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## OPTIMIZATION OF TOPOLOGICAL STRUCTURES OF CENTRALIZED LOGISTICS NETWORKS IN THE PROCESS OF REENGINEERING

The **subject** of research in the article is the topological structures of closed logistics networks. The **purpose** of the work is to create a mathematical model and methods for solving problems of optimization of topological structures of centralized logistics networks in the process of reengineering, taking into account many topological and functional constraints. The article solves the following **tasks**: analysis of the current state of the problem of system optimization of logistics networks and methods of its solution; formalization of the problem of system optimization of logistics networks as territorially distributed objects; development of a mathematical model of the problem of optimization of centralized three-level topological structures of logistics networks at the stage of reengineering; development of a method for solving the problem of optimization of centralized three-level topological structures of logistics networks at the reengineering stage; estimation of time complexity of the method of optimization of centralized three-level topological structures of logistics networks. The following **methods** are used: methods of systems theory, methods of utility theory, optimization and operations research. The following **results** were obtained: analysis of the current state of the problem of system optimization of logistics networks and methods of its solution; the problem of system optimization of logistics networks as territorially distributed objects has been formalized; developed a mathematical model of the problem of reengineering three-level topological structures of logistics networks in terms of cost and efficiency for the case of combined production and processing points; methods of directed search of variants of construction of a logistic network which use procedures of coordinate optimization and modeling of evolution on the basis of genetic algorithm are developed; estimates of the accuracy and time complexity of optimization methods of centralized three-level topological structures of logistics networks are obtained. **Conclusions**: Based on the results of the study of methods for solving the problem, an approximation of their accuracy and time complexity was performed. In practice, this will allow you to choose a more efficient method for solving large-scale practical problems, based on the required accuracy, available computing and time resources. The method based on the coordinate optimization procedure has a significantly higher accuracy, but it is more complex from a computational point of view. The accuracy of the evolutionary method based on a genetic algorithm can be increased by increasing the number of iterations. The practical use of the proposed mathematical model and methods of reengineering the topological structures of centralized closed logistics systems by jointly solving problems for direct and reverse flows will reduce the cost of transport activities of companies.

**Keywords:** closed logistics; logistics network; optimization; reengineering; structure; topology.

### Introduction

The processes of optimization of anthropogenic objects provide for the establishment of causal relationships and laws characteristic of the processes of movement of heterogeneous flows in them. This is done in order to identify and implement effective organizational forms and methods of object management. The effectiveness of such processes is largely determined by the quality of the relevant logistics [1]. One of the problems in managing the SCM supply chains (Supply Chain Management) is the design of SCND supplies networks (Supply Chain Network Design) [2]. Such networks are inherently facilities with significant territorial dispersal. The structural, cost and functional characteristics of logistics networks are largely determined by the topology (placement) of their elements (manufacturers, terminals, consumers). The topology of network elements, in turn, determines the topology of the corresponding flows.

Logistics networks operate in constantly changing conditions. This is due to the emergence of new consumers, changes in supply volumes, the introduction of environmental restrictions [3-4] and the like. As a result, at some stage, existing networks become inefficient or no longer meet new conditions. Attempts to upgrade them by solving traditional optimization problems do not guarantee efficient options. For more effective adaptation of such objects, a reengineering approach is increasingly used, which involves a fundamental analysis of existing options for their construction (modes of transport, structure,

topology, parameters, technology) and radical redesign [5]. This requires solving a set of problems of technological, structural, parametric, topological optimization, multifactorial evaluation and selection of system solutions taking into account their connections at all stages of the network life cycle [6]. At the same time, the principles of building a network are radically revised, which can lead to profound changes in the technology of its functioning, structure, parameters of elements, means of transport, topology. Given that logistics networks use a relatively small variety of alternative technologies, node (terminal) types and vehicle types, the main problem from a computational point of view is the optimization of their structure and topologies.

### Analysis of the problem and methods of its solution

Traditionally, logistics are macro and micro logical systems [7]. The first species systems operate within the interaction of several property objects are not related to territorial distribution and are used by international transcontinental companies or intermediary organizations. The second species system solves the problems of interaction of one or more enterprises with one economic interest or a separate enterprise [8]. In this case, in the transport micro logical of a large enterprise allocate systems of global and local transportation. The task of global transportation in the general case is reduced to the optimal placement of intermediate truck units to minimize cargo transportation costs [9].

Increasingly, the tasks of environmental logistics are

increasingly relevant, which are aimed at reducing the level of risk of economic losses due to deterioration of environmental quality and enhancing ecological and economic efficiency of enterprises [10-11].

The subject of reversal logistics is the ordering and systematization of reverse commodity-material, information, cash flows [12-17]. Usually, tasks of direct and reversing logistics are considered as conditionally independent, which does not allow to receive effective global solutions to the logistics problem as a whole. Their joint solution generates a plurality of new tasks that require the development of new mathematical models and effective research methods [18].

One of the stages of synthesis of the system of global transportation of products of one manufacturer is structural and topological optimization of the centralized transport and warehouse system [19]. It is considered as set: the location of the supply center, set of consumers  $I = \{i : i = \overline{1, n}\}$ , for each of which is determined by the matrix of the distance location, supply volumes and transport tariffs for the delivery of goods. It is necessary to determine the optimal number of  $l^o$  and location of terminals, as well as subsets of consumers serviced by each of the terminals  $I_j = \{i, j = \overline{1, l^o}\}$ . In this case, in global transportation systems, it can be assumed that terminals are located near consumers of products.

Target function of the task shows the cost of delivery of products to consumers:

$$C = \sum_{j=1}^l c_j + \sum_{i=1}^n \sum_{j=1}^l q_i r_i d_{ij} v_{ij} \rightarrow \min_{r, d, v} \quad (1)$$

where  $c_j$  – the costs of creation and use of the  $j$ -th terminal (including delivery and storage costs);  $l$  – number of terminals;  $n$  – number of consumers of products;  $q_i$  – volume of cargo for the  $i$ -th consumer;  $r_i$  – transport tariff for the delivery of cargo to the  $i$ -th consumer;  $d_{ij}$  – distance between the  $i$ -th consumer and the  $j$ -th terminal;  $v_{ij}$  – Boolean variable:  $v_{ij} = 1$ , if the  $i$ -th consumer is serviced by the  $j$ -th terminal,  $v_{ij} = 0$  – otherwise.

To solve the problem with the target function of the type (1), approximate methods of directed interruption in the direction of increasing the number of terminals of the network are quite effective [19]. Their time complexity has an order  $O[n^2] r$ , and the error of solutions for tasks with the number of consumers  $n \leq 50$  is less than 5%.

The tasks of structural and topological optimization of closed logistics networks differ consideration of reversible flows from consumers to centers of production or processing (utilization). They predict the optimization of the network, in which, in the first level are centers of production and processing, on the second - terminals, on the third - end consumers. The cost of the network in this case consist of the cost of delivery of a direct flow from

the center of production to terminals  $C_{PT}$ , processing in the terminals of direct  $C_{T1}$  and reverse  $C_{T2}$  flows, the delivery of a direct flow from terminals to consumers  $C_{TS}$ , the delivery of a return flow from consumers to terminals  $C_{ST}$  and the backflow delivery from terminals to the center [18]:

$$C = C_{PT} + C_{T1} + C_{TS} + C_{ST} + C_{T2} + C_{TP} \quad (2)$$

The mathematical model of such a task with the target function (2) in the case of coincidence of production and processing points can be presented in this form:

$$C = \sum_{i=1}^n c_i x_{ii} + \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} \rightarrow \min_x \quad (3)$$

$$X = [x_{ij}] = \begin{cases} x_{ij} \in \{0, 1\}, i, j = \overline{1, n}; \sum_{i=j}^n x_{ij} \geq 1, \forall j = \overline{1, n}; \\ \sum_{j=1}^n \sum_{i=j}^n x_{ij} = 2(n-1) + \sum_{i=1}^n x_{ii}, \end{cases} \quad (4)$$

where  $c_i$  – the costs of the terminal are organized on the base (in the immediate vicinity of)  $i$ -th consumer;  $n$  – number of network elements;  $X = [x_{ij}]$  – Matrix of network elements;  $x_{ij}$  – Boolean variable:  $x_{ij} = 1$ , if there is a direct link between  $i$ -th and  $j$ -th elements of the network;  $x_{ij} = 0$  – in other case;  $c_{ij}$  – shipping costs flow from the network item  $i$  to the item.

The system of restrictions in the problem (3) - (4) describes the entire set of permissible three-level centralized structures with direct and reverse flows.

The basic task in the network of local transportation is the problem of optimizing ring routes, which is reduced to the classical sales problem (SP) or its varieties. The most effective precise algorithms for solving the SP without additional restrictions are based on the method of branches and limits, and heuristic methods use different search schemes in the vicinity of a basic solution [21-22].

The peculiarity of routing tasks in closed logistics systems is that the direct stream can be delivered to the network points and the reversible stream may be collected from the network items by one and the same vehicle. Consideration of this feature requires changes to mathematical models and methods for solving problems of optimization of networks of ring routes [18].

In modern conditions there are relatively rapid changes in flows in all chains of logistics networks. With relatively minor changes in flows, adaptation of existing structures, which requires decision-making in the context of part-time data [23-24]. At the level of micro logistics, this is reduced to the operational solution of the problem of optimizing the network of ring routes at a minimum of costs [25]. With significant changes in flows, there is a need for reengineering macro logistic networks. One of the most important tasks in this case is the problem of optimizing their topological structures in terms of efficiency and expenses. This determines the relevance of

tasks of developing mathematical models and methods for solving problems of reengineering of topological structures of logistics networks.

The **purpose** of this article is the development of a mathematical model and the method of solving the problem of optimizing the topological structures of logistics networks at the stage of reengineering, taking into account the set of structural, topological and functional constraints. To achieve the goal, the following tasks are solved:

- formalization of the problem of systematic optimization of logistics networks;

- development of a mathematical model of the problem of optimizing centralized three-level topological structures of logistics networks at the stage of reengineering;

- development of the method of solving the problem of reengineering of topological structures of centralized three-level logistics networks;

- evaluation of the time complexity of the method of optimizing centralized three-level topological structures of logistics networks.

### Results of the study

Due to a significant territorial dispersion of elements of logistics networks, they are included in the class of territorially distributed objects [18]. Each of these objects as a system formally be submitted in the form of a tuple:

$$s = \langle E, R, G \rangle, \quad (5)$$

where  $E$  is a set of network elements;  $R = [r_{ij}]$  – the set of bonds between elements, which traditionally set the matrix of adjacency:  $r_{ij} = 1$ , if there is a direct connection between the  $i$ -th and  $j$ -th network elements, - otherwise;  $r_{ij} = 0$  – otherwise;  $G$  – a set of places where the network elements are located.

Based on a set of admissible locations of terminals of a system  $G^*$ , each of the variants of the network structure  $\langle E, R \rangle$  may have different topologies  $G \subseteq G^*$ . Each of the network  $\langle E, R, G \rangle$  topologies  $G = \langle G_E, G_R, G_A \rangle$  (where  $G_E$  – topology of elements;  $G_R$  – topology of connections between elements;  $G_A$  – topology of movement of flows) will correspond to its set of functional and consumables:

$$P = \varphi(E, R, G), \quad (6)$$

where  $\varphi$  – a certain reflection.

$$\xi_l(s) = \begin{cases} \bar{a} \cdot (b_l + 1) \cdot \left( 1 - \left( b_l / \left( b_l + \frac{\bar{k}(s)}{\bar{k}_a} \right) \right) \right), & 0 \leq \bar{k}(s) \leq \bar{k}_a; \\ \bar{a} + (1 - \bar{a}) \cdot (b_l + 1) \cdot \left( 1 - \left( b_l / \left( b_l + \frac{\bar{k}(s) - \bar{k}_a}{1 - \bar{k}_a} \right) \right) \right), & \bar{k}_a < \bar{k}(s) \leq 1, \end{cases} \quad (10)$$

where  $\bar{k}_a, \bar{a}$  – normalized values of the coordinates of the gluing point of the components of the function,

According to the results of the analysis of the purpose of creating a network  $s$ , the conditions for its functioning and desirable properties  $P'$  that are evaluated by a plurality of local criteria  $k_i(s)$ ,  $i = \overline{1, m}$ , a subset of elements  $E'(P')$ , connections between elements  $R'(P')$  and topologies on which it can theoretically be implemented can be determined. Taking into account economic, environmental, other restrictions are determined subsets of permissible elements  $E^* \subseteq E'(P')$ , connections  $R^* \subseteq R'(P')$  and topologies  $G'(P')$  that provide the possibility of obtaining necessary properties. Further the most effective variant must be determined on the permissible set of network construction options  $S^* = \{s\}$ :

$$s^o = \arg \max_{s \in S^*} P(s). \quad (7)$$

Due to the incomplete certainty of the requirements for the properties of the logistics network as an efficiency assessment (a function of general utility)  $P(s)$  in (2), it is proposed to use the function of belonging to the fuzzy set "Best Network Option" [26]:

$$\langle \text{Best Network Option} \rangle = \{ \langle s, P(s) \rangle, \quad (8)$$

where  $s \in S^*$  – network construction option;  $P(s)$  – the value of the function of the total utility of the option  $s$ , which assesses the degree of its belonging to a fuzzy set (8).

For quantitative evaluation of the efficiency of the variants of network construction, we use the most versatile additive-multiplicative model, built using Polynoma Kolmogorov Gabor:

$$P(s) = \sum_{i=1}^m \lambda_i \cdot \xi_i(s) + \sum_{i=1}^m \sum_{j=i}^m \lambda_{ij} \cdot \xi_j(s) \cdot \xi_i(s) + \dots, \quad (9)$$

where  $m$  – is a number of local criteria characterizing network properties;  $\lambda_i, \lambda_{ij}$  – weight coefficients of local criteria and their yields  $\lambda_i \geq 0, \lambda_{ij} \geq 0, \xi_l(s)$  – local criterion utility function  $k_l(x)$ ,  $l = i, j$ .

To assess the usefulness of individual partial criteria values, it is proposed to use the gluing function, which has advantages in terms of accuracy and number of computer operations for its calculation compared to the Gauss, Harrington and logistics functions [27]:

$0 \leq \bar{k}_a \leq 1$ ,  $0 \leq \bar{a} \leq 1$ ;  $b_1, b_2$  – parameters, nonlinearity of the initial and final components of the function.

The task of reengineering the topological structure of a centralized three-level closed logistics network with the united items of production and processing is considered in such a statement. Are specified:

- set of elements of the existing network  $I = \{i : i = \overline{1, n}\}$ ;

- an existing version of the topological structure of the network  $s' \in S^*$  given by places of consumers, terminals (coincide with places of location of elements), center (coincides with the place of location of the element  $i = 1$ ), as well as connections between elements, terminals and the center  $[s'_{ij}]$ ,  $i, j = \overline{1, n}$  ( $s'_{ij} = 1$  if there are direct connections between elements  $i$  and  $j$ , and  $s'_{ij} = 0$  otherwise);

- costs of creation (modernization), operation of terminals  $[c_i]$ ,  $i = \overline{1, n}$ , implementation of transportation  $i = \overline{1, n}$   $[c_{ij}]$ , cost of resources that can be reused (or realized) after dismantling of equipment of terminals or vehicles.

It is necessary to determine the best of the set of functional-cost indicators option of the topological structure of the logistics network  $s^o \in S^*$  (7).

Basic restrictions that are taken into account when optimizing topological structures of three-level centralized networks:

- each consumer of the network  $i$ ,  $i = \overline{1, n}$  must be connected to one of the terminals:  $\sum_{j=1}^i s_{ij} + \sum_{j=1}^n s_{ij} = 1$  for all  $i$  for which  $s_{ij} = 0, i = \overline{1, n}$  or directly to the manufacturer (center)  $s_{1j} = 1, i = \overline{1, n}$ ;

$$k_I(s', s) = \Delta C = \sum_{i=1}^n [c_i(1 - s'_{ii}) s_{ii} + d_i s'_{ii} s_{ii} + e_i(1 - s'_{ii}) s'_{ii} - g_i(1 - s_{ii}) s'_{ii}] + \sum_{j=1}^n \sum_{i=j}^n [c_{ij}(1 - s'_{ij}) s_{ij} + d_{ij} s'_{ij} s_{ij} + e_{ij}(1 - s_{ij}) s'_{ij} - g_{ij}(1 - s_{ij}) s'_{ij}] \rightarrow \min_{s \in S^*}, \quad (11)$$

where  $c_i$  – costs for creating the  $i$ -th terminal in a new structure,  $i = \overline{1, n}$ ;  $s'_{ii}, s_{ij}$  – elements of matrices of adjacency (connections) between elements in existing network  $x' = [x'_{ij}]$  and network after reengineering  $x = [x_{ij}]$ ;  $d_i$  – the cost of upgrading the  $i$ -th terminal;  $e_i$  – costs of dismantling the  $i$ -th terminal of the existing network;  $g_i$  – cost of resources, which can be reused (or implemented) after dismantling of equipment  $i$ -th terminal;  $c_{ij}$  – the given costs of transportation between the elements  $i$  and  $j$ ,  $i, j = \overline{1, n}$ .

- more than one consumer must be directly connected to each terminal  $\sum_{j=1}^i s_{ij} + \sum_{j=1}^n s_{ij} > 1$ , for all  $i$ , for

which  $s_{ii} = 1, i = \overline{1, n}$ ;

- each consumer  $i$  connects to terminal  $j$  by minimum of resulted expenses:

$$s_{ii} = 1 \wedge s_{ij} = 1 \rightarrow ij = \arg \min_{i \leq j \leq n} c_{ji} \quad \forall i, j = \overline{1, n};$$

- each of the terminals of the network  $j$  has a direct connection with the center  $s_{ij} = 1 \rightarrow s_{1j} = 1$ ;

- the number of terminals in the network is in the range  $1 \leq \sum_{i=1}^n s_{ii} \leq n/2$ ;

- the total number of direct links in the three-level network structure is equal to  $\sum_{j=1}^n \sum_{i=j}^n s_{ij} = n + \sum_{i=1}^n s_{ii}$ .

Formally set of permissible variants of network constructing can be shown in this form:

$$S = \{s\} = \begin{cases} [s_{ij}], s_{ij} \in \{0, 1\}, i, j = \overline{1, n}, s_{11} = 1; \\ \sum_{i=j}^n s_{ij} \geq 1, \forall j = \overline{1, n}; \\ \sum_{j=1}^n \sum_{i=j}^n s_{ij} = n + \sum_{i=1}^n s_{ii}; \\ s_{ii} = 1 \rightarrow s_{11} = 1 \quad \forall i = \overline{1, n}; \\ s_{ii} = 1 \wedge s_{ij} = 1 \rightarrow ij = \arg \min_{i \leq j \leq n} c_{ji} \quad \forall i, j = \overline{1, n}. \end{cases} \quad (10)$$

Target Function for reengineering of logistics network in the above designations may be shown in the form:

It is considered that the cost of modernization, dismantling and the cost of reused resources do not exceed the expenses of new terminals and the sale of transportation between the elements of the network:

$$e_i < c_i, \quad d_i < c_i, \quad g_i < c_i, \quad i = \overline{1, n}; \\ e_{ij} < c_{ij}, \quad d_{ij} < c_{ij}, \quad g_{ij} < c_{ij}, \quad i, j = \overline{1, n}. \quad (12)$$

Given the desire to minimize the maximum time of delivery of goods to consumers, the criterion of maximizing speed can be submitted in the form of:

$$k_2(s) = \tau(s) = \tau_{PT}(s) + \tau_T(s) + \tau_{TS}(s) \rightarrow \min_{s \in S^*}, \quad (13)$$

where  $\tau_{PT}(s)$  – maximum time of transportation of goods from the center of production to terminals;  $\tau_T(s)$  – maximum goods handling time in terminals;  $\tau_{TS}(s)$  – maximum time for delivery of goods from terminals to consumers  $C_{TS}$ .

Determination of weight coefficients of model (9) is proposed using technology of comparator identification of advantages of decision-making person [26].

In practice, for some types of cargo, the change in delivery time (13)  $\tau(s)$  is uncritical when changing the options for building a logistics network. Based on this, the two-criterion task (10) - (11), (13) is reduced to single-criterion (10) - (11) with an additional restriction on the maximum delivery time of the cargo:

$$\max_{2 < i \leq n} \tau_i(s) \leq \tau^*, \quad (14)$$

where  $\tau_i(s)$  – time of delivery goods to the  $i$ -th consumer;  $\tau^*$  – permissible time of goods delivery.

Analysis of the reported expenditure (11) provides the following conclusions.

If reengineering results in a topological structure that does not contain terminals and connections of a previously existing network, it requires maximum additional costs  $\Delta C$ . Such a topological structure can be obtained by solving a traditional network design problem. In this case, the cost function of the number of terminals in the network  $\Delta C(l)$  is unimodal [28–29].

In the general case of reengineering, the cost function of the number of nodes  $\Delta C(l)$  can be multi-extreme. Analysis of the components of the function (11) showed that the most significant are the costs of delivering goods between elements of the network. At the same time, in the process of reengineering, situations are possible when different parts of the existing topological structure are used with a change in the number of terminals  $1 \leq l \leq n/2$  at each step, which leads to the appearance of local extremes in the function determination interval (11).

The delivery time of the goods  $\tau(l)$ , which is determined by the ratio (13), increases with the number of terminals  $1 \leq l \leq n/2$ . To solve the problem of reengineering the topological structure, it is necessary to use methods that allow you to obtain effective solutions taking into account the monotonicity of the function  $\tau(l)$  (13) and the multi-extremity of the cost function (11) depending on the number of terminals in the network  $1 < l \leq n/2$ .

Total number of topological network reengineering options for number of terminals  $1 < l \leq n/2$  is:

$$N(n) = \frac{1}{2} \sum_{l=1}^n C_n^l = \frac{1}{2} \sum_{l=1}^n \frac{n!}{l!(n-l)!} = 2^n / 2, \quad (15)$$

where  $n$  – number of possible locations of terminals (system elements).

The solution of the problem by the method of complete search of all possible options for building a network for the number of consumers  $n > 50$  is impractical due to the high time complexity of such a method.

To simplify the procedure for finding a global optimal solution of the problem (10) - (11) we use the idea of a method of directed variance proposed for reengineering of territorially distributed objects in the work [28]. Its essence lies in the previous definition of the boundaries of the number of network terminals  $l_{min} \leq l \leq l_{max}$ , which guaranteed corresponds to the optimal solution of the problem. As the lower limit of the number of terminals, we choose  $l_{min} = 1$ , that corresponds to the hypothetical situation when the only terminal is located on the territory of the manufacturer. To determine the upper limit, it is necessary to determine the minimum of maximum additional costs (11):

$$\Delta C_{max}(s, l) \rightarrow \min_{s, l}. \quad (16)$$

With this aim, it is necessary to solve the problem of synthesis of optimal topology of the network (10) – (11) without taking into account existing terminals. Search for optimal task solution is offered to be carried out on a segment  $1 \leq l \leq l_{max}$ , by changing the number of network terminals by rule  $l := l + 1$ .

Thus, to solve the problem by the proposed method, it is necessary to find solutions of two auxiliary tasks to find a minimum of maximum expenses and search for a minimum additional cost of reengineering. Each of the tasks has approximately the same computing complexity as a classical problem of propyling the topological structure. In view of this, the timing of the proposed method has an order  $2O[t(n)]$  (where  $t(n)$  is the timing of the method of solving the classical problem of structural and topological optimization).

The accuracy of the proposed method is determined by the accuracy of methods for solving problems determining the number of terminals and places of their placement.

In order to implement a directional selection of options for optimizing the placement of terminals, we use a coordinatewise optimization method (COM) and an evolutionary method built on the basis of a genetic algorithm (EM-GA).

**Coordinatewise optimization method (COM)** [28, 29]. The essence of the method is to improve the initial variant of the network by alternating optimization of places of possible location of one terminal in fixed placed. This procedure is cyclically repeated until the local extrema of the target function is reached (11).

#### Algorithm

1. Incoming data settings: a set of places for possible terminal locations; number of terminals  $l := 1$ ; index of the current terminal –  $j := 1$ ; current iteration value –

$i := 0$ ; the value of the full pass count for all network elements  $u$ ; an initial value for the best arrangement of the terminals  $w^o$ ; best current criterion value  $\Delta C(w_i^u) := \infty$ .

2. To form an initial version of placement of terminals  $w_i^u$ , distribute consumers between terminals for the minimum transportation costs  $i, j = \arg \min_{i \leq j \leq n} c_{ji} \quad \forall i, j = 1, n$  and calculate the value of the criterion  $\Delta C(w_i^u)$  (11).

3. Increase the value of the meter of iterations  $i := i + 1$ ; for a terminal  $j$  at  $w_i^u$  to change its location at fixed terminal  $l-1$  placement.

4. Calculate the value of the criterion  $\Delta C(w_i^u)$  (11). If  $\Delta C(w_i^u) \leq \Delta C(w_{i-1}^u)$ , to save  $\Delta C(w^u) := \Delta C(w_i^u)$ ,  $w^o := w_i^u$  and go to step 5.

5. Increase the index of the current terminal  $j := j + 1$ . If  $j < l$ , go to step 3, otherwise go to step 6.

6. If  $u = 0$ , to save  $w_i^{u+1} := w_i^u$ ,  $u := u + 1$ ,  $j := 1$  and go to step 3, otherwise - to step 7.

7. If  $\Delta C(w^u) \leq \Delta C(w^{u-1})$ , to save  $w_i^{u+1} := w_i^u$ ,  $u := u + 1$ ,  $j := 1$  and go to step 3, otherwise - to step 8.

8. End of the algorithm operation: a decision was made to place terminals  $w^o$  with the minimum of the considered cost values  $\Delta C(w^o)$ .

**Evolutionary method based on genetic algorithm (EM-GA).** This method iteratively implements

the procedures of inheritance, mutation, selection, crossover [29-30]. Each chromosome of the genetic algorithm displays a plurality of terminal  $w_i$  locations on the iteration  $i$ . The terminal index code is used as the gene, and the function is used as the adaptability function (11).

#### Algorithm

1. Incoming data settings: a set of possible locations of terminals; chromosome population size; probability of mutation  $p$ ; number of iterations (populations)  $i := 0$ ; the quantities of iterations  $i^*$ ; the best current criterion value  $\Delta C(w_i) := \infty$ .

2. Transition to next iteration  $i := i + 1$ . Population formation.

3. Calculate the value of the adaptability function (11) for all chromosomes of the population and choose the best chromosome by the minimum value of the adaptability function  $\Delta C(w_i)$ ;  $w^o := w_i$ .

4. If  $\Delta C(w_{i-1}) > \Delta C(w_i)$ , then  $w^o := w_i$ .

5. If  $i \geq i^*$ , then go to step 9, otherwise go to step 6.

6. Chromosome selection.

7. Perform crossing operations and (with a given probability) mutation.

8. Form a new population and go to step 3.

9. End of algorithm operation: the decision to locate terminals with minimum cost considered  $w^o$  with the minimum of the considered cost values  $\Delta C(w^o)$ .

The results of the experimental study of accuracy and time of solving problems of optimization of topological structures of logistics networks with the number of elements  $n = 15 \div 40$  are given in table 1.

**Table 1.** Results of experimental research of methods

Dimension of the task, $n$	Relative error of the decision, $\delta C(n)$ , %		Time to solve the problem, $t(n)$ , c	
	COM	EN-GA	COM	EN-GA
15	0.17	1.86	0.52	0.22
20	0.32	2.54	3.03	0.71
25	0.56	3.12	5.24	1.86
30	0.72	3.61	11.86	3.91
35	0.88	4.81	21.35	8.13
40	0.99	6.84	35.26	14.01
Average value	0.61	3.86	12.88	4.81

In order to predict the assessments of the proposed methods in solving large-size tasks, approximation of data in table 1 is carried out.

Functions of errors for the proposed modifications of the method of directional selection of options are approximated by polynomials with reliability  $R^2$ :

- COM:  $\delta C(n) = 0.0339n + 0.3268$ ,  $R^2 = 0.9892$ ;

- EN-GA:  $\delta C(n) = 0.0066n^2 - 0.1786n + 3.2418$ ;  $R^2 = 0.9823$ .

The functions of time complexity for the proposed modifications of the method of directed variants are approximated by polynomials with reliability  $R^2$ :

- COM:  $t(n) = 0.0615n^2 - 2.0388n + 17.939$ ,  $R^2 = 0.9969$ ;

– EN-GA:  $\delta C(n) = 0.028n^2 - 0.0083n + 9.2995$  ;  
 $R^2 = 0.9966$  .

Approximations obtained according to table 1 data 1, allow to choose a more effective modification of the method for solving practical problems of large dimension, based on the necessary accuracy, available computing and time resources.

The method based on coordinate optimization procedure COM has substantially higher accuracy, but it is more complex from a computing point of view. The accuracy of the evolutionary method based on the genetic EN-GA algorithm can be increased by increasing the number of iterations.

### Conclusions

As a result of the study, the state of the problem of optimization of global and local transportation routes in macro and micro logistics in systems of ecological and reversing logistics is analyzed. It is established that the tasks of direct and reversing logistics are considered as conditionally independent, which does not allow to receive effective global solutions as a whole. In addition, because of the emergence of new consumers, the change in delivery volumes, the introduction of environmental restrictions there is a need to re-engine the logistics networks, which provides for their radical redevelopment. As a result of decomposition, problems are allocated tasks that require the development of new mathematical models and effective methods for solving their solutions.

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## ОПТИМІЗАЦІЯ ТОПОЛОГІЧНИХ СТРУКТУР ЦЕНТРАЛІЗОВАНИХ ЛОГІСТИЧНИХ МЕРЕЖ У ПРОЦЕСІ РЕІНЖІНІРИНГУ

**Предметом** дослідження в статті є топологічні структури мереж замкнутої логістики. **Мета** роботи – створення математичної моделі та методів вирішення завдань оптимізації топологічних структур централізованих логістичних мереж у процесі їхнього реінжинірингу з урахуванням множини топологічних і функціональних обмежень. У статті вирішуються наступні **завдання**: аналіз сучасного стану проблеми системної оптимізації логістичних мереж та методів її вирішення; формалізація проблеми системної оптимізації логістичних мереж як територіально розподілених об'єктів; розробка математичної моделі задачі оптимізації централізованих тривірневих топологічних структур логістичних мереж на етапі реінжинірингу; розробка методу розв'язання задачі реінжинірингу топологічних структур централізованих тривірневих



логістичних мереж; оцінка часової складності методу оптимізації централізованих трирівневих топологічних структур логістичних мереж. Використовуються такі **методи**: методи теорії систем, методи теорії корисності, оптимізації та дослідження операцій. Отримані такі **результати**: виконано аналіз сучасного стану проблеми системної оптимізації логістичних мереж та методів її вирішення; виконано формалізацію проблеми системної оптимізації логістичних мереж як територіально розподілених об'єктів; розроблено математичну модель задачі реінжинірингу трирівневих топологічних структур логістичних мереж за показниками витрат та оперативності для випадку об'єднаних пунктів виробництва і переробки; розроблено методи спрямованого перебору варіантів побудови логістичної мережі, які використовують процедури покоординатної оптимізації та моделювання еволюції на основі генетичного алгоритму; отримано оцінки точності та часової складності методів оптимізації централізованих трирівневих топологічних структур логістичних мереж. **Висновки**: За результатами дослідження запропонованих методів розв'язання задачі виконано апроксимацію показників їхньої точності та часової складності. На практиці це дозволить обирати більш ефективний метод для розв'язання практичних задач великої розмірності, виходячи з необхідної точності, наявних обчислювальних і часових ресурсів. Метод на основі процедури покоординатної оптимізації має суттєво вищу точність, проте він є більш складним з обчислювальної точки зору. Точність еволюційного методу на основі генетичного алгоритму може бути підвищена шляхом збільшення кількості ітерацій. Практичне використання запропонованих математичної моделі і методів реінжинірингу топологічних структур централізованих мереж замкнутої логістики за рахунок спільного розв'язання задач для прямих і зворотних потоків дозволить скорочувати витрати на реалізацію транспортної діяльності компаній.

**Ключові слова**: замкнена логістика; логістична мережа; оптимізація; реінжиніринг; структура; топологія.

## ОПТИМИЗАЦИЯ ТОПОЛОГИЧЕСКИХ СТРУКТУР ЦЕНТРАЛИЗОВАННЫХ ЛОГИСТИЧЕСКИХ СЕТЕЙ В ПРОЦЕССЕ РЕИНЖИНИРИНГА

**Предметом** исследования в статье является топологические структуры сетей замкнутой логистики. **Цель** работы – создание математической модели и методов решения задач оптимизации топологических структур централизованных логистических сетей в процессе их реинжиниринга с учетом множества топологических и функциональных ограничений. В статье решаются следующие **задачи**: анализ современного состояния проблемы системной оптимизации логистических сетей и методов ее решения; формализация проблемы системной оптимизации логистических сетей как территориально распределенных объектов; разработка математической модели задачи оптимизации централизованных трехуровневых топологических структур логистических сетей на этапе реинжиниринга; разработка метода решения задачи реинжиниринга топологических структур централизованных трехуровневых топологических структур логистических сетей; оценка временной сложности метода оптимизации централизованных трехуровневых топологических структур логистических сетей. Используются следующие **методы**: методы теории систем, методы теории полезности, оптимизации и исследования операций. Получены следующие **результаты**: выполнен анализ современного состояния проблемы системной оптимизации логистических сетей и методов ее решения; выполнена формализация проблемы системной оптимизации логистических сетей как территориально распределенных объектов; разработана математическая модель задачи реинжиниринга трехуровневых топологических структур логистических сетей по показателям затрат и оперативности для случая объединенных пунктов производства и переработки; разработаны методы направленного перебора вариантов построения логистической сети, которые используют процедуры покоординатной оптимизации и моделирования эволюции на основе генетического алгоритма; получены оценки точности и временной сложности методов оптимизации централизованных трехуровневых топологических структур логистических сетей. **Выводы**: По результатам исследования предложенных методов решения задачи выполнены аппроксимация показателей их точности и временной сложности. На практике это позволит выбирать более эффективный метод для решения практических задач большой размерности, исходя из требуемой точности, имеющихся вычислительных и временных ресурсов. Метод на основе процедуры покоординатной оптимизации имеет существенно более высокую точность, однако он является более сложным с вычислительной точки зрения. Точность эволюционного метода на основе генетического алгоритма может быть повышена путем увеличения количества итераций. Практическое использование предложенных математической модели и методов реинжиниринга топологических структур централизованных сетей замкнутой логистики за счет совместного решения задач для прямых и обратных потоков позволит сокращать расходы на реализацию транспортной деятельности компаний.

**Ключевые слова**: замкнутая логистика; логистическая сеть; оптимизация; реинжиниринг; структура; топологія.

### Бібліографічні описи / Bibliographic descriptions

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