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CONSTRUCTION OF STABILITY AREAS FOR CONTROLLED SYSTEMS WITH PARAMETRIC AND DYNAMIC UNCERTAINTY

The **subject** of research in the article is sigularly perturbed controllable systems of differential equations containing terms with a small parameters on the right-hand side, which are not completely known, but only satisfy some constraints. The **aim** of the work is to expand the study of the behavior of solutions of singularly perturbed systems of differential equations to the case when the system is influenced not only by dynamic (small factor at the derivative) but also parametric (small factor at the right side of equations) uncertainties and to determine conditions under which such systems will be asymptotically resistant to any perturbations, estimate the upper limit of the small parameter, so that for all values of this parameter less than the obtained estimate, the undisturbed solution of the system was asymptotically stable. The following **problems** are solved in the article: singularly perturbed systems of differential equations with regular perturbations in the form of terms with a small parameter in the right-hand sides, which are not fully known, are investigated; an estimate is made of the areas of asymptotic stability of the unperturbed solution of such systems, that is, the class of systems that can be investigated for stability is expanded, the formulas obtained that allow one to analyze the asymptotic stability of solutions to systems even under conditions of incomplete information about the perturbations acting on them. The following **methods** are used: mathematical modeling of complex control systems; vector Lyapunov functions investigation of asymptotic stability of solutions of systems of differential equations. The following **results** were obtained: an estimate was made for the upper bound of a small parameter for sigularly perturbed systems of differential equations with fully known parametric (fully known) and dynamic uncertainties, such that for all values of this parameter less than the obtained estimate, such an unperturbed solution is asymptotically stable; a theorem is proved in which sufficient conditions for the uniform asymptotic stability of such a system are formulated. **Conclusions:** the method of vector Lyapunov functions extends to the class of singularly perturbed systems of differential equations with a small factor in the right-hand sides, which are not completely known, but only satisfy certain constraints.

Keywords: asymptotic stability; Lyapunov vector functions; parametric uncertainty; small parameter.

Introduction

Most control systems are largely uncertain. Uncertainties significantly affect the performance of control systems and can lead to its loss. In this regard, a very important task in the study of the efficiency of control systems is the task of studying the stability of their movement. A control system is called coarse with respect to some of its properties, if sufficiently small deviations of parameters in the equations of motion of such a system do not lead to the loss of this property. In practice, the uncertainties (possible deviations of the parameters of the system under study) can be so large that it leads to a loss of stability.

When studying the properties of solutions of differential equations describing control systems, one of the most important tasks is to study different types of stability. First of all, this is due to the fact that in most technical problems, stable solutions are the most interesting. Second, when developing control systems, it is necessary to be aware of unstable solutions in order to avoid them. Third, the solutions can be quite sensitive to errors in the mathematical model of the control system.

In the classic setting of A.M. Lyapunov, problems with stability of motion are considered only perturbations of the initial conditions. However, practical problems lead to the need to study the dynamics of systems in the presence of perturbation of the right parts.

Most often, perturbations of the right parts (uncertainty) are formed in the form of a vector of uncertainty, which contains components due to uncertainties:

- coefficients of equations of motion,
- initial conditions,
- boundary conditions,
- undesirable for nonlinearity control systems,
- external influences.

In many cases, the influence of these factors (uncertainties), although they seem insignificant, can significantly change that information about the process. To avoid this, you need to develop an extended process model that takes into account those small factors that were not represented in the original model, and then explore the similarity of the solutions obtained from the simplified and extended models.

Analysis of the problem and existing methods

The problem of studying the stability of singularly perturbed equations is far from complete. Intensive development of the theory of singular perturbations began in the middle of the 20th century, thanks to the work of A.N. Tikhonov [1], which describes the formulation of the problem of the theory of singularly perturbed systems of differential equations. This theory was further developed in the works of Vasilieva A.B., Butuzova V.F. [2], Hoppensteadt F. [3], which investigates the behavior of solutions singularly of such systems. Methods of singular perturbations are widely studied in our time. In particular, in the works of Kachalov V.I. [4], [5] the method of obtaining solutions of singularly perturbed problems in the form of series that coincide in the usual sense by degrees of small parameter is presented. Works [6], [7] are devoted to the construction of an asymptotic schedule of singularly perturbed equations with singular points.

Singular perturbations are present in many classical and modern control systems based on low-order systems and those that ignore parasitic dynamics. This led to the development of methods of separation of movements. These methods have been found to be useful for high-gain

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feedback analysis and low-order models. These methods are used to model and control dynamic systems and certain classes of large-scale systems. In the works of Binning H.S., Goodell D.P. [8], Kodra K.; Gajic Z. [9] and Y. Li, Y.Y. Wang, D. Y. Yao [10] consider singularly perturbed control systems and investigate the behavior of their movements. The work of H. S. Liu, Y. Huang [11] is devoted to the study of the behavior of the trajectories of the manipulator robot, whose movements are described by means of singularly perturbed control systems. [12], [13] study the existing methods of the theory of singular perturbations extend to the class of controlled systems, on the right part of which parametric, not completely known perturbations additionally act.

[14], [15] are devoted to the study of the asymptotic stability of solutions of singularly perturbed systems of differential equations. The problems considered in these works are formulated for a class of singularly perturbed systems, which do not take into account external perturbations and uncertainties acting on the system. In addition, these works use special Lyapunov functions to study stability, which are suitable for studying the stability of solutions of equations of a particular class and usually cannot be applied to other types of equations. Meanwhile, the practice of automatic control requires the development of methods for studying the stability of motion for a wide class of control systems, which are described using nonlinear singularly perturbed systems of differential equations with incompletely known right-hand sides. In [16], [17], the study of the asymptotic stability of solutions of singularly perturbed systems of differential equations extends to the class of systems with parametric uncertainty.

When studying the behavior of control system solutions, it is important not only to investigate the stability of these solutions, but also to estimate the size of their areas of gravity, ie to conduct a large-scale study of stability. Due to the effects of perturbations on the control system in many cases it is impossible to ensure asymptotic stability of program movements. Therefore, the size of the program traffic around the state space, which is guaranteed to include solutions, is important.

The **aim** of this article is to estimate the region of gravity of solutions of singularly perturbed systems of differential equations with parametric (small factor in the right part of equations) and dynamic (small factor in the derivative) uncertainties, finding the upper limit of a small parameter such that for all values of this parameter than the obtained estimate, the undisturbed solution of a singularly perturbed system of differential equations is asymptotically stable. This problem is solved using Lyapunov vector functions. A theorem is proved in which sufficient conditions for uniform asymptotic stability of solutions of such a system are formulated.

The practical value is that the class of systems that can be tested for stability is expanding. Necessary researches are made and the formulas allowing to analyze stability of systems even at the conditions of the incomplete information on disturbances operating on them are received.

Task solving

Let’s consider a system of differential equations

$$\begin{aligned} \dot{x} &= f(x,z,t) + \varepsilon f_1(x,z,t) \\ \varepsilon \dot{z} &= g(x,z,t) + \varepsilon g_1(x,z,t) \end{aligned}, \qquad (1)$$

where f and f_1 – n-measurable vector functions, g and g_1 – m-measurable vector functions, ε – small parameter, $\varepsilon > 0$, f_1 and g_1 unknown and satisfy only some of the limitations discussed below. Members $\varepsilon f_1(x,z,t)$ and $\varepsilon g_1(x,z,t)$ make up *the parametric uncertainty* of the system, the second equation of the system (1) contains a small parameter when the derivative determines *the dynamic uncertainty.*

We set the initial conditions:

$$z(0,\varepsilon) = z^0, \qquad (2)$$

$$x(0,\varepsilon) = x^0. \qquad (3)$$

Let's explore the solution $(x(t,\varepsilon), z(t,\varepsilon))$ of tasks (1) - (3) on the interval $0 \le t \le T$. If we put in (1) $\varepsilon = 0$, we get the system

$$\dot{x}^s = f(x^s, z^s, t), \qquad (4)$$

$$0 = g(x^s, z^s, t), \qquad (5)$$

which in the terminology of A.N. Tikhonov is called a *degenerate system*?

The order of the system (4) - (5) is equal to *n*, ie is lower than the order of the original system. For the system (4) - (5) a smaller number of initial conditions is given, namely

$$x^s(0) = x^0. \qquad (6)$$

Let’s solve equation (5) with regard to $z^s(t)$, if such an operation is possible.

$$z^s = \phi(x^s, t). \qquad (7)$$

Due to the nonlinearity of the function $g(x^s, z^s, t)$, this operation is ambiguous and the choice of solution arises.

Substitution (7) in (3) gives

$$\dot{x}^s = f(x^s, \phi(x^s, t), t), \qquad (8)$$

$$x^s(0) = x^0, \qquad (9)$$

$z^s = \varphi(x^s, t)$, generally does not satisfy the initial condition (2) for z, that is, $z^s(0) \ne z^0$, and therefore, in some vicinity of the starting point $t = 0$, the solution $z^s(t)$ of the degenerate system will not be close to the solution $z(x,t)$ of the original system (1).

Let’s enter a new variable

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$$\eta = z - \phi(x,t). \tag{10}$$

Let the following conditions be met:

a) Functions $f(x,z,t)$ and $g(x,z,t)$ are continuous and satisfy the Lipschitz condition by x and z in some region G of the space of variables (x,z,t), i.e. for some positive N_1, N_2, N_3, N_4 inequalities are performed

$$\|f(x,z,t) - f(\tilde{x},z,t)\| \le N_1 \|x - \tilde{x}\|, \tag{11}$$

$$\|f(x,z,t) - f(x,\tilde{z},t)\| \le N_2 \|z - \tilde{z}\|, \tag{12}$$

$$\|g(x,z,t) - g(\tilde{x},z,t)\| \le N_3 \|x - \tilde{x}\|, \tag{13}$$

$$\|g(x,z,t) - g(x,\tilde{z},t)\| \le N_2 \|z - \tilde{z}\|, \tag{14}$$

where $\|y\| = \sqrt{y_1^2 + y_2^2 + \ldots y_n^2}$ - Euclidean norm.

b) Solution (3.6) has in some closed area $\overline{D}$ such properties:

1) $\varphi(x^s,t)$ - continuous function in $\overline{D}$,

2) $(x^s, \varphi(x^s,t), t) \in G$ for all $(x^s,t) \in \overline{D}$.

3) The root $z^s = \varphi(x^s,t)$ is isolated in $\overline{D}$, that is, there is $\lambda > 0$ such that $g(x^s, z^s, t) \ne 0$ when $\|z^s - \varphi(x^s,t)\| < \lambda$, $(x^s,t) \in \overline{D}$.

c) The system (8), (9) has a single solution $x^s(t)$ on the interval $0 \le t \le T$, besides in this interval the point $(x^s,t) \in D$ where D is the set of inner points $\overline{D}$. In addition, suppose that $f(x^s, \varphi(x^s,t), t)$ satisfies the Lipschitz condition at $x^s \in \overline{D}$.

That is, there is such a constant $L > 0$ that for any y_1 and y_2, the inequality is performed

$$\|f(x,y_1,t) - f(x,y_2,t)\| \le L\|y_1 - y_2\|.$$

Let's now introduce a connected system

$$\frac{d\tilde{z}}{d\tau} = g(x^s, \tilde{z}, t)\Big|_{t=\tau\varepsilon}. \tag{15}$$

In which x^s and t are considered as parameters, $\tau = \varepsilon^{-1} t$ (stretched time).

Obviously $\tau \ge 0$. According to condition b) 3, $\tilde{z}(\tau) = \varphi(x^s,t)$ is an isolated resting point of the system (15) at $(x^s,t) \in \overline{D}$.

Also, let

d) The resting point $\tilde{z}(\tau) = \varphi(x^s,t)$ of the system (15) is asymptotically stable according to Lyapunov even in regard to $(x^s,t) \in \overline{D}$. This means that $\forall \mu > 0$. $\exists \overline{\delta}(\mu) > 0$ (common to all $(x^s,t) \in \overline{D}$, such that for all solutions $\tilde{z}(\tau)$ of the equation (15) for which

$$\|\tilde{z}(\tau) - \varphi(x^s,t)\| < \mu, \quad (x^s,t) \in \overline{D}.$$

At $\tau \ge 0$ i $\tilde{z}(\tau) \to \varphi(x^s,t)$ when $\tau \to \infty$.

Let's consider a connected system (15) when $x^s = x^0$, $t = 0$:

$$\frac{d\tilde{z}}{d\tau} = g(x^0, \tilde{z}, t) \tag{16}$$

with the initial condition

$$\tilde{z}(0) = z^0. \tag{17}$$

Since the initial value z^0, in general, is not close to the resting point $\varphi(x^0,0)$, the solution $\tilde{z}(\tau)$ of the task (16), (17) may not go to $\varphi(x^0,0)$ when $\tau \to \infty$.

Let

e) The solution $\tilde{z}(\tau)$ problems (16) with initial conditions (17) satisfies the conditions

1. $\lim\limits_{n\to\infty} \tilde{z}(\tau) = \varphi(x^0,0)$,

2. point $(x^0, \tilde{z}(\tau), t) \notin G$, when $\tau \ge 0$.

f) Functions $f_1(x,z,t)$, $g_1(x,z,t)$ are continuous and satisfy the Lipschitz conditions by variables x and z, but there are summarized functions $M_1(t)$ and $M_2(t)$ with constants M_1^0 and M_2^0 such that in n area G there is inequality

$$\|f_1(x,z,t)\| \le M_1, \quad \|g_1(x,z,t)\| \le M_2, \tag{18}$$

$$\int_0^T M_1(t) \le M_1^0 T, \quad \int_0^T M_2(t) \le M_2^0 T. \tag{19}$$

Theorem. Let the following assumptions be fulfilled:

a) For systems (4) and (16) there are positively defined Lyapunov functions $V(x,t)$ and $W(x,z,t)$, accordingly, satisfying estimates that are peculiar to quadratic forms, such that

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x^s, \phi(x^s,t), t) \le -\alpha_1 \psi^2(x), \tag{20}$$

$$\frac{\partial W}{\partial z} g(x, \tilde{z}, t) \le -\alpha_2 \theta^2(\eta), \tag{21}$$

where $\psi(x)$ and $\theta(\eta)$ – functions that are zero at point 0 and different from zero for other argument values.

b) Interconnection conditions $V(x,t)$ and $W(x,z,t)$ satisfy inequalities

$$\frac{\partial W}{\partial t} + \frac{\partial V}{\partial x} f(x,z,t) \le c\psi(x)\theta(\eta), \tag{22}$$

$$\frac{\partial V}{\partial x}\left(f(x,z,t) - f(x^s, \phi(x^s,t), t)\right) \le \beta\psi(x)\theta(\eta), \tag{23}$$

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$$\frac{\partial W}{\partial z}\left(g(x,z,t)-g(x,\tilde{z},t)\right)\le \varepsilon k\psi(x)\theta(\eta), \quad (24)$$

$$\frac{\partial V}{\partial x} f_1(x,z,t)\le \gamma_1\psi^2(x), \quad (25)$$

$$\frac{\partial W}{\partial x} f_1(x,z,t)\le \gamma_2\psi^2(x), \quad (26)$$

$$\frac{\partial W}{\partial t}+\frac{\partial W}{\partial z} g_1(x,z,t)\le c\psi(x)\theta(\eta), \quad (27)$$

where $c,\beta,k,\gamma_1,\gamma_2,\gamma_3$ – positive constants. Then for any $0<d<1$ linear combination

$$U(x,z,t)=(1-d)V(x,t)+dW(x,z,t) \quad (28)$$

is a Lyapunov function of the system (1) and exists

$$\varepsilon^*(d)=\frac{\alpha_1}{\gamma_1+\frac{d}{1-d}\gamma_2+\frac{\left((1-d)\beta+d(c+k+\gamma_3)\right)^2}{4d(1-d)\alpha_2}} \quad (29)$$

such that for all $\varepsilon<\varepsilon^*(d)$ the solution $x=x^s(t)$, $\eta=0$ at the system (1) is resistant to constant disturbances.

Proof. The complete derivative of the function (28) in time along the trajectory is

$$\frac{dU(x,z,t)}{dt}=(1-d)\left(\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x} f(x,z,t)+\frac{\partial V}{\partial x}\varepsilon f_1(x,z,t)\right)+$$
$$+d\left(\frac{\partial W}{\partial t}+\frac{\partial W}{\partial x} f(x,z,t)+\frac{\partial W}{\partial x}\varepsilon f_1(x,z,t)+\frac{1}{\varepsilon}\frac{\partial W}{\partial z} g(x,z,t)+\frac{\partial W}{\partial z} g_1(x,z,t)\right).$$

Let's convert an expression to a view that allows to apply inequalities (20), (27):

From the above inequalities we get

$$\frac{dU(x,z,t)}{dt}=(1-d)(\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x} f\left(x^s,\phi(x^s,t),t\right)+$$
$$+\frac{\partial V}{\partial x}\left(f(x,z,t)-f\left(x^s,\phi(x^s,t),t\right)\right)+\frac{\partial V}{\partial x}\varepsilon f_1(x,z,t))+ \quad (30)$$
$$+d(\frac{\partial W}{\partial t}+\frac{\partial W}{\partial x} f(x,z,t)+\frac{\partial W}{\partial x}\varepsilon f_1(x,z,t)+$$
$$+\frac{1}{\varepsilon}\frac{\partial W}{\partial z}\left(g(x,z,t)-g(x,\tilde{z},t)+g(x,\tilde{z},t)\right)+\frac{\partial W}{\partial z} g_1(x,z,t)).$$

From the above inequalities we get

$$\frac{dU(x,z,t)}{dt}\le -(1-d)\alpha_1\psi^2(x)-\frac{d}{\varepsilon}\alpha_2\theta^2(\eta)+dc\psi(x)\theta(\eta)+$$
$$+(1-d)\beta\psi(x)\theta(\eta)+dk\psi(x)\theta(\eta)+\varepsilon(1-d)\gamma_1\psi^2(x)+ \quad (31)$$
$$+\varepsilon d\gamma_2\psi^2(x)+d\gamma_3\psi(x)\theta(\eta)=$$
$$=-\begin{pmatrix}\psi(x)\\ \theta(\eta)\end{pmatrix}^T T\begin{pmatrix}\psi(x)\\ \theta(\eta)\end{pmatrix}$$

where

$$T=\begin{pmatrix}(1-d)\alpha_1-\varepsilon(1-d)\gamma_1-\varepsilon d\gamma_2 & -\frac{(1-d)\beta+d(c+k+\gamma_3)}{2}\\ -\frac{(1-d)\beta+d(c+k+\gamma_3)}{2} & \frac{d}{\varepsilon}\alpha_2\end{pmatrix}. \quad (32)$$

For negative certainty $\dot{U}$, it is necessary that the matrix T be positively defined. We will demand that

$$(1-d)\alpha_1-\varepsilon(1-d)\gamma_1-\varepsilon d\gamma_2>0 \quad (33)$$

and

$$\det T>0. \quad (34)$$

From the inequality (33) it follows that

$$\varepsilon<\varepsilon_1(d)=\frac{\alpha_1}{\gamma_1+\frac{d}{1-d}\gamma_2}. \quad (35)$$

From (35)

$$\varepsilon<\varepsilon_2(d)=\frac{\alpha_1}{\gamma_1+\frac{d}{1-d}\gamma_2+\frac{\left((1-d)\beta+d(c+k+\gamma_3)\right)^2}{4d(1-d)\alpha_2}}. \quad (36)$$

Obviously, $\varepsilon_1<\varepsilon_2=\varepsilon^*(d)$. Thus, for all $\varepsilon<\varepsilon^*(d)$ matrix T positively defined, as a result of which the derivative $\dot{U}(x,z,t)$ is negatively defined. In addition, $\dot{U}(x,z,t)$ contains a term with a factor $\frac{1}{\varepsilon}$: $\frac{1}{\varepsilon}\frac{dW}{dz}g(x,\tilde{z},t)$, which in force (21) increases unlimitedly when $\varepsilon\to 0$, respectively negative derivative $\dot{U}(x,z,t)$ increases in absolute value. Hence the function $U(x,z,t)$, remaining negative, rapidly decreasing in magnitude and solving problems (4) and (16) in a short period of time approaching the origin, remaining in a small neighborhood of this point. This proves that an undrilled solution is resistant to perturbations that operate continuously.

Conclusions

In this work the following has been done:

- A method for studying the asymptotic stability of solutions of a singularly perturbed system of differential equations based on the use of Lyapunov vector functions that satisfy the estimates inherent in quadratic forms is considered;

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- The above method is extended to the case when the systems are affected not only by dynamic (small factor in the derivative), but also parametric (small factor in the right part of the equations) uncertainties. In addition, the right parts are not fully defined, but only satisfy some restrictions. These uncertainties are due to the uncertainties of the coefficients of the equations of motion, the uncertainties of the initial and boundary conditions, undesirable for control systems of nonlinearities and external influences. Uncertainties are formed in the uncertainty vector with a small factor in the right part of the system of singularly perturbed differential equations.

- An estimate of the upper limit of a small parameter is made such that for all values of this parameter, smaller than the obtained estimate, the undisturbed solution of a singularly perturbed system of differential equations with dynamic and parametric uncertainties is asymptotically stable.

- A theorem is proved in which sufficient conditions for uniform asymptotic stability of solutions of such a system are formulated.

- The class of systems that can be tested for stability has been expanded. Necessary researches are made and the formulas allowing to analyze stability of systems even at the conditions of the incomplete information on disturbances operating on them are received.

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ПОБУДОВА ОБЛАСТЕЙ СТІЙКОСТІ ДЛЯ КЕРОВАНИХ СИСТЕМ З ПАРАМЕТРИЧНОЮ ТА ДИНАМІЧНОЮ НЕВИЗНАЧЕНОСТЯМИ

Предметом дослідження в статті є сигулярно збурені керовані системи диференціальних рівнянь, що містять доданки з малим множником у правій частині, які не є повністю відомими, а лише задовольняють деяким обмеженням. **Мета** роботи — поширити дослідження поведінки розв’язків сигулярно збурених систем диференціальних рівнянь на випадок, коли на систему впливають не тільки динамічні (малий множник при похідній), а ще і параметричні (малий множник у правій частині рівнянь) невизначеності та визначити умови, за яких розв’язки таких систем будуть асимптотично стійкими до будь-яких збурень, оцінити верхню границю малого параметру, таким чином що для всіх значень цього параметру, менших ніж отримана оцінка, незбурений розв’язок системи є асимптотично стійким. В статті вирішуються наступні **завдання:** досліджуються сингулярно збурені системи диференціальних рівнянь, що мають регулярні збурення у вигляді доданків з малим множником у правих частинах, які не є повністю відомими; робиться оцінка областей асимптотичної стійкості незбуреного розв’язку таких систем, тобто розширюється клас систем, які можна досліджувати на стійкість, отримуються формули, що дозволяють аналізувати асимптотичну стійкість розв’язків систем навіть за умов неповної інформацій про збурення, що діють на них. Використовуються такі **методи:** математичне моделювання складних систем керування; векторні функції Ляпунова дослідження асимптотичної стійкості розв’язків систем диференціальних рівнянь. Отримано наступні **результати:** зроблена оцінка верхньої границі малого параметра для сигулярно збурених систем диференціальних рівнянь параметричними (неповністю відомими) і динамічними невизначеностями , така що для всіх значень цього параметру, менших ніж отримана оцінка, незбурений розв’язок такої є асимптотично стійким; доведена теорема, в якій сформульовані достатні умови рівномірної асимптотичної стійкості такої системи. **Висновки:** метод векторних функцій Ляпунова може бути поширеним на клас сингулярно збурених систем диференціальних рівнянь з малим множником у правих частинах, які не є повністю відомими, а лише задовольняють деяким обмеженням.

Ключові слова: асимптотична стійкість; векторні функції Ляпунова; параметрична невизначеність; малий параметр.

ПОСТРОЕНИЕ ОБЛАСТЕЙ УСТОЙЧИВОСТИ ДЛЯ УПРАВЛЯЕМЫХ СИСТЕМ С ПАРАМЕТРИЧЕСКОЙ И ДИНАМИЧЕСКОЙ НЕОПРЕДЕЛЕННОСТЯМИ

Предметом исследования в статье является сигулярно возмущенные управляемые системы дифференциальных уравнений, содержащих слагаемые с малым множителем в правой части, которые не являются полностью известными, а лишь удовлетворяют некоторым ограничениям. **Цель** работы – расширить исследование поведения решений сигулярно возмущенных систем дифференциальных уравнений на случай, когда на систему влияют не только динамические (малый множитель при производной), но и параметрические (малый множитель в правой части уравнений) неопределенности и определить условия, при которых решения таких систем будут асимптотически устойчивыми к любым возмущениям , оценить верхнюю границу малого параметра, таким образом что для всех значений этого параметра, меньших чем полученная оценка, невозмущенное решение системы являлось асимптотически устойчивым. В статье решаются следующие **задачи:** исследуются сингулярно возмущенные системы дифференциальных уравнений, имеющих регулярные возмущения в виде слагаемых с малым множителем в правых частях, которые не являются полностью известными; делается оценка областей асимптотической устойчивости невозмущенного решения таких систем, то есть расширяется класс систем, которые можно исследовать на устойчивость, получены формулы, позволяющие анализировать асимптотической устойчивости решений систем даже в условиях неполной информаций о возмущения, действующие на них. Используются такие **методы:** математическое моделирование сложных систем управления; векторные функции Ляпунова исследования асимптотической устойчивости решений систем дифференциальных уравнений. Получены следующие **результаты:** сделана оценка верхней границы малого параметра для сигулярно возмущенных систем дифференциальных уравнений с параметрическими (не полностью известными) и динамическими неопределенностями, такая что для всех значений этого параметра, меньших чем полученная оценка, невозмущенное решение такой является асимптотически устойчивым; доказана теорема, в которой сформулированы достаточные условия равномерной асимптотической устойчивости такой системы. **Выводы:** метод векторных функций Ляпунова может быть расширен на класс сингулярно возмущенных систем дифференциальных уравнений с малым множителем в правых частях, которые не являются полностью известными, а лишь удовлетворяют некоторым ограничениям.

Ключевые слова: асимптотическая устойчивость; векторные функции Ляпунова; параметрическая неопределенность; малый параметр.

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