 28]. At the first stage on a set of effective, it is necessary to allocate a subset of a small number of the most informative options $S' \subseteq S^E$. On it DM, proceeding from requirements to a logistic network and qualitative estimations of variants, forms the binary relation of strict advantage:

$$R(S') = \{ \langle s, v \rangle : s, v \in S', s \succ v \}. \quad (19)$$

For the established relation of strict advantage using the function of scalar estimation of variants (17) we make a system of inequalities:

$$P(\lambda, s) > P(\lambda, v) : \forall \langle s, v \rangle \in R(S'), \quad (20)$$

where λ – vector of general utility model parameters (17).

If you enter a notation:

$$\begin{aligned} \xi_j(s) \cdot \xi_l(s) &= \xi_{m+j}(s), \quad \lambda_{j,l} = \lambda_{m+j}, \\ \xi_j(s) \cdot \xi_2(s) &= \xi_{m+2}(s), \quad \lambda_{j,2} = \lambda_{m+2}, \dots, \end{aligned} \quad (21)$$

then the universal model (17) can be presented in additive form:

$$P(\lambda, s) = \sum_{i=1}^N \lambda_i \xi_i(s). \quad (22)$$

The maximum number of such a model additions is $N = C_{m+p}^p - 1$ (where p is the degree of the polynomial,

$$\begin{cases} \eta_j(\lambda) \equiv \sum_{i=1}^N \lambda_i \xi_i(s) - \sum_{i=1}^N \lambda_i \xi_i(v) > 0, < s, v > \in R(S'), j = \overline{1, n'}, \\ \eta_{n'+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \lambda_i \geq 0, i = \overline{1, N}, \end{cases} \quad (23)$$

where $n' = \text{Card } R(S')$ – power set ratio of strict advantage (19).

Stable estimates of the vector of model parameters (22) are the solution of the Chebyshev point search problem [7, 28]:

$$\begin{cases} \eta_j(\lambda) + \lambda_{N+1} > 0, j = \overline{1, n'}, \\ \eta_{n'+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \lambda_i \geq 0, i = \overline{1, N}, \\ \lambda_{N+1} \rightarrow \min. \end{cases} \quad (24)$$

Using the obtained parameter values $[\lambda_i^o], i = \overline{1, N}$ the next step is to calculate the scalar quality estimates $P(s)$ (22) for all options from the subset of effective $s \in S^E$, which determine their ordering by value.

At the last stage, based on the established estimates of options $P(s), s \in S^E$, the selection of a subset

which is determined by the required accuracy of DM). The problem of parametric synthesis of the general utility function (22) is to determine the vector $[\lambda_i], i = \overline{1, N}$ which satisfies the system of inequalities (20) and the conditions for normalization of parameters:

$S^o \in S^E$ of a given number n^o of the best options. The DM then makes the final choice of the best option $s^o \in S^o$.

It is experimentally established that the insufficiently substantiated choice of options for establishing the binary relation (19) reduces the accuracy of determining the advantages of DM, which is set by the values of the weights $\lambda_i, \lambda_{ij}, \lambda_{ijl}, \dots$.

Consider the problem of parametric synthesis of the general utility model (22) on the set of 30 effective options for building a logistics network $S^E = \{s\}$, estimated by the values of the utility functions of local criteria $\xi_j(s), j = \overline{1, 4}$. Based on the equivalence of local criteria, we calculate the value of the general utility function $P(s)$ for all options, which establishes such an order among the options (table 3):

$$S^E = \{s_1 \succ s_2 \succ \dots \succ s_{29} \succ s_{30}\}. \quad (25)$$

Table 3. Characteristics of network construction options

s	$\xi_1(s)$	$\xi_2(s)$	$\xi_3(s)$	$\xi_4(s)$	$P(s)$	$P_1(s)$	$P_2(s)$	$P_3(s)$
1	2	3	4	5	6	7	8	9
s_1	0,871	0,754	0,999	0,887	0,878	0,880	0,875	0,877
s_2	0,819	0,858	0,913	0,882	0,868	0,866	0,867	0,868
s_3	0,851	0,974	0,595	0,978	0,850	0,842	0,855	0,851
s_4	0,926	0,707	0,791	0,943	0,842	0,844	0,844	0,842
s_5	0,958	0,701	0,787	0,878	0,831	0,837	0,833	0,831
s_6	0,972	0,886	0,845	0,593	0,824	0,838	0,825	0,824
s_7	0,651	0,862	0,789	0,965	0,817	0,803	0,816	0,817
s_8	0,985	0,642	0,861	0,738	0,807	0,820	0,807	0,806
s_9	0,609	0,959	0,678	0,939	0,796	0,780	0,797	0,797
s_{10}	0,458	0,834	0,975	0,817	0,771	0,756	0,764	0,769
s_{11}	0,799	0,915	0,902	0,435	0,763	0,774	0,760	0,762
s_{12}	0,711	0,956	0,984	0,364	0,754	0,764	0,748	0,752
s_{13}	0,559	0,927	0,468	0,987	0,735	0,714	0,739	0,737
s_{14}	0,934	0,874	0,983	0,094	0,721	0,752	0,717	0,720

The end **Table 3.**

1	2	3	4	5	6	7	8	9
s_{15}	0,747	0,988	0,964	0,106	0,701	0,721	0,695	0,700
s_{16}	0,973	0,823	0,171	0,791	0,690	0,693	0,703	0,694
s_{17}	0,988	0,801	0,197	0,732	0,680	0,687	0,692	0,684
s_{18}	0,984	0,525	0,351	0,815	0,669	0,678	0,679	0,671
s_{19}	0,519	0,938	0,695	0,497	0,662	0,657	0,660	0,662
s_{20}	0,892	0,062	0,819	0,851	0,656	0,670	0,655	0,655
s_{21}	0,385	0,435	0,811	0,957	0,647	0,629	0,642	0,645
s_{22}	0,221	0,412	0,943	0,917	0,623	0,600	0,613	0,620
s_{23}	0,433	0,247	0,803	0,959	0,611	0,597	0,606	0,608
s_{24}	0,104	0,835	0,952	0,494	0,596	0,576	0,584	0,593
s_{25}	0,171	0,317	0,845	0,983	0,579	0,552	0,570	0,576
s_{26}	0,989	0,345	0,306	0,632	0,568	0,587	0,578	0,570
s_{27}	0,981	0,767	0,275	0,216	0,560	0,586	0,569	0,563
s_{28}	0,997	0,201	0,582	0,417	0,549	0,580	0,553	0,550
s_{29}	0,993	0,382	0,254	0,524	0,538	0,560	0,548	0,541
s_{30}	0,277	0,982	0,354	0,494	0,527	0,508	0,528	0,528

We will choose a subset containing 8 variants for random examination $S' = \{s_4, s_8, s_9, s_{10}, s_{18}, s_{22}, s_{24}, s_{25}\}$. To determine the ratio of strict advantage $R(S')$ (19), we establish an order on it:

$$s_4 \succ s_8 \succ s_9 \succ s_{10} \succ s_{18} \succ s_{22} \succ s_{24} \succ s_{25}. \quad (26)$$

Based on the results of solving problem (24), the best parameters of the model were determined: $\lambda_1 = 0,295$, $\lambda_2 = 0,234$, $\lambda_3 = 0,256$, $\lambda_4 = 0,215$ and the value of the general utility function was calculated. With the obtained values of the parameters, the set order (26) is reproduced, but there is a violation of the set order on the whole set. $S^E = \{s_1 \succ s_2 \succ \dots \succ s_{29} \succ s_{30}\}$: $s_4 \succ s_3$, $s_6 \succ s_5$, $s_8 \succ s_7$, $s_{11} \succ s_{10}$, $s_{14} \succ s_{13}$, $s_{20} \succ s_{19}$, $s_{26} \succ s_{25}$, $s_{27} \succ s_{25}$.

To increase the accuracy of restoring the benefits of DM on the set of effective S^E , we select a subset of the best options based on the results of solving problems:

$$s' = \arg \max_{s \in S^E} \min_{1 \leq i \leq m} \xi_i(s). \quad (27)$$

The subset $S' = \{s_2, s_1, s_4, s_5, s_7, s_8, s_9, s_3\}$ is defined in this way. Based on the same advantages of DM, let's establish order on it:

$$s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_7 \succ s_8 \succ s_9. \quad (28)$$

The system of advantages (28) corresponds to the parameters of the model $\lambda_1 = 0,262$, $\lambda_2 = 0,252$, $\lambda_3 = 0,232$, $\lambda_4 = 0,254$ and the value of the utility function $P_2(s)$ (table 3). Such values of parameters completely reproduce the established order (28) and give

only 2 violations of the set order on all set (30): $s_{16} \succ s_{15}$, $s_{26} \succ s_{25}$.

It was possible to completely restore order (25), including in the subset for preliminary examination variants of pairs with minimal deviations of characteristics $\xi_j(s)$, $j = \overline{1,4}$. The order received on them has such structure:

$$\begin{aligned} s_3 \succ s_4, s_5 \succ s_6, s_6 \succ s_7, s_{10} \succ s_{11}, \\ s_{18} \succ s_{19}, s_{19} \succ s_{20}, s_{26} \succ s_{27}. \end{aligned} \quad (28)$$

It corresponds to the vector of parameters $\lambda_1 = 0,253$, $\lambda_2 = 0,252$, $\lambda_3 = 0,244$, $\lambda_4 = 0,251$ and the value of the utility function $P_3(s)$ (table 3).

The proposed two-stage procedure allows to correctly reduce the set of effective options for building logistics networks for the final choice of many indicators, taking into account factors that are difficult to formalize, knowledge and experience of DM.

Conclusions

In the system approach, logistics networks are considered as geographically distributed objects, the optimization problems of which are combinatorial in nature and are solved on a variety of functional and cost indicators in conditions of incomplete definition of goals and data. This creates many tasks to support multi-criteria decision-making that require formalization. In particular, it is established that the solution of the main problems of optimization of topological structures of networks involves automatic generation and analysis of powerful sets of options for their network construction, and the final decision is made by DM, in which only a small number of

options can be analyzed. In order to harmonize the optimization procedures, it is proposed to correctly reduce the subsets of effective options for ranking at both stages of the examination, taking into account factors that are difficult to formalize, knowledge and experience of DM.

Due to the methodological complexity of the problem under consideration, its solution is proposed in the framework of aggregative-decompositional approach, which involves dividing the description of the network into hierarchical levels and aspects, and the optimization process – into groups of optimization procedures. The procedures defined in this way allow to obtain and transform descriptions of tasks of selected levels (aspects) and their aggregation to obtain the best option for building a network.

A mathematical model of the general problem of network optimization in terms of economy, efficiency, reliability and survivability is proposed, from which by excluding some local criteria and constraints it is possible to obtain models of optimization problems with different local criteria and constraints using universal functions of general usefulness and usefulness.

To reconcile the interaction between automatic and interactive network optimization procedures, it is proposed to use a combined method of ranking options, which involves the consistent implementation of stages of formation of a subset of effective alternatives, determining

the benefits of DM and parametric synthesis of the utility function. To implement the method, mathematical models of problems of the procedure of ranking variants in technologies to support project decision-making have been developed, which allow combining the advantages of technologies of ordinalistic and cardinalistic ordering of decisions.

The developed set of mathematical models expands the methodological principles of automation of processes for supporting multicriteria solutions for optimization of logistics networks, allows to correctly reduce the set of effective options for their construction for the final choice, taking into account factors that are difficult to formalize, knowledge and experience DM.

The practical use of the proposed mathematical models and procedures will reduce the time and capacity complexity of decision support technologies, and through the use of proposed selection procedures - to improve their quality for a variety of functional and cost indicators.

Further research in this direction may be aimed at taking into account in the models of network optimization incomplete certainty of their functional and cost characteristics using the apparatus of interval or fuzzy analysis, as well as effective methods of network optimization in terms of incomplete definition of goals and data.

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МАТЕМАТИЧНІ МОДЕЛІ ПІДТРИМКИ ПРИЙНЯТТЯ РІШЕНЬ В ЗАДАЧАХ ОПТИМІЗАЦІ ЛОГІСТИЧНИХ МЕРЕЖ

Предметом дослідження в статті є процес підтримки прийняття рішень в задачах оптимізації логістичних мереж. **Мета** роботи – розроблення комплексу математичних моделей задач оптимізації логістичних мереж для підвищення ефективності систем підтримки прийняття рішень шляхом узгодження взаємодії між автоматичними й інтерактивними процедурами систем автоматизованого проектування. У статті вирішуються наступні **завдання**: огляд і аналіз сучасного стану проблеми підтримки прийняття рішень в задачах оптимізації логістичних мереж; декомпозиція проблеми підтримки прийняття рішень з оптимізації логістичних мереж; розробка математичної моделі загальної задачі оптимізації мережі за показниками економічності, оперативності, надійності та живучості; розробка комплексу математичних моделей технології коректного скорочення множини ефективних варіантів побудови логістичних мереж для остаточного вибору з урахуванням факторів, що важко піддаються формалізації, знань і досвіду особи, що приймає рішення (ОПР). Використовуються такі **методи**: теорії систем, теорії корисності, оптимізації та дослідження операцій. **Результати**. Аналіз сучасного стану проблеми оптимізації логістичних мереж дозволив встановити існування проблеми коректного скорочення підмножин ефективних варіантів їхньої побудови для ранжування з урахуванням факторів, що важко піддаються формалізації, а також знань і досвіду ОПР. Виконана декомпозиція проблеми на задачі: визначення принципів побудови мережі; вибору структури мережі; визначення

топології елементів мережі; вибору технології функціонування мережі; визначення параметрів елементів і зв'язків (засобів доставки вантажів); багатокритеріальної оцінки та вибору найкращого варіанту побудови мережі. Запропоновано математичну модель загальної задачі оптимізації мережі за показниками економічності, оперативності, надійності та живучості. Для узгодження взаємодії між автоматичними й інтерактивними процедурами оптимізації мереж запропоновано використати комбінований метод ранжування варіантів, який дозволяє визначати та коректно скорочувати підмножин ефективних варіантів для ранжування ОПР. Для реалізації методу розроблено математичні моделі задач процедури ранжування варіантів в технологіях підтримки прийняття проектних рішень, які дозволяють об'єднати переваги технологій ординалістичного та кардиналістичного підходів. **Висновки.** Розроблений комплекс математичних моделей розширює методологічні засади автоматизації процесів підтримки багатокритеріальних рішень з оптимізації логістичних мереж, дозволяє здійснювати коректне скорочення множини ефективних варіантів їх побудови для остаточного вибору з урахуванням факторів, що важко піддаються формалізації, знань і досвіду ОПР. Практичне використання запропонованих математичних моделей і процедур дозволить скорочувати часову й емісню складності технологій підтримки прийняття рішень, а за рахунок використання запропонованих процедур відбору варіантів – підвищити їх якість за всією множиною функціонально-вартісних показників.

Ключові слова: логістична мережа; оптимізація; багатокритеріальне оцінювання; ефективний варіант; підтримка прийняття рішень.

МАТЕМАТИЧЕСКИЕ МОДЕЛИ ПОДДЕРЖКИ ПРИНЯТИЯ РЕШЕНИЙ В ЗАДАЧАХ ОПТИМИЗАЦИИ ЛОГИСТИЧЕСКИХ СЕТЕЙ

Предметом исследования в статье является процесс поддержки принятия решений в задачах оптимизации логистических сетей. **Цель** работы – разработка комплекса математических моделей задач оптимизации логистических сетей для повышения эффективности систем поддержки принятия решений путем согласования взаимодействия между автоматическими и интерактивными процедурами систем автоматизированного проектирования. В статье решаются следующие **задачи:** обзор и анализ современного состояния проблемы поддержки принятия решений в задачах оптимизации логистических сетей; декомпозиция проблемы поддержки принятия решений по оптимизации логистических сетей; разработка математической модели общей задачи оптимизации сети по показателям экономичности, оперативности, надежности и живучести; разработка комплекса математических моделей технологии корректного сокращения множества эффективных вариантов построения логистических сетей для окончательного выбора с учетом трудноформализуемых факторов, знаний и опыта лица, принимающего решения (ЛПР). Используются следующие **методы:** теории систем, теории полезности, оптимизации и исследования операций. **Результаты.** Анализ современного состояния проблемы оптимизации логистических сетей позволил установить существование проблемы корректного сокращения подмножества эффективных вариантов их построения для ранжирования с учетом трудно поддающихся формализации факторов, а также знаний и опыта ЛПР. Выполнена декомпозиция проблемы на задачи: определения принципов построения сети; выбора структуры сети; определение топологии элементов сети; выбора технологии функционирования сети; определения параметров элементов и связей (средств доставки грузов); многокритериальной оценки и выбора наилучшего варианта построения сети. Предложена математическая модель общей задачи оптимизации сети по показателям экономичности, оперативности, надежности и живучести. Для согласования взаимодействия между автоматическими и интерактивными процедурами оптимизации сетей предложено использовать комбинированный метод ранжирования вариантов, позволяющий определять и корректно сокращать подмножество эффективных вариантов для ранжирования ЛПР. Для реализации метода разработаны математические модели задач процедуры ранжирования вариантов в технологиях поддержки принятия проектных решений, позволяющих объединить преимущества технологий ординалистического и кардиналистического подходов. **Выводы.** Разработанный комплекс математических моделей расширяет методологические основы автоматизации процессов поддержки многокритериальных решений по оптимизации логистических сетей, позволяет осуществлять корректное сокращение множества эффективных вариантов их построения для окончательного выбора с учетом трудно поддающихся формализации факторов, знаний и опыта ЛПР. Практическое использование предложенных моделей и процедур позволит сокращать временную и емкостную сложности технологий поддержки принятия решений, а за счет использования предложенных процедур отбора вариантов – повысить их качество по всему множеству функционально-стоимостных показателей.

Ключевые слова: логистическая сеть; оптимизация; многокритериальная оценка; эффективный вариант; поддержка принятия решений.

Бібліографічні описи / Bibliographic descriptions

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