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METHODS OF COMPARATIVE STATICS AND DYNAMICS IN THE THEORY OF ECONOMIC CYCLES

The **subject** of this work is the problem of describing the dynamic behavior of the price in the market of one product. The typical balance of interaction of supply and demand functions depending on the price is considered. The dynamic model of price evolution is based on the assumption that the demand function at a given time depends on the supply function at all previous points in time, i.e. there is a process with an aftereffect. The core of the integral transformation is a characteristic of the second order, which can initiate periodic regimes in price variables. The **aim** of the work is to synthesize a mathematical model of price changes in the market of one product and study the stability of its equilibrium states with the manifestation of the structure of marginal cycles. The **task** of the study is to demonstrate the degree of connection between the problem of stability of equilibrium and the problem of obtaining fruitful results in comparative statics. This duality is the principle of conformity of P. Samuelson. The basic mathematical model of the studied process of price dynamics is a system of two nonlinear differential equations of the first order. The research **methods** are the nonlinear theory of analysis of dynamical systems, the mathematical theory of stability of systems of differential equations, the conceptual apparatus of analysis of typical bifurcations of birth (death) of the boundary cycle, known as the Andronov-Hopf bifurcation. As a **result** of a detailed analysis of the properties and parameters of self-oscillating modes, a double cycle is revealed, i.e. there is a fact of coexistence around the equilibrium state of stable and unstable limit cycles. Subsequent mathematical transformations prove that the line of demarcation of these two cycles is completely determined by the static parameters of the studied system, which illustrates the mechanism of action of the principle of conformity of P. Samuelson. **Conclusions:** on the example of the functioning of the labor market, a comprehensive analysis of the stability of two positive equilibrium states, which characterize the effects of substitution and income. Computer simulations are used to perform computational experiments that demonstrate self-oscillating modes of price changes. As a result of the analysis of the obtained numerical results, it is possible to draw a conclusion about the stability of the limit cycle in the vicinity of the equilibrium state, which corresponds to the substitution effect.

Keywords: market; demand; supply; price; dynamics; self-oscillation; bifurcation; stability; principle of conformity.

Introduction

In each problem of economic theory, some variables, such as prices and quantities, are unknown, which must be determined. Their values arise as solutions of the given set of relations for finding the necessary unknowns. The most important thing is to conduct the analysis in such a way as to determine what quantitative and qualitative changes occur with our variables when the external data shifts in the functional ratios of economic balance sheets. The usefulness of our approach arises from the fact that with the help of analytical methods we can determine the nature of the evolution of unknown variables caused by the growth of one or more parameters. This is the narrative of the method of comparative statics, which means the study of changes in the economic system in the presence of a transition from one equilibrium to another without taking into account the transition process associated with its establishment. The task of comparative statics is to study the process of determining the equilibrium points of unknown variables with known functional relations and various parameters. Thus, in the simplest market of one product, two independent ratios of supply and demand by their intersection determine the equilibrium prices and volumes.

If we consider the pricing process as dynamic, then the problem of stability of equilibrium states of economic systems and objects comes to the fore. The interaction of these conditions with the problems of comparative statics is the phenomenon which, in connection with the historical tradition, is called the principle of conformity of P. Samuelson. But the problem of stability of the equilibrium state cannot be realized without recourse to the dynamic theory of systems. That is, comparative

statistical analysis must be substantiated by dynamic considerations within the synergetic paradigm.

Analysis of previous publications

In the modern economic literature, it is reasonably believed that a significant contribution to the coverage of this issue belongs to the Nobel Laureate in Economics P. Samuelson, which is reflected in a large number of author's publications of the second half of the previous century. The most significant can be considered a series of works [1-3], which formulates the methodological principles of comparative statics and its relationship with the problem of stability of the equilibrium of the microeconomic system. The following development and the corresponding generalizations are reflected in the fundamental monograph [4], which is a classic work on this issue. It should be noted the works of Milgrom, Shannon [5], Sato, Yano [6, 7] and Savvateev, Kukushkin [8], which consider in addition to the ordinal approach to the problem of optimality and the relationship of advantage, the problem of stochastic description of models of comparative dynamics in the further development of methodology Samuelson. From domestic works it is possible to specify works of Alimpiev [9] and Matselyukh [10] who use in the researches the corresponding theoretical device.

As a result of the analysis of the considered works it can be considered that they use linearized mathematical models of economic systems, which do not take into account the cyclical nature of economic development, as well as nonlinearity, no equilibrium and multivariate evolution of complex systems. In addition, it should be noted the actual absence of scientific papers, which

consider the comparative dynamics and mathematical theory of economic cycles from the standpoint of a single formalism.

The **aim** of the work is to synthesize a mathematical model of price changes in the market of one product and study the stability of its equilibrium states with the manifestation of the structure of marginal cycles.

Research methods are focused on the conceptual apparatus of the theory of nonlinear dynamical systems and the use of the methodology of economic synergetics.

Results of the study.

The presentation of the main research results is focused on establishing a functional relationship between the principle of conformity of Samuelson and the theory of economic cycles. There are many interesting and fruitful aspects of dynamics, which are not always directly related to cyclicity, but important for understanding the processes classified within the general economic theory. The most significant among the various sectors of the dynamics was the one that examines the fluctuations of employment, income and general business activity.

In our time, the use of the terms "exogenous" and "endogenous" has become acceptable for the development of the theory of business cycles. The first term refers to theories that explain the reason for the existence of a cycle in some external non-economic phenomena with quasi-periodic changes that give rise to cyclical behavior of economic quantities. From an analytical point of view, the exogenous theory in its extreme form is similar to "forced" periodic motion, where the economic system reacts instantly to external perturbation.

On the other side is the endogenous theory of the cycle. This category includes various theories that distinguish between monetary factors, inventories, the principle of acceleration, etc. The components of the economic system are determined by dynamic equations that link different time periods with the corresponding generation of periodic fluctuations. If the cycle is realized, then the rise changes to decline, depression changes to recovery, etc. But it is important that purely endogenous theories cannot explain the exact or approximate value of the cycle amplitude other than the influence of the initial conditions. This interpretation of the parameters of the periodic process is characteristic of linear systems. An alternative to this may be to abandon the assumption of linearity, despite the associated certain mathematical difficulties. It is essential to us that only nonlinear systems lead to a theory that explains fluctuations of a fixed amplitude independent of the initial deviation. For example, a nonlinear system may have an unstable equilibrium state such that with a small deviation from the position of the stationary level, increasing oscillations begin. But instead of galloping to infinity, a constant amplitude is eventually achieved and maintained. This periodic movement can be stable in the sense that each subsequent deviation can lead to a movement that will go to the given periodic mode, either from below or from above. A classic example of such behavior is a nonlinear "web" model of a single product market.

Let's consider in more detail the dynamic model of interaction of supply and demand in the market of one product. That is, the basic variable is the price of the product $p = p(t)$, where t – is a continuous time. There are also supply $D = D(p)$ and demand $S = S(p)$ functions, depending on the price p .

The dynamics of the interaction of supply and demand functions is determined using the following functional equation:

$$D(p(t)) = \int_0^t K(t-\tau)S(p(\tau))d\tau, \quad (1)$$

where $K(t-\tau)$ is the given function of two arguments.

Equation (1) means that demand at the appropriate time depends on the weighted supply for all previous time. The function $K(t,\tau)$ has the meaning of the weight function "dynamic memory" of previous moments of time and is descending. As a weight function we choose a linear combination of two exponents, which has a representation in operator form:

$$K(\lambda) = \frac{b_1\lambda + b_0}{\lambda^2 + a_1\lambda + a_0}. \quad (2)$$

To ensure normalization conditions $\int_0^{+\infty} K(x)dx = 1$ equality $b_0 = a_0$ must be fulfilled.

Thus, all coefficients a_0, a_1, b_1 should be considered positive numbers, and the function $K(\lambda)$ is the operator form of the second-order process. If λ we can consider a differential operator, then (1) will take a form:

$$\begin{aligned} \frac{d^2(D(p(t)))}{dt^2} + a_1 \frac{d(D(p(t)))}{dt} + a_0 D(p(t)) = \\ = b_1 \frac{d(S(p(t)))}{dt} + a_0 S(p(t)). \end{aligned} \quad (3)$$

Functional relation (3) is the basis for building a mathematical model of price changes in the market of one product, if there are explicit expressions for the supply $S(p)$ and demand $D(p)$ function.

With respect to the demand function $D(p)$, we can assume that there is a linear decreasing function of the price argument $D(p) = d_0 - d_1 p$

The supply function $S(p)$ has a specific nonlinear nature of behavior in different price segments of the market and has intervals of growth and decline, which are due to the behavior of sellers (or producers) of goods. We will choose an explicit form in the form of a cubic parabola:

$$S(p) = -\frac{S_3}{6} p^3 + S_2 \frac{p^2}{2} + S_1 p - S_0, \quad (5)$$

where parameters S_0, S_1, S_2, S_3 , are also positive numbers.

Functional equation (3) has stationary solutions that can be determined by the relation:

$$D(p) = S(p).$$

If we use explicit expressions for $S(p)$ and $D(p)$, i.e. formulas (4) and (5), we obtain an algebraic cubic equation with respect to the equilibrium price:

$$S_3 \frac{p^3}{6} - S_2 \frac{p^2}{2} - (S_1 + d_1)p + S_0 + d_0 = 0. \quad (6)$$

Equation (6) can have one or three real solutions depending on the ratios between the coefficients.

To further develop the mathematical model of price evolution in the market of one product, it is advisable to introduce a new variable $\tilde{p}(t) = p(t) - p^*$, which is the

$$\ddot{\tilde{p}} + \left(a_1 + b_1 \cdot \frac{S'}{d_1} \right) \dot{\tilde{p}} + a_0 \left(1 + \frac{S'}{d_1} \right) \tilde{p} + a_0 \frac{S''}{d_1} \cdot \frac{\tilde{p}^2}{2} + b_1 \frac{S''}{d_1} \cdot \tilde{p} \cdot \dot{\tilde{p}} + a_0 \frac{S'''}{d_1} \cdot \frac{\tilde{p}^3}{6} + b_1 \frac{S'''}{d_1} \cdot \frac{\tilde{p}^2 \cdot \dot{\tilde{p}}}{2} = 0, \quad (7)$$

or

$$\ddot{\tilde{p}} + (a_1 - b_1 \cdot \eta_1) \dot{\tilde{p}} + a_0(1 - \eta_1) \tilde{p} - a_0 \eta_2 \cdot \frac{\tilde{p}^2}{2} - b_1 \eta_2 \cdot \tilde{p} \cdot \dot{\tilde{p}} - a_0 \eta_3 \cdot \frac{\tilde{p}^3}{6} - b_1 \eta_3 \cdot \frac{\tilde{p}^2 \cdot \dot{\tilde{p}}}{2} = 0, \quad (8)$$

where $\eta_1 = -\frac{S'}{d_1}$, $\eta_2 = -\frac{S''}{d_1}$, $\eta_3 = -\frac{S'''}{d_1}$.

The parameter η_1 is essentially the relative elasticity of "supply" to "demand", and η_2 and η_3 , accordingly, its first and second derivative at price.

$$\begin{cases} \dot{p}_1 = p_2; \\ \dot{p}_2 = a_0(\eta_1 - 1)p_1 + (b_1\eta_1 - a_1) \cdot p_2 + a_0\eta_2 \cdot \frac{p_1^2}{2} + b_1\eta_2 p_1 p_2 + a_0\eta_3 \cdot \frac{p_1^3}{6} - b_1\eta_3 \cdot \frac{p_1^2 \cdot p_2}{2}. \end{cases} \quad (9)$$

The matrix of the linear part (9) has the form:

$$A = \begin{pmatrix} 0 & 1 \\ a_0(\eta_1 - 1) & b_1\eta_1 - a_1 \end{pmatrix}.$$

And satisfies the characteristic equation:

$$\lambda^2 + (a_1 - b_1\eta_1)\lambda + a_0(1 - \eta_1) = 0. \quad (10)$$

Solutions (10) $\lambda_{1,2}$ have a really negative part under the conditions:

$$a_1 - b_1\eta_1 > 0, \quad a_0(1 - \eta_1) > 0.$$

In this case, we can assume that the equilibrium state p^* is stable. Equilibrium point – a stable node or focus.

Let's consider in more detail the situation when $a_1 \approx b_1\eta_1$. Suppose, for example, that there is a connection $a_1 = b_1\eta - \mu$ where μ – is the small alternating value. Thus, the quadratic equation (10) takes the form:

$$\lambda^2 - \mu\lambda + \omega^2 = 0, \quad (11)$$

deviation of the price from its equilibrium state p^* . The value p^* is determined by the cubic equation (6). We also introduce the necessary notation of the derivatives of the supply function at the equilibrium point:

$$S'(p) = -S_3 \frac{p^{*3}}{2} + S_2 p^* + S_1, \\ S''(p^*) = -S_3 p^* + S_2, \quad S'''(p^*) = -S_3.$$

After the necessary transformations of the functional relation (3) taking into account formulas (4), (5), (6) we obtain a nonlinear differential equation of the second order with respect to the variable $\tilde{p}(t)$:

With the help of new variables $p_1 = \tilde{p}$, $p_2 = \dot{\tilde{p}}$, differential equation (8) will be transformed into a system of two first-order differential equations.

where $\omega^2 = a_0(1 - \eta_1) > 0$.

Obviously, the state of equilibrium can be both stable and unstable focus. For the existence of a periodic regime such as a limit cycle in the vicinity of the equilibrium state p^* , it is necessary to check the conditions of Hopf's theorem [11].

At $\mu = 0$ characteristic numbers $\lambda_{1,2} = \pm i\omega, i^2 = -1$. (11)

Let's find the derivative of equation (11) by parameter μ :

$$2\lambda \frac{d\lambda^*}{d\mu} - \mu \frac{d\lambda}{d\mu} - \lambda = 0.$$

If $\mu = 0$, then $\frac{d\lambda}{d\mu} = \frac{1}{2} \neq 0$.

This means that in system (9) there may be a limit cycle in the vicinity of the equilibrium state p^* , which is a composite focus.

Before applying bifurcation formulas to determine the required properties of the limit cycle, it is necessary to bring the system of two differential

equations (9) to the form of a model system in the normal form of Poincare. To do this, use replacements $p_1 = y_1$, $p_2 = -\omega \cdot y_2$, $\tau = \omega \cdot t$. After the necessary transformations, the system (9) is transformed into a form:

$$\begin{cases} \frac{dy_1}{d\tau} = -y_2; \\ \frac{dy_2}{d\tau} = y_1 + c_{20}y_1^2 + c_{11} \cdot y_1 \cdot y_2 + c_{30} \cdot y_1^3 + c_{21}y_1^2y_2, \end{cases} \quad (12)$$

where

$$c_{20} = \frac{2\eta_2}{\eta_1 - 1}, \quad c_{11} = \frac{b_1\eta_2}{\omega}, \quad c_{30} = \frac{2b_1\eta_3}{\eta_1 - 1}, \quad c_{21} = \frac{2b_1\eta_3}{\omega}.$$

To determine the stability conditions of the investigated limit cycle, it is necessary to find the so-called first Lyapunov quantity $l_1(0) = \frac{b_1\eta_2}{\omega}$. Using the appropriate formulas, we obtain the following [12]:

$$l_1(0) = \frac{1}{8}(c_{20} \cdot c_{11} + c_{21}). \quad (13)$$

Taking into account the existing form of coefficients, expression (13) will be transformed into a form.

$$l_1(0) = \frac{b_1}{16\omega} \left(\eta_3 + \frac{\eta_2^2}{\eta_1 - 1} \right). \quad (14)$$

From relation (14) we can draw the following conclusions: if $\eta_3 < \frac{\eta_2^2}{1 - \eta_1}$, then the limit cycle is stable in

the vicinity of the equilibrium state, otherwise $\eta_3 > \frac{\eta_2^2}{1 - \eta_1}$

– there is an unstable limit cycle.

In the first case, the mode of occurrence of self-oscillations is called "soft", and in the second - "hard" with a catastrophic loss of stability of equilibrium.

The case when $l_1(0) = 0$ or $\eta_3 = \frac{\eta_2^2}{1 - \eta_1}$ is the most interesting in terms of the diversity of the dynamic behavior of the system (9) in the vicinity of the steady state p^* . For further analysis, we need an analytical expression for the second Lyapunov quantity $l_2(0)$. From [12] we obtain:

$$l_2(0) = \frac{c_{21}}{48}(2c_{11}^2 - 11 \cdot c_{30}), \quad (15)$$

$$l_2(0) = \frac{b_1\eta_3}{96\omega} \left(\frac{2b_1^2\eta_2^2}{\omega^2} + \frac{11 \cdot \eta_2^2}{6(1 - \eta_1)^2} \right). \quad (16)$$

Suppose that $l_1 = \nu -$ is a small variable in the vicinity of zero. In expression (16) for l_2 the sign is determined by the parameter η_3 . If $\eta_3 = \frac{S_3}{d_1} > 0$, then $l_2 > 0$ for all parameters of the formula (16).

In polar coordinates, there are two independent differential equations for the amplitude ρ and phase Ψ of cycles [12]:

$$\begin{cases} \dot{\rho} = \rho(\mu + \nu\rho^2 + l_2\rho^4), \\ \dot{\Psi} = \omega. \end{cases} \quad (17)$$

The positive stationary solutions for the first equation (17) satisfy the biquadratic equation

$$\mu + \nu\rho^2 + l_2\rho^4 = 0. \quad (18)$$

Equation (18) may have zero, one or two positive solutions for the amplitudes of the cycles.

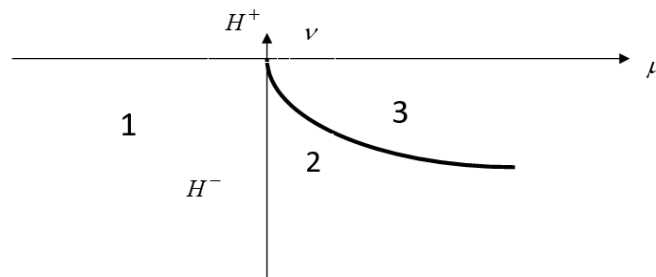


Fig. 1. Bifurcation diagram of a double cycle

Fig. 1 shows the corresponding bifurcation diagram. The line $H_1 = \{(\mu, \nu) : \mu = 0\}$ refers to a typical bifurcation of the birth (death) of the boundary cycle, known as the Andronov-Hopf bifurcation [12]. The state of equilibrium is stable at $\mu < 0$ and unstable for $\mu > 0$. If we move along the line $\mu = 0$ to the points where $\nu < 0$, then on the phase plane from the composite focus

of the second order an unstable boundary cycle (rough) will be born, and the focus itself becomes non-rough stable. When we leave, crossing the line H^- , to region 2, a stable boundary cycle is born from a stable composite focus. In region 2, two cycles coexist simultaneously: stable and unstable, which merge and disappear on the line $T = \{(\mu, \nu) : \nu^2 = 4l_2\mu, \nu < 0\}$. The T line

corresponds to the bifurcation of the double cycle. Further in region 3 boundary cycles are absent [13].

Fig. 2 shows the region of coexistence of two boundary cycles.

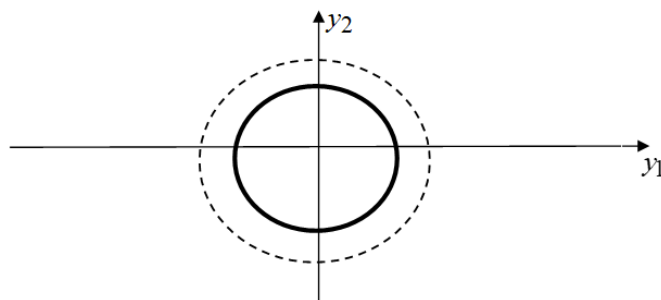


Fig. 2. Stable and unstable boundary cycles

The outer cycle is unstable and the inner cycle is stable.

Let us return to the relation that ensures the equality of zero of the first Lyapunov quantity, i.e.

$$\eta_3 = \frac{\eta_2}{1 - \eta_1}. \quad (19)$$

Using the explicit form η_1, η_2, η_3 , it is easy to obtain:

$$-S'''(S' + d_1) = (S'')^2. \quad (20)$$

Given the dependence of all derivatives of the supply function on the equilibrium price p^* , we have a quadratic algebraic equation to find p^* .

$$\left(p - \frac{S_2}{S_3} \right)^2 = \frac{1}{3} \left[\left(\frac{S_2}{S_3} \right)^2 + \frac{2(S_1 + d_1)}{S_3} \right]. \quad (21)$$

We introduce the following notation $\frac{S_2}{S_3} = \gamma, \frac{S_1 + d_1}{S_3} = \beta, \frac{S_0 + d_0}{S_3} = \alpha$. In this case, equation (21) is transformed into a form:

$$(p - \gamma)^2 = \frac{1}{3}(\gamma^2 + 2\beta), \quad (22)$$

and the cubic equation (6) is written in the form.

$$p^3 - 3\gamma p^2 - 6\beta\gamma + 6\alpha = 0. \quad (23)$$

Let $\bar{p} = p - \gamma$. Then (22) and (23) are transformed as follows:

$$\bar{p}^2 = \frac{1}{3}(\gamma^2 + 2\beta); \quad (24)$$

$$\bar{p}^3 - 3(\gamma^2 + 2\beta)\bar{p} + 2(3\alpha - 3\beta\gamma - \gamma^3) = 0. \quad (25)$$

From the two equations (24) and (25) it is quite easy to obtain an analytical solution for \bar{p} and to find the relationship between the positive coefficients α, β, γ . If we put $\beta = \gamma^2$, we get the first root $\bar{p}_1^* = \gamma$ leading to the

connection $\alpha = \frac{8\gamma^3}{3}$, and equation (25) can be factorized as follows:

$$(\bar{p} - \gamma)(\bar{p}^2 + \gamma\bar{p} - 8\gamma^2) = 0. \quad (26)$$

Solutions (26) are:

$$\bar{p}_1^* = 2\gamma, \quad p_{2,3}^* = \gamma \left(\frac{-1 \pm \sqrt{33}}{2} \right). \quad (27)$$

Returning to the variable p , we have.

$$\bar{p}_1^* = 2\gamma, \quad p_{2,3}^* = \frac{\gamma(1 \pm \sqrt{33})}{2}. \quad (28)$$

Among the solution (27) there is one negative, i.e. having no economic meaning. This fact can be explained by simple formalisms in the task of the analytical structure of supply and demand functions. Thus, two positive equilibrium states remain for further consideration:

$$\bar{p}_1^* = 2\gamma, \quad p_2^* = \frac{\gamma(1 + \sqrt{33})}{2}, \quad p_1^* < p_2^*.$$

If, for example, the existence of these two stationary prices is interpreted in relation to the labor market, the following remarks should be made. For geometric reasons, it is clear that the supply function changes its behavior depending on the price $p(t)$ (in this case, the level of wages). At the first stage, wage growth initiates an increase in labor supply (the number of people willing to work increases, labor intensity increases). There is a demonstration of the so-called "substitution effect". The logic of employee behavior is as follows: the higher the price of labor, the more profitable to sell it, reducing dependence on other income. The supply line $S(p)$, after passing its maximum, changes its bow in the other direction. There is a slow decline in labor supply with a continuous increase in its price, which can be explained by the "income effect". The essence of this phenomenon is that the income received by the employee exceeds the amount needed to meet basic living needs. The demand function $D(p)$ is a "traditionally" decreasing price function, which gives two positive points of intersection

of a line D and a parabola S . That is, we have an equilibrium price $\bar{p}_1^* = 2\gamma$ that corresponds to the

"substitution effect" and a price $p_2^* = \frac{\gamma(1 \pm \sqrt{33})}{2}$, that describes the "income effect".

It is here, analyzing the line separating the two graphical cycles, we found that its description depends entirely on the static parameters of the supply and demand functions. This is a clear connection between the methodology of comparative statics and the stability of nonlinear models of economic dynamics [14, 15].

For a system of two differential equations (12) for parameter values: $c_{20} = \frac{4}{3\gamma}$, $c_{11} = -\gamma \frac{b_1}{\omega}$, $c_{30} = -\frac{1}{9\gamma^2}$,

$$c_{21} = \frac{b_1}{\omega}, \omega = \gamma, S_3 = d_1.$$

Computer calculations of transients and phase portraits of the dynamics of price changes at different values of parameters γ and b_1 were performed.

In fig. 3. a, 3.b, 3.c the values of the price, speed of change of the price and dependence of speed of change of the price on the price at parameters $\gamma = \frac{1}{2}$, $b_1 = \frac{8}{5}$ are given.

Fig. 4. a, 4.b, 4.c provide the characteristics of the limit cycle for the parameters $\gamma = \frac{1}{2}$, $b_1 = \frac{16}{5}$.

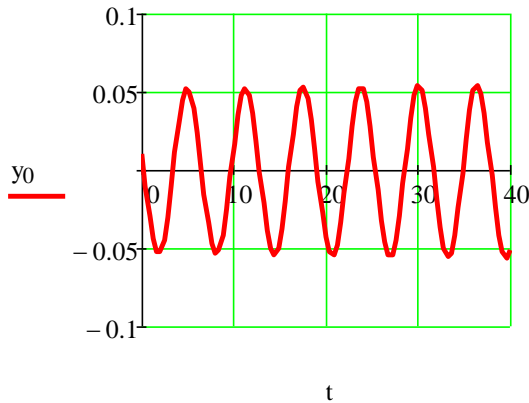


Fig. 3. a. Price fluctuations

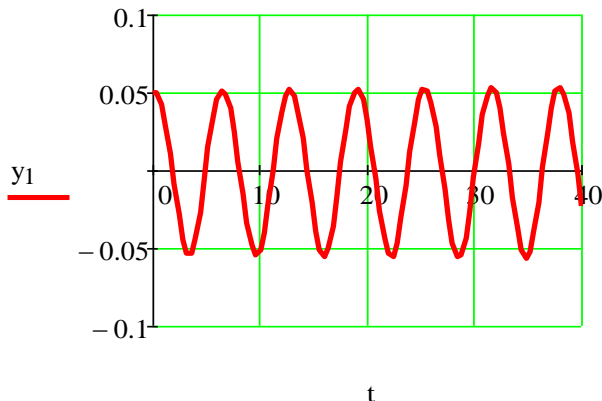


Fig. 3. b. Fluctuations in the speed of price changes

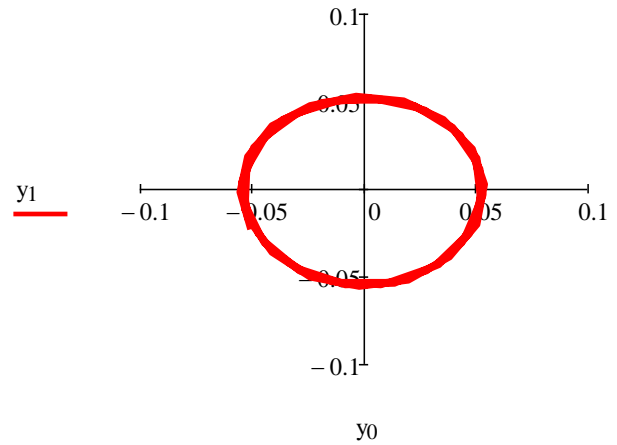


Fig. 3. c. Phase portrait of the boundary cycle

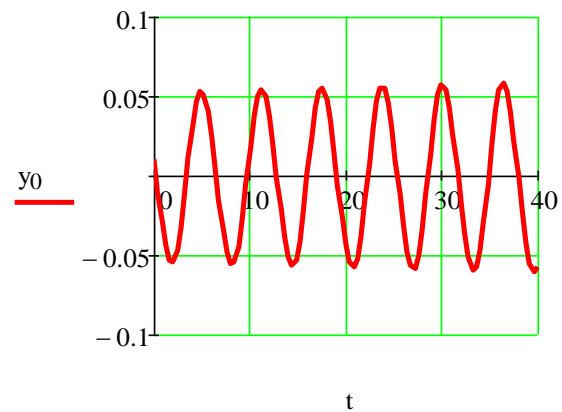


Fig. 4. a. Price fluctuations

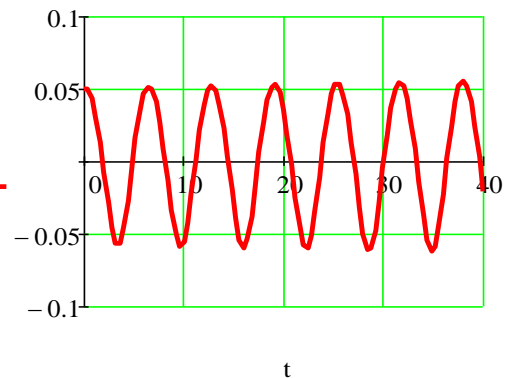


Fig. 4. b. Fluctuations in the speed of price changes

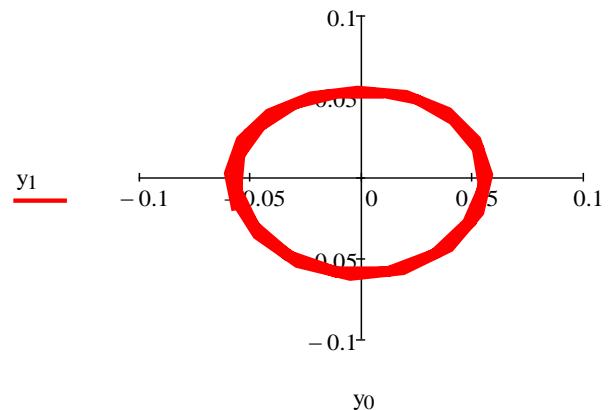


Fig. 4. c. Phase portrait of the boundary cycle

The graphical illustrations of self-oscillating modes in the vicinity of the equilibrium state, which corresponds to the substitution effect, demonstrate the stability of the observed two-fold limit cycle at different dynamic parameters.

Conclusions

Rejecting the assumption of linearity, this paper examines in detail the evolution of price changes in the market of one product within the model mechanism of "supply and demand". Using the methodology of stability analysis of nonlinear dynamical systems, the limit cycles

around the equilibrium states are identified and their basic characteristics are given. The connection between the existence of double cycles and statistical parameters of equilibrium prices is found. Appropriate computer calculations of self-oscillating modes are made.

In further research, it is advisable to introduce an autonomous component into the demand-side model, which can be a characteristic of seasonal market fluctuations. In this case, the system under study becomes nonstationary, which provides a theoretical opportunity to observe the so-called bifurcations of the doubling of the period and chaotic behavior in price evolution.

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МЕТОДИ ПОРІВНЯЛЬНОЇ СТАТИКИ І ДИНАМІКИ У ТЕОРІЇ ЕКОНОМІЧНИХ ЦИКЛІВ

Предметом наданої роботи є проблема опису динамічної поведінки ціни на ринку одного товару. Розглянуто типовий баланс взаємодії функцій попиту та пропозиції у залежності від ціни. Динамічна модель цінової еволюції базується на припущенні, що функція попиту у даний момент часу залежить від функції пропозиції в усі попередні моменти часу, тобто має місце процес з післядією. У якості ядра інтегрального перетворення вибрана характеристика другого порядку, яка може ініціювати періодичні режими у цінових змінних. **Метою** роботи є синтез математичної моделі цінових змін на ринку одного товару та дослідження стійкості її рівноважних станів з виявом структури граничних циклів. **Завданням** наступного дослідження є демонстрування ступеню зв'язку проблеми стійкості рівноваги з проблемою отримання плідних результатів у порівняльній статистиці. Ця дуальність являє собою принцип відповідності П. Самуельсона. Базовою математичною моделлю досліджуваного процесу цінової динаміки є система двох нелінійних диференціальних рівнянь першого порядку. **Методами** досліджень є нелінійна теорія аналізу динамічних систем, математична теорія стійкості систем диференціальних рівнянь, понятійний апарат аналізу типових біфуркацій народження (загибелі) граничного циклу, відома як біфуркація Андронова-Хопфа. У **результаті** детального аналізу властивостей та параметрів автоколивальних режимів виявлений дворазовий цикл, тобто є факт співіснування навколо рівноважного стану стійкого та нестійкого граничних циклів. Подальшими математичними перетвореннями доведено, що лінія розмежування цих двох циклів повністю визначається статичними параметрами досліджуваної системи, яка ілюструє механізм дії принципу відповідності П. Самуельсона. **Висновки:** на прикладі функціонування ринку праці виконаний вичерпний аналіз стійкості двох додатних рівноважних станів, які характеризують ефекти заміщення та доходу. За допомогою комп'ютерного моделювання реалізовані обчислювальні експерименти, які демонструють автоколивальні режими еволюції цінових змін. У наслідку аналізу отриманих чисельних результатів можна зробити висновок про стійкість граничного циклу у околі рівноважного стану, який відповідає ефекту заміщення.

Ключові слова: ринок; попит; пропозиція; ціна; динаміка; автоколивання; біфуркація; стійкість; принцип відповідності.

МЕТОДЫ СРАВНИТЕЛЬНОЙ СТАТИКИ И ДИНАМИКИ В ТЕОРИИ ЭКОНОМИЧЕСКИХ ЦИКЛОВ

Предметом предоставленной работы является проблема описания динамического поведения цены на рынке одного товара. Рассмотрен типичный баланс взаимодействия функций спроса и предложения в зависимости от цены. Динамическая модель ценовой эволюции базируется на предположении, что функция спроса в данный момент времени зависит от функции предложения во все предыдущие моменты времени, то есть имеет место процесс с последствием. В качестве ядра интегрального преобразования выбрана характеристика второго порядка, которая может инициировать периодические режимы в ценовых изменениях. **Целью** работы является синтез математической модели ценовых изменений на рынке одного товара и исследования устойчивости ее равновесных состояний с определением структуры предельных циклов. **Задачей** настоящего исследования является демонстрация степени связи проблемы устойчивости равновесия с проблемой получения плодотворных результатов в сравнительной статике. Эта дуальность представляет собой принцип соответствия П. Самуельсона. Базовой математической моделью изучаемого процесса ценовой динамики является система двух нелинейных дифференциальных уравнений первого порядка. **Методами** исследования является нелинейная теория анализа динамических систем, математическая теория устойчивости систем дифференциальных уравнений, понятійний апарат анализа типичных биуркаций рождения (гибели) предельного цикла, известная как биуркация Андронова-Хопфа. В **результате** детального анализа свойств и параметров автоколебательных режимов обнаружен двукратный цикл, то есть факт сосуществования вокруг равновесного состояния устойчивого и неустойчивого предельных циклов. Последующими математическими преобразованиями доказано, что линия разграничения этих двух циклов полностью определяется статическими параметрами исследуемой системы, иллюстрирующая механизм действия принципа соответствия П. Самуельсона. **Выводы:** на примере функционирования рынка труда выполнен исчерпывающий анализ устойчивости двух положительных равновесных состояний, характеризующих эффекты замещения и дохода. С помощью компьютерного моделирования реализованы вычислительные эксперименты, демонстрирующие автоколебательные режимы эволюции ценовых изменений. В результате анализа полученных численных расчетов можно сделать вывод об устойчивости предельного цикла в окрестности равновесного состояния, которое соответствует эффекту замещения.

Ключевые слова: рынок; спрос; предложение; цена; динамика; автоколебания; биуркация; устойчивость; принцип соответствия.

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