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A METHOD FOR SOLVING THE CANONICAL PROBLEM OF TRANSPORT LOGISTICS IN CONDITIONS OF UNCERTAINTY

Subject. The canonical task of transport logistics in the distributed system "suppliers - consumers" is considered. **Goal.** Development of an accurate algorithm for solving this problem according to the probabilistic criterion in the assumption of the random nature of transportation costs has been done. **Tasks.** 1. Development of an exact method for solving the problem of finding a plan that minimizes the total cost of transportation in conditions when their costs are given by their distribution densities. 2. Development of a method for solving the problem when the distribution density of the cost of transportation is not known. **Methods.** A computational scheme for solving the problem is proposed, which is implemented by an iterative procedure for sequential improvement of the transportation plan. The convergence of this procedure is proved. In order to accelerate the convergence of the computational procedure to the exact solution, an alternative method is proposed based on the solution of a nontrivial problem of fractional nonlinear programming. The method reduces the original complex problem to solving a sequence of simpler problems. The original problem is supplemented by considering a situation that is important for practice when, in the conditions of a small sample of initial data, there is no possibility of obtaining adequate analytical descriptions for the distribution densities of the random costs of transportation. To solve the problem in this case, a minimax method is proposed for finding the best transportation plan in the most unfavorable situation, when the distribution densities of the random cost of transportation are the worst. To find such densities, the modern mathematical apparatus of continuous linear programming was used. **Results.** A mathematical model and a method for solving the problem of transport logistics in conditions of uncertainty of the initial data are proposed. The desired plan is achieved using the solution of the fractional nonlinear programming problem. **Conclusions:** The problem of forming a transportation plan is considered, provided that their costs are random values. Also, a method for solving the problem of optimization of transportation for a situation where the density of distribution of random cost cannot be correctly determined is considered.

Keywords: transport linear programming problem; random cost of transportation; fractional nonlinear optimization.

Introduction

In the totality of management tasks of the logistics complex "production – delivery - consumption", transport tasks take the central place [1-3].

The transport linear programming problem is traditionally formulated as follows [1-3]. There are m suppliers of a homogeneous product and n consumers of this product. A vector $A = \begin{pmatrix} a_1 & a_2 & ... & a_i & ... & a_m \end{pmatrix}$ is known, the components of which fix the capabilities of suppliers, a vector $B = \begin{pmatrix} b_1 & b_2 & ... & b_j & ... & b_n \end{pmatrix}$, the components of which determine the demand of consumers, as well as a matrix $C = \begin{pmatrix} c_{ij} \end{pmatrix}$ that specifies the cost of delivering a unit of a product from suppliers to consumers. It is required to find a non-negative matrix $X = \begin{pmatrix} x_{ij} \end{pmatrix}$ that specifies a plan for the transportation of a product from suppliers to consumers, minimizing the total cost of transportation

$$L_1(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (1)$$

and satisfying the constraints

$$\sum_{j=1}^{n} x_{ij} \le a_i, \ i = 1, 2, ..., m, \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, \ j = 1, 2, ..., n, \qquad (3)$$

$$x_{ij} \ge 0, \ i = 1, 2, ..., m, \ j = 1, 2, ..., n. \qquad (4)$$

Methods and materials. Methods for solving this problem are well known and implemented in widely used mathematical packages (Mathcad, Excel, etc.)

It should be noted that the given model of the transport problem does not fully satisfy the natural requirements for the level of its adequacy. The point is that in real conditions the parameters of this problem are not deterministic values. In practice, the possibilities of suppliers, the needs of consumers, and the cost of transportation are random. It is clear that in this case the solution of the problem by traditional methods that do not take into account its probabilistic nature cannot be accurate [4-8]. In this regard, the above canonical mathematical model of the transport problem should be properly modernized. The obvious applied nature of the problem determines the relevance of the study.

Methods and materials

Methods for solving problems, the parameters of which are random, are combined into a specific subclass of general methods of mathematical programming, called stochastic programming [9-13]. The technology for solving such problems consists in constructing for each of them a corresponding deterministic analogue using further standard methods of mathematical programming. At the same time, the mathematical expectation or variance of the random value of the total cost of transportation is usually used as an objective function in such problems [11-13].

The solution to this problem using a more informative objective function was proposed in [14]. In this case, it was assumed that the cost of transportation are

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random values. Further, it is assumed that there is enough real data to formulate plausible hypotheses about the distribution laws of these quantities. At the same time, since a large number of various factors independently affect the cost of transporting goods, it is usually believed that their joint influence, in accordance with the central limit theorem of the theory of probability, leads to the normality of the resulting random variables. In this regard, a set of densities is introduced

$$\phi_{ij}\left(c_{ij}\right) = \frac{1}{\sqrt{2\pi}\sigma_j}\exp\left\{-\frac{\left(c_{ij}-m_{ij}\right)^2}{2\sigma_{ij}^2}\right\},$$

$$i = 1,2,...m, \quad j = 1,2,...,n,$$

Of random values c_{ij}. Therefore, the distribution density of the random total cost of transportation $R = L(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$ has the form

$$f(R) = f\left(L(X)\right) = \frac{1}{\sqrt{2\pi}\sigma_{\Sigma}}\exp\left\{-\frac{\left(R-m_{\Sigma}\right)^2}{2\sigma_{\Sigma}^2}\right\},$$

where

$$m_{\Sigma} = \sum_{i=1}^{m}\sum_{j=1}^{n}m_{ij}x_{ij}, \quad \sigma_{\Sigma}^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2.$$

The problem now consists in finding a set $X = \left(x_{ij}\right)$ that delivers the extreme value of some naturally chosen function of density $f(R)$ and satisfies constraints (2) - (4). For example, a certain threshold value R_{Π} of the total cost of transportation is set, the excess of which is regarded as a sign of inefficiency of the corresponding transportation plan. As the objective function of the problem, it is natural to choose the probability that the total cost of transportation will exceed the threshold. This probability is equal to

$$P\left(R \ge R_n\right) = \int_{R_{\Pi}}^{\infty} f(R)dR = \int_{R_{\Pi}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{\Sigma}}\exp\left\{-\frac{\left(R-m_{\Sigma}\right)^2}{2\sigma_{\Sigma}^2}\right\}dR \underset{\frac{R-m_{\Sigma}}{\sigma_{\Sigma}}=u}{=} \frac{1}{\sqrt{2\pi}}\int_{\frac{R_{\Pi}-m_{\Sigma}}{\sigma_{\Sigma}}}^{\infty} e^{-\frac{u^2}{2}}\,du. \tag{5}$$

Thus, the original problem is transformed to the following: find a transportation plan satisfying the constraints X, minimizing (5), or maximizing the lower limit in this integral, which is equal to

$$R(X) = \frac{R_{\Pi}-m_{\Sigma}}{\sigma_{\Sigma}} = \frac{R_{\Pi}-\sum_{i=1}^{m}\sum_{j=1}^{n}m_{ij}x_{ij}}{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2\right)^{\frac{1}{2}}}. \tag{6}$$

If the transportation plan X satisfies constraints (2) as equalities, then (6) can be transformed to the form

$$R(X) = \frac{\frac{R_n}{A}\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n}m_{ij}x_{ij}}{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2\right)^{\frac{1}{2}}} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{R_n}{A}-m_{ij}\right)x_{ij}}{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2\right)^{\frac{1}{2}}} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}}{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2\right)^{\frac{1}{2}}}, \tag{7}$$

$$i = 1,2,...m, \quad j = 1,2,...,n.$$

The maximization problem (7) while satisfying (2) - (4) is a complex problem of fractional nonlinear programming. An approximate solution to this problem can be obtained as follows.

As

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2\right)^{\frac{1}{2}} < \left[\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}x_{ij}\right)^2\right]^{\frac{1}{2}} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}x_{ij},$$

then

$$R(X) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}}{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2\right)^{\frac{1}{2}}} > \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}x_{ij}} = T(X). \tag{8}$$

At the same time, the task is reduced to finding a plan X that satisfies restrictions and maximizes $T(X)$. To solve the problem, it is proposed to use the following iterative procedure.

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A random plan $X^{(0)}$ is chosen that satisfies constraints (2) - (4). If this plan does not maximize $T(X)$, then there must be some other plan $X^{(1)}$ for which

$$T\left(X^{(1)}\right) - T\left(X^{(0)}\right) > 0. \qquad (9)$$

Since $\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}x_{ij} > 0$ in any task plan, it follows from (9) that

$$\left(T\left(X^{(1)}\right) - T\left(X^{(0)}\right)\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}x_{ij}^{(1)}\right) =$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}^{(1)} - T\left(X^{(0)}\right)\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}x_{ij}^{(1)} =$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(d_{ij} - T\left(X^{(0)}\right)\sigma_{ij}\right)x_{ij}^{(1)} = \sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}^{(1)}x_{ij}^{(1)} > 0.$$

Now the task is to find a plan $X^{(1)}$, that maximizes

$$S\left(X^{(1)}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}^{(1)}x_{ij}^{(1)} \qquad (10)$$

and satisfies constraints (2) - (4). An ordinary transport linear programming problem is obtained. If the plan $X^{(1)}$ calculated as a result of its solution does not maximize $T(X)$, then there must be a new plan $X^{(2)}$ for which

$T\left(X^{(2)}\right) - T\left(X^{(1)}\right) > 0$.

The computational procedure is naturally to stop when the inequality $\sum_{i=1}^{m}\sum_{j=1}^{n}\left|x_{ij}^{(k+1)} - x_{ij}^{(k)}\right| < \varepsilon$ is completed, where ε is a small number.

A brief analysis of publications on the problem of solving transport problems in conditions of uncertainty allows us to draw the following conclusion.

In the problem of finding a transportation plan under conditions of their random cost, there is no method for obtaining an exact solution. The quality of the approximate solution obtained using (8) and the rate of its convergence cannot be estimated.

The purpose of the article is to develop mathematical models and effective methods for solving transport problems in conditions of uncertainty. To achieve the goal, it is necessary to solve the following tasks.

1. Development of effective (accurate and fast) methods for solving transport problems with a probabilistic description of the cost of transportation;

2. Development of a method for solving transport problems for the case when the distribution density of the random cost of transportation is not known.

Thus, the mathematical model of the transport problem under uncertainty has a canonical representation (1) - (5). The corresponding optimization problem is formulated as follows: find a plan X that maximizes the fractional – quadratic function (11) and satisfies constraints (2) - (4).

Main result

Methods for solving transport problems in the context of a probabilistic description of the cost of transportation.

Any exact solution to problem (8), (2) - (4) can be obtained by solving a recurrent sequence of nonlinear programming problems as follows. It is clear that the maximum $R(X)$ is reached on the same set as the maximum $R^2(X)$. In accordance with this, we introduce

$$F(X) = R^2(X) = \frac{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}\right)^2}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2}. \qquad (11)$$

Thus, the problem is reduced to the following: to find a plan X that maximizes the fractional-quadratic function (11) and satisfies constraints (2) - (4). This problem is proposed to be solved as follows.

Let's introduce columns $\overline{C}, X$ and a matrix G

$$\overline{C} = \begin{pmatrix} d_{11} \\ d_{12} \\ ... \\ d_{1n} \\ d_{21} \\ . \\ . \\ d_{mn} \end{pmatrix}; \quad X = \begin{pmatrix} x_{11} \\ x_{12} \\ ... \\ x_{1n} \\ x_{21} \\ . \\ . \\ x_{mn} \end{pmatrix};$$

$$G = \begin{pmatrix} G_{11}^2 & & & & & & \\ & G_{12}^2 & & & & & \\ & & \cdot & & & & 0 \\ & & & \cdot & & & \\ & & & & G_{1n}^2 & & \\ & & & & & G_{21}^2 & \\ & 0 & & & & & \cdot \\ & & & & & & \cdot \\ & & & & & & & G_{mn}^2 \end{pmatrix}.$$

Then relation (11) takes the form

$$L(X) = \frac{X^T \overline{C} \cdot \overline{C}^T X}{X^T G X} = \frac{X^T C X}{X^T G X}, \quad C = \overline{C}\,\overline{C}^T. \qquad (12)$$

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Now the problem is reduced to finding a set X that maximizes (12) and satisfies

$$AX = B , \qquad (13)$$

where (13) is a matrix analogue (2) – (4).

$$X \geq 0 . \qquad (14)$$

The solution to this problem is achieved by implementing an iterative procedure similar to that described above. An arbitrary vector $X^{(0)}$ is chosen that is a solution to the system of equations $AX = B$ and satisfies the constraint $X \geq 0$. If this vector is not a solution to the problem, that is, does not maximize (12), then there must be some other vector $X^{(1)}$ for which

$$L(X^{(1)}) - L(X^{(0)}) > 0 . \qquad (15)$$

Since $X^T DX > 0$, it follows from (15) that

$$(L(X^{(1)}) - L(X^{(0)}))X^{(1)T} GX^{(1)} =$$
$$\qquad (16)$$
$$= X^{(1)T} CX^{(1)} - X^{(1)T} GX^{(1)} L(X^{(0)}) \geq 0.$$

Taking into account (16), the problem is reduced to finding a vector $X^{(1)}$ satisfying (13) - (14) and maximizing

$$R(X^{(1)}) = X^{(1)T} (C - L(X^{(0)})G)X^{(1)} .$$

Thus, the original problem is reduced to an iterative procedure for finding a sequence of vectors satisfying (13) - (14)

$$X^{*(1)}, X^{*(2)}, ..., X^{*(k)}, X^{*(k+1)} ,$$

for which recurrence ratio is performed

$$X^{*(k+1)T} CX^{*(k+1)} - X^{*(k+1)T} GX^{*(k+1)} L(X^{*(k)}) =$$

$$= \max_{X^{(k+1)}} \left[X^{(k+1)T} CX^{(k+1)} - X^{(k+1)T} GX^{(k+1)} L(X^{*(k)}) \right] =$$
$$\qquad (17)$$
$$= \max_{X^{(k+1)}} X^{(k+1)T} (C - GL(X^{*(k)}))X^{(k+1)} =$$

$$= \max_{X^{(k+1)}} X^{(k+1)T} G_k X^{k+1}.$$

It is natural to stop the computational procedure when the inequality $\left\| X^{(k+1)} - X^{(k)} \right\| < \varepsilon$ is completed $\left\| X^{(k+1)} - X^{(k)} \right\| < \varepsilon$, where ε is some fairly small number. This ensures the required accuracy of solving the problem.

Each of the sequence of problems (17) is easier than the original problem, since here the maximization of the fractional-quadratic functional (12) is replaced by the optimization of the usual quadratic functional (17). In this case, at each iteration, it is necessary to solve problems of the form: find a vector X that maximizes

$$f(x) = X^T GX \qquad (18)$$

And satisfies

$$AX = B , \qquad (19)$$

$$X \geq 0 . \qquad (20)$$

Lagrange function is generated:

$$F(X, \Lambda) = f(x) - \Lambda^T (AX - B) = X^T GX - \Lambda^T (AX - B) . \quad (21)$$

As it is known, if X^* is the optimal solution of the problem (18) - (21), then there must be a vector Λ^* such that X^* and Λ^* satisfy the relations

$$\left. \frac{dF(X, \Lambda)}{dX} \right|_{\substack{X=X^* \\ \Lambda=\Lambda^*}} = 2(X^*)^T G - \Lambda^{*T} A \leq 0 , \qquad (22)$$

$$\frac{dF(X^*, \Lambda^*)}{dX} X^* = 0 , \qquad (23)$$

$$\frac{dF(X^*, \Lambda^*)}{d\Lambda} X^* = B - AX = 0 . \qquad (24)$$

Transposing (22), we get

$$2GX^* - A^T \Lambda \leq 0 \qquad (25)$$

or

$$A^T \Lambda - 2GX^* \geq 0 .$$

Enter vector $V^* = A^T \Lambda - 2GX^* \geq 0$.
Then, taking into account (25),

$$2GX^* - A^T \Lambda + V^* = 0 . \qquad (26)$$

Further, we write (23) as follows

$$(V^*)^T X^* = 0$$

or

$$(X^*)^T V^* = 0 . \qquad (27)$$

Thus, if X^* is a solution to the problem, then there are $V^* \geq 0$ and Λ^* such that relations (24), (26), (27) are satisfied. On the other hand, if there exist $X \geq 0$, $V^* \geq 0$, Λ which satisfy the conditions

$$AX = B , \; 2GX - A^T \Lambda + V = 0 , \; X^T V = 0 , \quad (28)$$

then the vector X is the optimal solution to the problem (18) - (20).

A significant drawback of this technique is the slow convergence of the procedure for finding a sequence of

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solutions $X^{*(1)}$, $X^{*(2)}$,..., $X^{*(k)}$ in problem (15) - (17). This noted feature is explained by the fact that at each iteration of the procedure for sequential improvement of the objective function (12), it is necessary to solve a quadratic programming problem, which is difficult in itself. In this case, only an approximate estimate of the number of iterations required to obtain an admissible solution in terms of accuracy is possible. In this regard, let's consider another method for solving the problem.

It is convenient to replace the problem of maximizing function (11) by the equivalent problem of minimizing the function

$$G(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 x_{ij}^2}{\left(\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}\right)^2}. \quad (29)$$

To solve the fractional-quadratic programming problem obtained in this form, we introduce a new variable

$$y_0 = \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij}},$$

whence

$$y_0\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}x_{ij} = 1. \quad (30)$$

Let us formulate a new set of variables

$$y_{ij} = y_0 x_{ij}, \ i = 1, \ 2,...,m, \ j = 1, \ 2,...,n. \quad (31)$$

In this case, relations (2) - (4), (29), (30) take the form

$$G(y) = \frac{1}{y_0^2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 \cdot \frac{y_{ij}^2}{y_0^2} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 y_{ij}^2, \quad (32)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}y_{ij} = 1, \quad (33)$$

$$\sum_{j=1}^{n}x_{ij} = \frac{1}{y_0}\sum_{j=1}^{n}y_{ij} = a_i, \ i = 1, \ 2,...,m,$$

$$\sum_{i=1}^{m}x_{ij} = \frac{1}{y_0}\sum_{i=1}^{m}y_{ij} = b_j, \qquad j = 1, \ 2,...,n-1, \quad (34)$$

whence

$$\sum_{j=1}^{n}y_{ij} = y_0 a_i, \ i = 1, \ 2,...,m, \quad (35)$$

$$\sum_{i=1}^{m}y_{ij} = y_0 b_j, \ j = 1, \ 2,....,n-1. \quad (36)$$

Now the original problem has been reduced to the form: find a set $Y = (y_{ij})$, $i = 1, \ 2,...,m$, $j = 1, \ 2,...,n$, minimizing (32) and satisfying constraints (33) - (36). Let's solve the resulting quadratic programming problem.

Let's introduce the Lagrange function

$$\Phi(y) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}^2 y_{ij}^2 - \sum_{i=1}^{m}\lambda_i\left(\sum_{j=1}^{n}y_{ij} - y_0 a_i\right) - \sum_{j=1}^{n-1}\mu_j\left(\sum_{i=1}^{m}y_{ij} - y_0 b_j\right) - v\left(\sum_{i=1}^{m}\sum_{j=1}^{n}d_{ij}y_{ij} - 1\right) \quad (37)$$

Further

$$\frac{d\Phi(y)}{dy_{ij}} = 2\sigma_{ij}^2 y_{ij} - \lambda_i - \mu_j - vd_{ij} = 0, \qquad i = 1, \ 2,...,m, \qquad j = 1, \ 2,...,n,$$

$$y_{ij} = \frac{1}{2\sigma_{ij}^2}(\lambda_i + \mu_j + vd_{ij}), \ i = 1, \ 2,...,m, \qquad j = 1, \ 2,...,n. \quad (38)$$

Substituting (38) into (33) - (36), we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{d_{ij}}{2\sigma_{ij}^2}(\lambda_i + \mu_j + vd_{ij}) = 1,$$

$$\sum_{j=1}^{n}\frac{1}{2\sigma_{ij}^2}(\lambda_i + \mu_j + vd_{ij}) = y_0 a_i, \ i = 1, \ 2,...,m,$$

$$\sum_{i=1}^{m}\frac{1}{2\sigma_{ij}^2}(\lambda_i + \mu_j + vd_{ij}) = y_0 b_j, \ j = 1, \ 2,...,n-1.$$

Solving the resulting system of linear algebraic equations, we obtain expressions for $\{\lambda_i\}$, $\{\mu_j\}$, v through y_0 and the initial data $\{a_i\}$, $\{b_j\}$. Substituting these expressions in (38), we obtain the relations for y_{ij} through y_0. Now we find the value y_0 from (32). Finally, we calculate the values of the initial variables x_{ij} using (31). Problem is solved.

The fundamental advantage of the proposed method for solving a transport problem in conditions of uncertainty is the ability to obtain an accurate and fast

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result due to a single solution of the fractional – quadratic programming problem.

The real complication of the problem arises if the statistical data for an adequate reconstruction of the distribution density of the random costs of transportation is not enough, but a correct statistical estimate of the mathematical expectations and variances of these random variables is possible. Let's consider a method for solving the original problem under these conditions. Let m_{ij} and σ_{ij}^2 – statistical estimates of the mathematical expectation and variance of the random cost of transporting a unit of a product from the i-th supplier to the i-th consumer. To solve the problem, the following two-stage procedure is proposed.

At the first stage, we will find a transportation plan X that minimizes the average total cost of transportation

$$M\left[L(x)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n} m_{ij}x_{ij} \qquad (39)$$

and satisfying constraints (2) - (4). This is a common transportation problem and let $X^{(0)}$ be the solutions.

At the second stage, we will set the task of finding a plan that minimizes the compromise criterion made up of four terms. The first term determines the variance of the total cost of transportation, the second – a measure of deviation of the desired plan from the plan $X^{(0)}$, the third term provides a solution satisfying (2), the fourth – satisfying (3). Thus, the composite criterion has the form

$$L(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \sigma_{ij}^2 x_{ij}^2 + \sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij} - x_{ij}^{(0)}\right)^2 + \sum_{i=1}^{m}\lambda_i\left(\sum_{j=1}^{m} x_{ij} - a_i\right) + \sum_{j=1}^{n-1}\mu_j\left(\sum_{i=1}^{m} x_{ij} - b_j\right). \qquad (40)$$

Further

$$\frac{dL(x)}{dx_{ij}} = 2\sigma_{ij}^2 x_{ij} + 2\left(x_{ij} - x_{ij}^{(0)}\right) + \lambda_i + \mu_j = 0,\ i = 1,\ 2,...,m\ ,\quad j = 1,\ 2,...,n-1\ .$$

Hence

$$x_{ij} = \frac{2x_{ij}^{(0)} - \lambda_i - \mu_j}{2\left(\sigma_{ij}^2 + 1\right)}\ ,\ i = 1,\ 2,...,m\ ,\ j = 1,\ 2,...,n-1\ . \qquad (41)$$

Substituting (41) into (2) - (3), we obtain the system of equations

$$\sum_{j=1}^{n-1} \frac{2x_{ij}^{(0)} - \lambda_i - \mu_j}{2\left(\sigma_{ij}^2 + 1\right)} = a_i\ ,\ i = 1,\ 2,...,m,$$

$$\sum_{i=1}^{m} \frac{2x_{ij}^{(0)} - \lambda_i + \mu_j}{2\left(\sigma_{ij}^2 + 1\right)} = b_j\ ,\ j = 1,\ 2,...,n-1\ .$$

The solution to this system of linear algebraic equations defines the sets $\{\lambda_i\}$, $i = 1,\ 2,...,m$, $\{\mu_j\}$, $j = 1,\ 2,...,n-1$. Substitution of these sets in (41) gives the desired solution to the problem.

We now make an important remark. The results of solving the original problem naturally depend on the nature of the distribution density of the random variables c_{ij}, $i = 1,\ 2,...,m$, $j = 1,\ 2,...,n$. The set $\left(x_{ij}\right)$ obtained in problem (39), (40), (2), (3) corresponds to the hypothesis that this distribution is Gaussian with parameters $\left(m_{ij},\ \sigma_{ij}^2\right)$. The real distribution can be very different from the Gaussian one. Therefore, it is advisable to consider the solution of this problem under the assumption of the worst distribution density of the random cost of transportation, having previously found this worst

distribution. The problem of finding the worst distribution is solved under the condition that, based on the results of statistical processing of the available data for each "supplier – consumer" pair (i, j), estimates of the mathematical expectation and variance of the corresponding random value c_{ij} are obtained. Let us assume that the worst density corresponds to the maximum probability that a random value of the cost c_{ij} will exceed a given acceptable threshold c_n. Thus, this problem can be formulated as follows: find the distribution density of the random value $f\left(c_{ij}\right)$ that maximizes

$$p\left(c_{ij} > c_n\right) = \int_{c_n}^{\infty} f\left(c_{ij}\right)dc_{ij} \qquad (42)$$

and satisfies the accepted constraints

$$\int_{-\infty}^{\infty} f\left(c_{ij}\right)dc_{ij} = 1\ , \qquad (43)$$

$$\int_{-\infty}^{\infty} c_{ij} f\left(c\right)dc = m_{ij}\ , \qquad (44)$$

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$$\int_{-\infty}^{\infty} c^2 f(c)\, dc = \sigma_{ij}^2 + m_{ij}^2. \qquad (45)$$

The resulting problem is a special case of the continuous linear programming problem, the general methods of solving which are presented in [15]. It is shown that the solution to problem (42) - (45) must be sought in the form of a linear combination δ of the Dirac functions. The solution procedure is iterative and consists

in the sequential calculation of the components $(x_1, x_2,...,x_m)$, $(\theta_1, \theta_2,...,\theta_m)$, the basic plan – function

$$f(\theta) = \sum_{j=1}^{m} x_j \delta(\theta - \theta_j). \qquad (46)$$

The plan-function defined on each iteration is checked for optimality. If the sign of optimality formulated in [15] is not fulfilled, then another step is taken to improve the solution. This procedure will result in a function for task (42) - (45)

$$f(\theta) = \frac{(c_n - m_{ij})^2}{\sigma^2 + (c_n - m_{ij})^2} \delta\left(\theta - \frac{m_{ij}^2 + \sigma^2 - m_{ij}c_n}{c_n - m_{ij}}\right) + \frac{\sigma^2}{\sigma^2 + (c_n - m_{ij})^2} \delta(\theta - c_n), \qquad (47)$$

moreover, the probability of getting into critical area $c > c_n$ is equal to coefficient at second term in expression (47). Thus

$$P(c > c_n) = \frac{\sigma^2}{\sigma^2 + (c_n - m_{ij})^2}. \qquad (48)$$

Now, using (48), an additive criterion of efficiency (expediency) of a transportation plan X can be formed, taking into account the worst conditions for its implementation, which has the form

$$L(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\sigma_{ij}^2}{\sigma_{ij}^2 + (c_n - m_{ij})^2} x_{ij}. \qquad (49)$$

The directions of further research are associated with the development of methods for solving the considered problems for cases that are deeper than the theoretical and probabilistic uncertainty of the initial data. This uncertainty in the context of small samples of observations can be described in terms of fuzzy [16 - 18] or imprecise [19 - 20] mathematics. Possible solutions in this case can be obtained using the methods proposed in [21].

Conclusions

1. Considered the problem of forming a transportation plan in the "supplier - consumers" system in conditions when the values of the cost of transportation are random values. An exact method is proposed for solving this problem according to the criterion - the probability that the random total cost of implementing the plan will exceed the threshold, acceptable value. The solution is achieved using an iterative procedure, at each step of which a quadratic programming problem is solved.

In order to speed up this procedure, an alternative method is proposed based on a single solution of the fractional - quadratic programming problem.

2. A method for solving the problem of optimization of transportation for a situation where the density of distribution of a random cost cannot be correctly determined is considered. In this case, the solution to the problem was obtained under the assumption of the worst distribution density of the random cost of transportation. The solution is implemented using the theory of continuous linear programming.

3. The theoretical and applied significance of the results obtained is determined by the everyday nature of the optimization problems arising in practice, the parameters of which are not precisely determined.

References

1. Birman, I. Ya. (1962), *Transport Problem of Linear Programming* [*Transportnaya zadacha lineynogo programmirovaniya*], Moscow : Ed. eq. lit., 262 p.
2. Nesterov, E. P. (1962), *Transport Problem of Linear Programming* [*Transportnaya zadacha lineynogo programmirovaniya*], Moscow : Ed. Ministry of Railways, 189 p.
3. Yudin, D. B., Golstein, E. G. (1969), *Transport-type linear programming problems* [*Zadachi lineynogo programmirovaniya transportnogo tipa*], Moscow : "Sov. radio", 382 p.
4. Chumakov, V. B. (2006), "Solving the problems of organizing road transport in the context of the uncertainty of the state of regional transport systems" ["Resheniye zadach organizatsii avtomobil'nykh perevozok v usloviyakh neopredelennosti sostoyaniya regional'nykh transportnykh sistem"], Novocherkassk : SKSTU, P. 22–26.
5. Malakhov, V. P., Kondratenko, G. V. (2002), "Simulation models and algorithms for forming routes and trajectories based on fuzzy logic" ["Imitatsiyni modeli ta alhorytmy formuvannya marshrutiv i trayektoriy na osnovi nechitkoyi lohiky"], *Proceedings of the OPU*, Issue 1 (17), P. 1–10.
6. Ryan, G. (2005), "Rosandich. Quantification on Uncertainty in Transportation Infrastructure Projects", *Santiago: Economics planning*, P. 5–14.
7. Borisova, E. A., Finaev, V. I. (2006), "Triaxial distribution problem with fuzzy parameters" ["Triaksial'naya raspredelitel'naya zadacha s nechetkimi parametrami"], *Izvestia TRTU, Issue "Intelligent CAD"*, No. 8, P. 17–21.

8. Borisova, E. A., Finaev, V. I. (2007), *Three-index distribution problems with fuzzy parameters* [*Trekhindeksnyye raspredelitel'nyye zadachi s nechetkimi parametrami*], Taganrog : TTI SFU, 190 p.

9. Ermoliev, Yu. M. (1976), *Stochastic programming methods* [*Metody stokhasticheskogo programmirovaniya*], Moscow : Nauka, 239 p.

10. Kaplinsky, A. I., Poznyak, A. S., Propoy, A. I. (1971), "On some methods for solving stochastic programming problems" ["O nekotorykh metodakh resheniya zadach stokhasticheskogo programmirovaniya"], *Automation and Remote Control*, No. 10, P. 87–94.

11. Porotsky, S. M. (1977), "Stochastic problems of transport type" ["Stokhasticheskiye zadachi transportnogo tipa"], *Izv. Academy of Sciences of the USSR. Tech. Cybernetics*, No. 6, P. 34–39.

12. Yudin, D. B., Yudin, A. D. (2009), *Extreme models in economics* [*Ekstremal'nyye modeli v ekonomike*], Moscow : LIBROKOM, 312 p.

13. Yudin, D. B. (1979), *Problems and methods of stochastic programming* [*Zadachi i metody stokhasticheskogo programmirovaniya*], Moscow : "Sov. radio ", 385 p.

14. Seraya, O. V. *Multidimensional models of logistics in conditions of uncertainty* [*Mnogomernyye modeli logistiki v usloviyakh neopredelennosti*], Kharkiv : VOP Stetsenko, 512 p.

15. Raskin, L. G., Kirichenko, I. O. (2005), *Continuous linear programming* [*Kontinual'noye lineynoye prgrammirovaniye*], Kharkiv : VIVV, 175 p.

16. Zadeh, L. (1965), "Fuzzy Sets", *Information and Control*, Vol. 8, P. 338–353.

17. Dubois, D., Prade, A. (1990), *Theory of Opportunities. Application to the representation of knowledge in computer science* [*Teoriya vozmozhnostey. Prilozheniye k predstavleniyu znaniy v informatike*] : Per. from French, Moscow : Radio and communication, 286 p.

18. Raskin, L. G., Gray, O. V. (2008), *Fuzzy mathematics* [*Nechetkaya matematika*], Kharkiv : Parus, 352 p.

19. Pawlak, Z. (1982), "Rough sets", *International Journal of Information and Computer Sciences*, Vol. 11, No. 5, P. 341–356.

20. Pawlak, Z. (1991), *Rough Sets: Theoretical Aspects of Reasoning about Data*, Dordrecht : Kluwer Academic Publisher, 284 p.

21. Raskin, L., Sira, O. (2016), "Method of solving fuzzy problems of mathematical programming", *Eastern–european journal of enterprise technologies*, Vol. 5, No. 4 (83). DOI: http://dx.doi.org/10.15587/1729–4061.2016.81292

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МЕТОД ВИРІШЕННЯ КАНОНІЧНОЇ ЗАДАЧІ ТРАНСПОРТНОЇ ЛОГІСТИКИ В УМОВАХ НЕВИЗНАЧЕНОСТІ

Предмет. Розглянуто канонічну задачу транспортної логістики у розгалуженій системі "постачальники – споживачі". **Ціль.** Розробка точного алгоритму вирішення цієї задачі за імовірнісним критерієм в припущенні про випадковий характер вартостей транспортувань. **Задачі.** 1. Розробка точного методу вирішення задачі відшукання плану, мінімізуючого сумарну вартість транспортувань в умовах, коли їх вартості задані власною щільністю розподілу. 2. Розробка методу вирішення задачі, коли щільності розподілу вартості перевезень невідомі. **Методи.** Запропоновано обчислювальну схему вирішення задачі, яка реалізується ітераційною процедурою послідовного покращення плану транспортувань. Збіжність цієї процедури доведено. З ціллю прискорення збіжності обчислювальної процедури до точного рішення запропоновано альтернативний метод, заснований на рішенні нетривіальної задачі дробно-нелінійного програмування. Метод зводить вихідну складну задачу до рішення послідовності більш простих задач. Вихідна задача доповнена розглядом важливої для практики ситуації, коли в умовах малої вибірки похідних даних відсутня можливість отримання адекватних аналітичних описів для щільностей розподілення випадкових вартостей транспортувань. Для рішення задачі в такому випадку запропоновано мінімаксний метод відшукання найкращого плану транспортувань у найбільш несприятливій ситуації, коли щільності розподілу випадкової вартості транспортувань будуть найгіршими. Для відшукання таких щільностей використано сучасний математичний апарат континуального лінійного програмування. **Результати.** Запропоновані математична модель та метод рішення задачі транспортної логістики в умовах невизначеності похідних даних. Шуканий план досягається з використанням рішення задачі дробно-нелінійного програмування. **Висновки:** Розглянуто задачу формування плану транспортувань за умови, що їх вартості – випадкові величини. Також розглянуто метод вирішення задачі оптимізації транспортувань для випадку, у якому щільність розподілення випадкової вартості не може бути коректна визначена.

Ключові слова: транспортна задача лінійного програмування; випадкова вартість транспортувань; дробно–нелінійна оптимізація.

МЕТОД РЕШЕНИЯ КАНОНИЧЕСКОЙ ЗАДАЧИ ТРАНСПОРТНОЙ ЛОГИСТИКИ В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ

Предмет. Рассмотрена каноническая задача транспортной логистики в распределенной системе "поставщики – потребители". **Цель.** Разработка точного алгоритма решения этой задачи по вероятностному критерию в предположении о случайном характере стоимостей транспортировок. **Задачи.** 1. Разработка точного метода решения задачи отыскания плана, минимизирующего суммарную стоимость транспортировок в условиях, когда их стоимости заданы своими плотностями распределения. 2. Разработка метода решения задачи, когда плотности распределения стоимости перевозок не известны. **Методы.** Предложена вычислительная схема решения задачи, которая реализуется итерационной процедурой последовательного улучшения плана транспортировок. Сходимость этой процедуры доказана. С целью ускорения сходимости вычислительной процедуры к точному решению предложен альтернативный метод, основанный на решении нетривиальной задачи дробно-нелинейного программирования. Метод сводит исходную сложную задачу к решению последовательности более простых задач. Исходная задача дополнена рассмотрением важной для практики ситуации, когда в условиях малой выборки исходных данных отсутствует возможность получения адекватных аналитических описаний для плотностей распределения случайных стоимостей транспортировок. Для решения задачи в этом случае предложен минимаксный метод отыскания наилучшего плана транспортировок в наиболее неблагоприятной ситуации, когда плотности распределения случайной стоимости транспортировок являются наихудшими. Для отыскания таких плотностей использован современный математический аппарат континуального линейного программирования. **Результаты.** Предложены математическая модель и метод решения задачи транспортной логистики в условиях неопределенности исходных данных. Искомый план достигается с использованием решения задачи дробно-нелинейного программирования. **Выводы:** Рассмотрена задача формирования плана транспортировок при условии, что их стоимости - случайные величины. Также рассмотрен метод решения задачи оптимизации транспортировок для ситуации, когда плотность распределения случайной стоимости не может быть корректно определена.

Ключевые слова: транспортная задача линейного программирования; случайная стоимость транспортировок; дробно–нелинейная оптимизация.

Бібліографічні описи / Bibliographic descriptions

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