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## THE TASK OF MINIMAX ADAPTIVE MANAGEMENT OF INNOVATIVE PROCESSES AT AN ENTERPRISE WITH RISK ASSESSMENT

The **subject** matter of the article is a discrete dynamic system that consists of an object whose dynamics is described by a vector linear discrete recurrent relation and is affected by control parameters (managements) and uncontrolled parameters (the vector of risks or interference). It is supposed that the phase conditions of the object, management actions and the vector of risks of the considered dynamic system at any moment of time are constrained by given finite or convex polyhedral sets in corresponding finite-dimensional vector spaces. The **objective** of the article is to model a task of adaptive management of an enterprise innovative processes (EIP) under risks, which requires to complete the following **tasks**: to develop a software model of managing EIP under risks; to formalize the task of optimizing the EIP adaptive management and general paradigm of its solving as a guaranteed result based on minimax (optimizing a guaranteed result at a given final moment of time considering risks). In such a case, risks in the system of EIP management are thought of as factors that negatively or even catastrophically affect the results of the processes considered in it. In view of this, it is suggested to use the deterministic approach based of the **methods** of the theory of optimal management and dynamic optimization. The **result** of the research is a recurrent algorithm which reduces the initial multi-step task to the implementation of finite sequence of tasks of minimax software management of EIP. In turn, the implementation of each task is reduced to the implementation of finite sequence of only one-step optimizing operations as the tasks of linear convex mathematical and discrete optimization. The following **conclusions** are made: the suggested method makes it possible to work out efficient numerical procedures that enable computer modelling the dynamics of the target task, developing adaptive minimax management of EIP and obtaining an optimal guaranteed result. The results demonstrated in the work can be used for economic and mathematical modelling and solving other tasks of optimizing processes of data prediction and management under the lack of information and under risks as well as for developing corresponding software and hardware complexes to support efficient managerial decisions in practice.

**Keywords:** innovative process, economic and mathematical model, risks, dynamical model, optimization, process of management, minimax adaptive management, guaranteed result.

### Introduction

To achieve the set tasks under increasing competition among Ukrainian enterprises leads to an increase in the amount and complexity of production processes, analysis, planning, management, internal and external relations with suppliers, intermediaries, etc. Effective implementation of tasks linked with these processes is impossible without the appropriate economic and mathematical modelling of managing an enterprise innovative processes (EIP) as a computer information system.

However, innovative activity in the process of dynamic development of production relations cannot be considered fully justified and adapted without using modern approaches of economic and mathematical modelling as an effective tool for theoretical processing and practical generalization of mechanisms and tools for managing the innovative activity of an enterprise that is a complex, open, capable to self-organization and self-development economic system with dynamically changing nondeterministic and conflicting characteristics.

Modelling in EIP management provides for the solution of tasks of software and adaptive control. The result of the EIP software management is forecast values for a certain prospective period of time. But when an innovation process is introduced in each period of time, the model parameters can change (technological processes are compromised, financial indicators, types and suppliers of raw materials change and so on). In addition, in order to obtain a guaranteed result, the task of optimizing the EIP software management under risks takes into account risks that can lead to maximum losses. But risks that lead to a maximum damage can affect a real process. In this case, in order to take into account changes in the economic environment and the current state of an innovation

process, the procedure for adapting the model to the current conditions should be specified on the basis of the results of the EIP software management. Thus, in order to take into account the instability of the innovation process, which is characterized by various "disturbances" as changes in the current state of the production process and the economic environment, an adaptive control model should be developed, which enables correcting and considering the dynamics of the main production characteristics within the innovation process

It should be noted that the problem of economic and mathematical modelling of the EIP adaptive management under uncertainty and risks at enterprises have not been solved yet by scientific researches that deal with the problems of enterprise management, by various economic and mathematical models and techniques for finding out optimal solutions as well as by methods of business process modelling; so this problem remains a burning topic for researching.

### Analysis of literature sources

A number of reputed Ukrainian and foreign scientists deal with the problems of economic and mathematical modelling of production and financial processes, among them are: N.N. Krasovsky [1], A.F. Shorikov [2], A.V. Lotov [3], A.I. Propoy [4], A.V. Ter-Krikorov [5] and others. However, some issues require further elaboration. So, at this stage, there are practically no economic and mathematical models that consider the specificity of production process dynamics, and take into account the impact of risks while managing innovative processes at enterprises and optimizing these processes.

Research and solving the task of the EIP managing requires the development of a dynamic economic and

mathematical model that takes into account control actions, uncontrolled parameters (risks, modelling errors, etc.) and the lack of information. At the same time, available approaches to solving similar problems are based mainly on static models and stochastic modelling apparatus to use which it is necessary to know the probabilistic characteristics of the model main parameters and special conditions for performing the process under consideration. It should be noted that very severe conditions, which usually are unachievable beforehand, are necessary for using the stochastic modelling apparatus.

Economic and mathematical models of such problems are presented, for example, in [6–8]. This article continues the studies presented in [9], the concepts and notation that are introduced in it are used in this paper without additional explanations.

### Developing a generalized EIP management model under risks

Let a multi-step dynamic system be considered for a given integer time,  $\overline{0, T} = \{0, 1, \dots, T\}$  ( $T > 0$ ); this system consists of one controlled object –  $I$  (the subject of management as it is managed by player  $P$ ), whose motion is described by a linear discrete recurrent vector equation:

$$\begin{aligned} \bar{x}(t+1) &= A(t)\bar{x}(t) + B(t)\bar{u}(t) + c(t)\bar{w}(t) + D(t)\bar{v}(t), \\ \bar{x}(0) &= \bar{x}_0. \end{aligned} \quad (1)$$

Here  $t \in \overline{0, T-1}$ ,  $\bar{x} \in \mathbf{R}^{\bar{n}}$  is the phase vector of object  $I$ , which consists of  $\bar{n} = n + m + 2$  coordinates for the model of the EIP management [2], that is  $\bar{x}(t) = (x_1(t), x_2(t), \dots, x_n(t), y_1(t), y_2(t), \dots, y_m(t), Z(t), k(t))' \in \mathbf{R}^{\bar{n}}$ , where, according to the notions in [2],  $x(t) = (x_1(t), x_2(t), \dots, x_n(t))' \in \mathbf{R}^n$  is the vector of amounts of production residues stored at the warehouses of the enterprise over the period of time  $t$ ;  $y(t) = (y_1(t), y_2(t), \dots, y_m(t))' \in \mathbf{R}^m$  is the vector of amounts of residues of production resources stored at the warehouses of the enterprise over the period of time  $t$ ;  $Z(t)$  is the enterprise total costs over the period of time  $t$ ;  $k(t)$  is the amount of available financial resources accumulated before the beginning of period  $t$ ;  $n, m \in \mathbf{N}$ ;  $\mathbf{N}$  is the set of all natural numbers; for  $k \in \mathbf{N}$ ,  $\mathbf{R}^k$  is  $K$ -dimensional Euclidean vector space of column vectors, even if they are written as a string for saving the space);  $\bar{u}(t) = (\bar{u}_1(t), \bar{u}_2(t), \dots, \bar{u}_n(t))' \in \mathbf{R}^{\bar{n}}$  is the vector of innovative management of the intensity of production over the period of time  $t$  ( $t \in \overline{0, T-1}$ ), where each  $j$ -th coordinate  $u_j(t)$  is the value of the  $j$ -th production amount ( $j \in \overline{1, n}$ ) constrained by the given restriction:

$$\bar{u}(t) \in U_1(t) = U_{N_t}(t) \subset \mathbf{R}^{\bar{p}} \quad (\bar{p} \in \mathbf{N}: \bar{p} = n), \quad (2)$$

$U_{N_t}(t)$  is the finite set of vectors for each  $t \in \overline{0, T-1}$ , i.e. the finite set consisting of  $N_t$  ( $N_t \in \mathbf{N}$ ) of vectors in  $\mathbf{R}^n$ , defining all possible implementations of different management scenarios at the moment of time  $t$ ;  $\bar{w}(t) = (w_1(t), w_2(t), \dots, w_{\bar{m}}(t))' \in \mathbf{R}^{\bar{m}}$  ( $\bar{m} = m$ ) is the vector of intensity of replenishment of storage resources over the period of time  $t$  ( $t \in \overline{0, T-1}$ ), which depends on the permissible implementation of management  $\bar{u}(t) \in U_1(t)$  and must meet the following specified limit:

$$\bar{w}(t) \in W_1(\bar{u}(t)) = W_{M_t}(\bar{u}(t)) \subset \mathbf{R}^{\bar{m}} \quad (\bar{m} \in \mathbf{N}: \bar{m} = m); \quad (3)$$

$W_{M_t}(\bar{u}(t))$  is the finite set of vectors for every moment of time  $t \in \overline{0, T-1}$  and management  $\bar{u}(t) \in U_{N_t}(t)$ , i.e. the finite set consisting of  $M_t(i)$  ( $M_t(i) \in \mathbf{N}$ ,  $i \in \overline{1, N_t}$ ) vectors in space  $\mathbf{R}^{\bar{m}}$ , defining all possible realizations of various scenarios of replenishment of the warehouse resources at the moment of time  $t$ .

It is also assumed that for every  $t \in \overline{0, T-1}$ , each permissible realization of the phase vector  $\bar{x}(t) = (x_1(t), x_2(t), \dots, x_n(t), y_1(t), y_2(t), \dots, y_m(t), Z(t), k(t)) \in \mathbf{R}^{\bar{n}}$ , meets the following phase constraint

$$\begin{aligned} \bar{x}(t) &= (x_1(t), x_2(t), \dots, x_n(t), y_1(t), y_2(t), \dots, y_m(t), Z(t), k(t)) \in X_1(t) = \\ &= \begin{cases} x_j(t) \geq 0, & x_j(0) = 0, & j \in \overline{1, n}; \\ y_i(t) \geq 0, & y_i(0) = b_i, & i \in \overline{1, m}; \\ k(t) \geq 0, & k(0) = G + G_0 \geq 0; \\ Z(t) \geq 0, & Z(0) = 0. \end{cases} \end{aligned} \quad (4)$$

where  $G$  is the amount of financial resources of a bank loan intended for investments to the expansion of production within the initial period of management (when  $t = 0$ );  $G_0$  is the amount of own financial resources, deducted from net profit and directed to the expansion of production (when  $t = 0$ );  $\bar{v}(t) = (v(t), v'(t), v''(t))' \in \mathbf{R}^q \times \mathbf{R}^l \times \mathbf{R}^r$  is the generalized vector of risks ( $v(t) = (v_1(t), v_2(t), \dots, v_q(t))' \in \mathbf{R}^q$  is the vector of risks describing possible unfavorable implementations of the vector of priori indefinite factors or the vector combining the modeling error of the considered process, affecting the release of a product unit over the period of time  $t$ ;  $v'(t) = (v'_1(t), v'_2(t), \dots, v'_l(t))' \in \mathbf{R}^l$  is the vector of risks that affects the state of a unit of available resources over the period of time  $t$ ;  $v''(t) = (v''_1(t), v''_2(t), \dots, v''_r(t))' \in \mathbf{R}^r$  is the vector of financial risks affecting a unit of total costs of the enterprise over the period of time  $t$ ;  $q, l, r \in \mathbf{N}$ ) which depends on the permissible implementation of the management  $\bar{u}(t) \in U_1(t)$  during the EIP management over the period of time  $t$  ( $t \in \overline{0, T-1}$ ) and must meet the following specified limit:

$$\bar{v}(t) \in V_1(\bar{u}(t)) \subset \mathbf{R}^{\bar{q}} \quad (\bar{q} \in \mathbf{N}: \bar{q} = q + l + r). \quad (5)$$

Matrices  $\bar{A}(t)$ ,  $\bar{B}(t)$ ,  $\bar{C}(t)$  and  $\bar{D}(t)$  in the vector equation (1) for the economic and mathematical model describing the dynamics of the EIP management are real matrices of orders  $(\bar{n} \times \bar{n})$ ,  $(\bar{n} \times \bar{p})$ ,  $(\bar{n} \times \bar{m})$  and  $(\bar{n} \times \bar{q})$  respectively, and such ones that for all  $t \in \overline{0, T-1}$  matrix

$$\bar{A}(t) = \begin{pmatrix} a_{11}(t) & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & a_{22}(t) & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{mm}(t) & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & r_{11}(t) & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & r_{22}(t) & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & r_{mm}(t) & 0 & 0 \\ z_1(t) & z_2(t) & \dots & z_n(t) & p_1(t) & p_2(t) & \dots & p_m(t) & 1 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix};$$

$$\bar{B}(t) = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 \\ -b_{11} & -b_{12} & \dots & -b_{(1,(n-1))} & -b_{1n} \\ -b_{21} & -b_{22} & \dots & -b_{(2,(n-1))} & -b_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ -b_{m1} & -b_{m2} & \dots & -b_{(m,(n-1))} & -b_{mn} \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}; \quad \bar{C}(t) = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix};$$

$$\bar{D}(t) = \begin{pmatrix} -c_{11} & -c_{12} & \dots & -c_{1q} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ -c_{11} & -c_{12} & \dots & -c_{1q} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ -c_{n1} & -c_{n2} & \dots & -c_{nq} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & -c'_{11} & -c'_{12} & \dots & -c'_{1l} & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & -c'_{21} & -c'_{22} & \dots & -c'_{2l} & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & -c'_{m1} & -c'_{m2} & \dots & -c'_{ml} & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & -c''_1 & -c''_2 & \dots & -c''_r \end{pmatrix}.$$

It should be pointed out that for all  $t \in \overline{0, T-1}$  set  $U_1(t)$  in the restriction (2) is not empty and is the finite set consisting of  $N_t$  ( $N_t \in \mathbf{N}$ ) vectors of space  $\mathbf{R}^{\bar{p}}$ ; for all  $t \in \overline{0, T-1}$  and vectors  $\bar{u}(t) \in U_1(t)$ , set  $W_1(\bar{u}(t))$  in the restriction (3) is not empty and is the finite set consisting of  $M_t(i)$  ( $M_t(i) \in \mathbf{N}$ ,  $i \in \overline{1, N_t}$ ) vectors of space  $\mathbf{R}^{\bar{m}}$ ; set  $X_1(t)$ , according to its definition (4) is not empty and is a convex, closed and bounded polyhedron (with the finite number of vertices) in space  $\mathbf{R}^{\bar{n}}$ ; it is assumed that set  $V_1(\bar{u}(t))$  in the restriction (5) is not

$\bar{A}(t)$  is nondegenerate, i.e. there exists an inverse matrix  $\bar{A}^{-1}(t)$  corresponding to it, and the rank of matrix  $\bar{B}(t)$  equals  $\bar{p}$  (the dimension of vector  $\bar{u}(t)$ ).

For the considered EIP management process [2], these matrices have the following specific form:

empty and is a convex, closed, and bounded polyhedron (with the finite number of vertices) in space  $\mathbf{R}^{\bar{q}}$ .

Let us describe the information capabilities of player  $P$  in the process of minimax adaptive (according to the feedback principle) management of EIP for a discrete dynamical system (1) – (5).

It is assumed that while managing EIP for any moment of time  $\tau \in \overline{1, T}$  and the corresponding integer time interval  $\overline{0, \tau} \subseteq \overline{0, T}$  ( $0 < \tau$ ) up to the moment of time  $\tau$  player  $P$  has measured and stored the following values:  $\bar{x}(0) = \bar{x}_0$ , i.e. the initial phase state of object I;  $\bar{u}(\cdot) = \{\bar{u}(t)\}_{t \in \overline{0, \tau-1}}$ , i.e. the history of the implementation

of player  $P$  management over the period  $\overline{0, \tau}$ ;  $\bar{w}(\cdot) = \{\bar{w}(t)\}_{t \in \overline{0, \tau-1}}$ , i.e. the history of implementation of the vector of the intensity of replenishment of the warehouse resources over the period of time  $\overline{0, \tau}$ ;  $\bar{v}(\cdot) = \{\bar{v}(t)\}_{t \in \overline{0, \tau-1}}$ , i.e. the history of the implementation of the vector of risks over the period of time  $\overline{0, \tau}$ . Equation (1) and constraints for it (2) – (5) are also known.

The considered process of the EIP management is estimated by the value of the convex functional  $\tilde{F} : \mathbf{R}^{\bar{n}} \rightarrow \mathbf{R}^1$  defined at possible implementations of the phase vector  $\bar{x}(T) \in \mathbf{R}^{\bar{n}}$  of the system (1) – (5) at the final moment of time  $T$ .

Then, for system (1) – (5) from the point of view of player  $P$  the goal of optimal adaptive management can be formulated as follows: for a given time interval  $\overline{0, T}$  player  $P$  should organize management  $\bar{u}(\cdot) = \{\bar{u}(t)\}_{t \in \overline{0, T-1}}$  (for all  $t \in \overline{0, T-1}$ :  $\bar{u}(t) \in U_1(t)$ ) according to the feedback principle (as the implementation of the minimax adaptive strategy [1], [3], [4] from the selected class of admissible adaptive strategies), using all the available information about this process in such a way that possible maximum value of the functional  $\tilde{F}$  defined on vector  $\bar{x}(T) \in \mathbf{R}^{\bar{n}}$  (where  $\bar{x}(T)$  is the implementation of the phase vector of object  $I$  at the moment of time  $T$  which corresponds to management  $\bar{u}(\cdot)$ ) was minimal.

In this case it is assumed that the worst (largest) values of functional  $\tilde{F}$  can be implemented with respect to possible unfavorable realizations  $\bar{v}(\cdot) = \{\bar{v}(t)\}_{t \in \overline{0, T-1}}$  (for all  $t \in \overline{0, T-1}$ :  $\bar{v}(t) \in V_1(\bar{u}(t))$ ) of generalized risk vector; while the implementations  $\bar{w}(\cdot) = \{\bar{w}(t)\}_{t \in \overline{0, T-1}}$  (for all  $t \in \overline{0, T-1}$ :  $\bar{w}(t) \in W_1(\bar{u}(t))$ ) of the vector of the intensity of the replenishment of the warehouse resources further  $P$  player's goals, i.e. their selection (according to player  $P$ ) is aimed at minimizing the functional  $\tilde{F}$  according to the strategy he has selected.

### Formalizing the task of optimization of the EIP adaptive management

It should be noted that definitions and notations which were introduced in work [2] are straightly used in this section while formalizing and solving the task of the EIP minimax software management since the considered dynamic model (1) – (5) coincides with the model for this task in [2].

To assess the quality of the EIP management by player  $P$  under adaptive management in the dynamic system (1) – (5) over a time interval, a vector terminal functional (the process quality index)  $F_{\tau, T} = (F_{\tau, T}^{(1)}, F_{\tau, T}^{(2)}, \dots, F_{\tau, T}^{(r)})$  is introduced  $\tau, T \subseteq \overline{0, T}$  similarly to the formalization described in work [2]. This functional is a collection

of  $r$  convex functionals of  $F_{\tau, T}^{(k)} : \hat{G}(\tau) \times U(\tau, T) \times W(\tau, T; \bar{u}(\cdot)) \times V(\tau, T; \bar{u}(\cdot)) \rightarrow \mathbf{R}^1$  ( $k \in \overline{1, r}$ ) such as. to implement the set  $(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) \in \hat{G}(\tau) \times U(\tau, T) \times W(\tau, T; \bar{u}(\cdot)) \times V(\tau, T; \bar{u}(\cdot))$ , where  $g(\tau) = \{\tau, \bar{x}(\tau)\} \in \hat{G}(\tau)$ , their values are determined by the following relation:

$$F_{\tau, T}^{(k)}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = F_{\tau, T}^{(k)}(\bar{x}_{\tau, T}(T; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))) = F_{\tau, T}^{(k)}(\bar{x}(T)), \quad k \in \overline{1, r}, \quad (6)$$

where  $F_{\tau, T}^{(k)} : \mathbf{R}^{\bar{n}} \rightarrow \mathbf{R}^1$  is the convex functional for each  $k \in \overline{1, r}$ ;  $\bar{x}(T) = \bar{x}_{\tau, T}(T; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))$ .

On the basis of the vector functional  $F_{\tau, T} = (F_{\tau, T}^{(1)}, F_{\tau, T}^{(2)}, \dots, F_{\tau, T}^{(r)})$  introduced by relation (6), to assess the quality of the process of optimizing the considered EIP management, the scalar objective function  $F_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))$  is introduced; its values for all admissible implementations of  $(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) \in \hat{G}(\tau) \times U(\tau, T) \times W(\tau, T; \bar{u}(\cdot)) \times V(\tau, T; \bar{u}(\cdot))$  over the time interval  $\tau, T$ , where  $g(\tau) = \{\tau, \bar{x}(\tau)\} \in \hat{G}(\tau)$ ,  $\bar{u}(\cdot) = \{\bar{u}(t)\}_{t \in \overline{\tau, T-1}} \in U(\tau, T)$ ,  $\bar{w}(\cdot) = \{\bar{w}(t)\}_{t \in \overline{\tau, T-1}} \in W(\tau, T; \bar{u}(\cdot))$ ,  $\bar{v}(\cdot) = \{\bar{v}(t)\}_{t \in \overline{\tau, T-1}} \in V(\tau, T; \bar{u}(\cdot))$ , are determined according to the following relation:

$$F_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = \sum_{k=1}^r \mu_k \cdot F_{\tau, T}^{(k)}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = \sum_{k=1}^r \mu_k \cdot F_{\tau, T}^{(k)}(\bar{x}_{\tau, T}(T; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))) = \sum_{k=1}^r \mu_k \cdot F_{\tau, T}^{(i)}(\bar{x}(T)) = \tilde{F}(\bar{x}(T)).$$

$$\forall k \in \overline{1, r}: \mu_k \geq 0, \quad \sum_{k=1}^r \mu_k = 1, \quad (7)$$

where  $\bar{x}(T) = \bar{x}_{\tau, T}(T; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))$ , and  $\tilde{F}$  is the convex functional introduced earlier.

The objective function (functional)  $F_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))$  is a convex scalar convolution of the vector functional  $F_{\tau, T} = (F_{\tau, T}^{(1)}, F_{\tau, T}^{(2)}, \dots, F_{\tau, T}^{(r)})$ , i.e. it is formed according to the method of scalarization of vector objective functions (e.g.[6]), with nonnegative weighting factors  $\mu_k$ ,  $k \in \overline{1, r}$ , which can be determined, for example, by expertise or on the basis of statistical information on the history of the implementation of the main parameters of the considered EIP management.

Assume that player  $P$  having selected management  $\bar{u}(t) \in \bar{U}_1(t)$ ,  $t \in \overline{0, T-1}$  in the dynamic system (1) – (5) for a given period of time  $\overline{0, T}$  ( $T > 0$ ) is under agreed awareness conditions. Then, on the basis of stated above, from the position of player  $P$  his goal in the task of the

EIP minimax adaptive management for the dynamic system (1) – (5) can be formulated as follows.

Player P over the period of time  $\overline{0, T}$  is supposed to arrange the selection of his management  $\bar{u}(\cdot) = \{\bar{u}(t)\}_{t \in \overline{0, T-1}}$  (for all  $x \ t \in \overline{0, T-1}$ :  $u(t) \in \hat{U}_1(t)$ ) of object I in the adaptive mode (according to the feedback principle) knowing his  $\tau$  – position  $g(t) = \{t, \bar{x}(t)\} \in \hat{G}(t)$  at every moment of time  $t \in \overline{0, T-1}$  so that functional  $F_{\overline{0, T}}$ , determined by relation (7) when  $\tau = 0$  has the smallest possible value when the implementation of the EIP management is completed. It should be taken into consideration that the worst values of the vector function  $\bar{v}(\cdot) \in V(\overline{0, T}; \bar{u}(\cdot))$  can be realized, i.e. maximizing the given functional, and the realization of the vector function  $\bar{w}(\cdot) \in W(\overline{0, T}; \bar{u}(\cdot))$  furthers player's P goal.

Then, using the above arguments and similarly to [3], [4] the achievement of this goal of player P can be formalized in the following way.

The permissible strategy of the EIP adaptive management  $U_a$  of player P for a discrete dynamical system (1) – (5) over the time interval  $\overline{0, T}$  can be mapping  $U_a: \hat{G}(\tau) \rightarrow U_1(\tau)$  that assigns set  $U_a(g(\tau)) \subseteq U_1(\tau)$  of  $\bar{u}(\tau) \in U_1(\tau)$  management of player P to each moment of time  $\tau \in \overline{0, T-1}$  and to possible realization of  $\tau$  – position  $g(\tau) = \{\tau, \bar{x}(\tau)\} \in \hat{G}(\tau)$  ( $g(0) = g_0$ ). The set of all admissible management strategies for player P for the process considered through  $U_a^*$  is denoted.

Further, the group of motions of object I over the time interval  $\overline{0, T}$ , corresponding to the equation of motion (1), the initial position P  $g_0 = \{0, \bar{x}_0\} \in \hat{G}_0$  of player P, the permissible strategy  $U_a = U_a(g^*(t)) \in U_a^*$ ,  $t \in \overline{0, T-1}$ ,  $g^*(t) = \{t, \bar{x}^*(t)\} \in \hat{G}(t)$ , and the admissible software implementation of the intensity of the replenishment of the warehouse resources  $\bar{w}(\cdot) \in W(\overline{0, T}; \bar{u}_a(\cdot))$ , where  $\bar{u}_a(\cdot) = \{\bar{u}_a(t)\}_{t \in \overline{0, T-1}} \in U(\overline{0, T})$  any admissible management of player P over the time interval  $\overline{0, T}$  generated by strategy  $U_a$  will be called as follows:

$$\begin{aligned} X(\cdot; \overline{0, T}, g_0, U_a, \bar{w}(\cdot)) &= \{\bar{x}^*(\cdot): \bar{x}^*(\cdot) \in S_n(\overline{0, T}), \exists \bar{u}^*(\cdot) \in U(\overline{0, T}), \\ &\quad \exists \bar{v}^*(\cdot) \in V(\overline{0, T}; \bar{u}^*(\cdot)), \\ &\quad \forall t \in \overline{0, T}, \bar{x}^*(t) = \bar{x}_{\overline{0, T}}(t; \bar{x}_0, \bar{u}_t^*(\cdot), \bar{w}_t^*(\cdot), \bar{v}_t^*(\cdot)), \\ &\quad g^*(t) = \{t, \bar{x}^*(t)\} \in \hat{G}(0, g_0, t, \bar{u}_t^*(\cdot), \bar{w}_t^*(\cdot)) \subseteq \hat{G}(t), \quad g^*(0) = g_0, \\ &\quad \bar{u}_t^*(\cdot) = \{\bar{u}(\tau)\}_{\tau \in \overline{0, t-1}}, \quad \forall t \in \overline{0, T-1}, \quad \bar{u}^*(t) \in U_a(g^*(t)), \\ &\quad \bar{w}_t^*(\cdot) = \{\bar{w}(\tau)\}_{\tau \in \overline{0, t-1}}, \quad \bar{v}_t^*(\cdot) = \{\bar{v}(\tau)\}_{\tau \in \overline{0, t-1}}\}. \end{aligned} \quad (8)$$

Then the following nonlinear multi-step problem of the EIP minimax adaptive management for the dynamical system (1) – (5) can be formulated.

Task 1. For the given time interval  $\overline{0, T}$  ( $T > 0$ ) and initial position  $g_0 = \{0, \bar{x}_0\} \in \hat{G}_0$  of player P in the discrete dynamical system (1) – (5), the strategy of the EIP minimax adaptive management  $U_a^{(e)} = U_a^{(e)}(g(t)) \in U_a^*$ ,  $g(t) = \{t, \bar{x}(t)\} \in \hat{G}(t)$ ,  $t \in \overline{0, T-1}$ , ( $g(0) = g_0$ ), should be found, which meets the relation

$$\begin{aligned} F_{\overline{0, T}}^{(e,a)} &= \min_{\bar{w}(\cdot) \in W(\overline{0, T}; \bar{u}_a^{(e)}(\cdot))} \max_{\bar{v}(\cdot) \in V(\overline{0, T}; \bar{u}_a^{(e)}(\cdot))} F_{\overline{0, T}}(g_0, U_a^{(e)}, \bar{w}(\cdot), \bar{v}(\cdot)) = \\ &= \min_{U_a \in U_a^*} \min_{\bar{w}(\cdot) \in W(\overline{0, T}; U_a(\cdot))} \max_{\bar{v}(\cdot) \in V(\overline{0, T}; U_a(\cdot))} F_{\overline{0, T}}(g_0, U_a, \bar{w}(\cdot), \bar{v}(\cdot)) = \\ &= \min_{U_a \in U_a^*} \min_{\bar{w}(\cdot) \in W(\overline{0, T}; U_a(\cdot))} \max_{\bar{x}(T) \in X(T; \overline{0, T}, g_0, U_a, \bar{w}(\cdot))} \tilde{F}(\bar{x}(T)) = \\ &= \max_{\bar{x}(T) \in X(T; \overline{0, T}, g_0, U_a^{(e)}, \bar{w}^{(e)}(\cdot))} \tilde{F}(\bar{x}(T)) = c_F^{(e,a)}(\overline{0, T}, g_0), \end{aligned} \quad (9)$$

as the realization of the finite sequence of only one-step operations.

Here functional  $F_{\overline{0, T}}$  is determined according to the relation (7);  $\bar{u}_a(\cdot) = \{\bar{u}_a(t)\}_{t \in \overline{0, T-1}} \in U(\overline{0, T})$  is any admissible management of player P over the time interval  $\overline{0, T}$  generated by strategy  $U_a$ ;  $\bar{u}_a^{(e)}(\cdot) = \{\bar{u}_a^{(e)}(t)\}_{t \in \overline{0, T-1}} \in U(\overline{0, T})$  is any admissible management of player P over the time interval  $\overline{0, T}$  generated by the strategy  $U_a^{(e)}$ .

The number  $c_F^{(e,a)}(\overline{0, T}, g_0) = F_{\overline{0, T}}^{(e,a)}$  will be called the optimal guaranteed (minimax) result of the minimax adaptive management of P player's EIP over the time interval  $\overline{0, T}$  for the discrete dynamical system (1) – (5) concerning its initial position  $g_0$  and functional  $F_{\overline{0, T}}$ .

It should be noted that the above conditions for the parameters of the system (1) – (5) and the results of works [3], [4] demonstrate that there is the solution for this task.

Further, for any realizations of management  $\bar{u}_a^{(e)}(\cdot) = \{\bar{u}_a^{(e)}(t)\}_{t \in \overline{0, T-1}}$ ,  $\forall t \in \overline{0, T-1}$ :  $\bar{u}_a^{(e)}(t) \in U_a^{(e)}(w^{(e)}(t))$  of player P generated by strategy  $U_a^{(e)} \in U_a^*$ , for vector functions  $\bar{w}_a^{(e)}(\cdot) \in W(\overline{0, T}; \bar{u}_a^{(e)}(\cdot))$  and  $\bar{v}_a(\cdot) \in V(\overline{0, T}; \bar{u}_a^{(e)}(\cdot))$ , for the group of motions  $\bar{x}^{(e)}(\cdot) = \bar{x}_{\overline{0, T}}(\cdot; \bar{x}_0, \bar{u}_a^{(e)}(\cdot), \bar{w}_a^{(e)}(\cdot), \bar{v}_a(\cdot)) \in$

$X(\cdot; \overline{0, T}, g_0, U_a, \bar{w}_a^{(e)}(\cdot))$  corresponding to it, on the basis of relations (6) – (9), it is not difficult to show the validity of the following equation:

$$\begin{aligned} &F_{\overline{0, T}}(g_0, u_a^{(e)}(\cdot), \bar{w}_a^{(e)}(\cdot), \bar{v}_a(\cdot)) = \\ &= \tilde{F}(\bar{x}_{\overline{0, T}}(T; \bar{x}_0, u_a^{(e)}(\cdot), \bar{w}_a^{(e)}(\cdot), \bar{v}_a(\cdot)) = \tilde{F}(\bar{x}_a^{(e)}(T)) \leq \\ &\leq c_F^{(e,a)}(\overline{0, T}, g_0) = F_{\overline{0, T}}^{(e,a)} \leq F_{\overline{0, T}}^{(e)} = c_F^{(e)}(\overline{0, T}, g_0), \end{aligned} \quad (10)$$

where  $g_0 = \{0, \bar{x}_0\} \in \hat{G}_0$ ;  $c_F^{(e)}(\overline{0, T}, g_0)$  is the optimal guaranteed (minimax) result of solving the problem of the EIP minimax software management.

It should be noted that the relations (6) demonstrate that the result of solving task 1 can only improve the result of solving the task of the EIP minimax software management, i.e. the EIP minimax adaptive management is more promising in comparison with the minimax software management for the considered process.

Thus, in this section we formalize the task of the EIP minimax adaptive management for the dynamical system (1) – (5).

It should be noted that task 1 is the main one in this chapter, but its formalization and solution are based on the task of the EIP minimax software management [2].

### General pattern for solving task 1

The general pattern for solving task 1 on the basis of the results of [2] – [4] is suggested.

Using the solution of the task of the EIP minimax software management considered in the previous chapter, for all the moments of time  $\tau \in \overline{0, T-1}$  and all  $\tau$ -positions  $g^{(e)}(\tau) = \{\tau, \bar{x}^{(e)}(\tau)\} \in \hat{G}(\tau)$  ( $g^{(e)}(0) = g_0 = \{0, \bar{x}_0\} \in \hat{G}_0$ ) of player  $P$ , where  $\bar{x}^{(e)}(\tau) = \bar{x}_{0, \tau}(\tau; \bar{x}_0, \bar{u}^{(e)}(\cdot), \bar{w}^{(e)}(\cdot), \bar{v}(\cdot))$ ,  $\bar{u}^{(e)}(\cdot) \in U_F^{(e)}(\overline{0, T}, g^{(e)}(0))$ ,  $\bar{w}^{(e)}(\cdot) \in W(\overline{0, T}; \bar{u}^{(e)}(\cdot))$  the following sets can be developed:

$$\tilde{U}^{(e)}(g^{(e)}(\tau)) = \{\tilde{u}^{(e)}(\tau): \tilde{u}^{(e)}(\tau) \in U_1(\tau), \tilde{u}^{(e)}(\tau) = \bar{u}^{(e)}(\tau), \bar{u}^{(e)}(\cdot) \in U_F^{(e)}(\overline{\tau, T}, g^{(e)}(\tau)), \tau \in \overline{0, T-1}\}, \quad (11)$$

where  $U_F^{(e)}(\overline{\tau, T}, g^{(e)}(\tau))$  is the set of minimax software managements developed from the solution of the corresponding task of the EIP minimax software management considered in the previous chapter.

Then the management strategy  $\tilde{U}_a^{(e)} = \tilde{U}_a^{(e)}(g(\tau)) \in U_a^*$ ,  $\tau \in \overline{0, T-1}$ ,

$$\begin{aligned} \tilde{c}_F^{(e,a)}(\overline{0, T}, g_0) &= F_{0, T}^{(e,a)}(g_0, \bar{u}_a^{(e)}(\cdot), \bar{w}_a^{(e)}(\cdot), \bar{v}(\cdot)) = \\ &= F_{T-1, T}^{(e,a)}(g_a^{(e)}(T-1), \bar{u}_a^{(e)}(T-1), \bar{w}_a^{(e)}(T-1), \bar{v}(T-1)) \leq \\ &\leq F_{T-1, T}^{(e,a)}(g_a^{(e)}(T-1), \bar{u}_a^{(e)}(T-1), \bar{w}_a^{(e)}(T-1), \bar{v}^{(e)}(T-1)) = \\ &= \max_{\bar{v}^{(e)}(T-1) \in V(\overline{T-1, T}; \bar{u}_a^{(e)}(\cdot))} F_{T-1, T}^{(e,a)}(g_a^{(e)}(T-1), \bar{u}_a^{(e)}(T-1), \bar{w}_a^{(e)}(T-1), \bar{v}^{(e)}(T-1)) \leq \\ &\leq \max_{v(\cdot) \in V(\overline{0, T}; \bar{u}_a^{(e)}(\cdot))} F_{0, T}^{(e,a)}(g_0, \bar{u}_a^{(e)}(\cdot), \bar{w}_a^{(e)}(\cdot), v(\cdot)) \leq c_F^{(e,a)}(\overline{0, T}, g_0) \leq c_F^{(e)}(\overline{0, T}, g_0). \end{aligned} \quad (14)$$

On the basis of the results of [2] – [4] and relations (11) – (14), the following statement, which is the main result of this paper, can be justified.

Statement 1. For the given initial position  $g(0) = g_0 = \{0, \bar{x}_0\} \in \hat{G}_0$  of player  $P$  in the discrete dynamical system (1) – (5) the strategy of the EIP management  $\tilde{U}_a^{(e)} \in U_a^*$  over the period of time  $\overline{0, T}$  which is determined by relations (11) – (13), is the minimax adaptive management strategy for task 1, i.e.

$g(\tau) \in \hat{G}(\tau)$  ( $g(0) = g_0$ ) of player  $P$  for the considered EIP minimax adaptive management in a discrete dynamical system over the time interval  $\overline{0, T}$  from all admissible management strategies  $U_a^*$  is determined; it is formally described by the following relations:

For all  $\tau \in \overline{0, T-1}$  and  $\tau$ -positions

$$g^{(e)}(\tau) = \{\tau, \bar{x}^{(e)}(\tau)\} \in G(0, g_0, \tau, \bar{u}_\tau^{(e)}(\cdot), \bar{w}_\tau^{(e)}(\cdot))$$

( $g^{(e)}(0) = g_0$ ) assume that

$$\tilde{U}_a^{(e)}(g^{(e)}(\tau)) = \tilde{U}^{(e)}(g^{(e)}(\tau)) \subseteq U_1(\tau). \quad (12)$$

For all  $\tau \in \overline{0, T-1}$  and  $\tau$ -positions

$$g^*(\tau) = \{\tau, x^*(\tau)\} \in$$

$$\in \{\hat{G}(\tau) \setminus G(0, g_0, \tau, \bar{u}_\tau^{(e)}(\cdot), \bar{w}_\tau^{(e)}(\cdot))\} \quad (g^*(0) \neq g_0) \text{ assume}$$

$$\text{that } \tilde{U}_a^{(e)}(g^*(\tau)) = U_1(\tau), \quad (13)$$

where  $\bar{u}_\tau^{(e)}(\cdot) = \{\bar{u}^{(e)}(t)\}_{t \in \overline{0, \tau-1}}$ ,  $\bar{u}^{(e)}(\cdot) \in U_F^{(e)}(\overline{0, T}, g_0)$ ;  $\bar{w}_\tau^{(e)}(\cdot) = \{\bar{w}^{(e)}(t)\}_{t \in \overline{0, \tau-1}}$ ,  $\bar{w}^{(e)}(\cdot) \in W(\overline{0, T}; \bar{u}^{(e)}(\cdot))$ .

Let  $\bar{u}_a^{(e)}(\cdot) = \{\bar{u}_a^{(e)}(t)\}_{t \in \overline{0, T-1}} \in U(\overline{0, T})$  be the realization of the management of player  $P$  over the time interval  $\overline{0, T}$ , which is developed as a result of using strategy  $\tilde{U}_a^{(e)} \in U_a^*$  over this interval of time, and the realization of vector functions  $\bar{w}_a^{(e)}(\cdot) \in W(\overline{0, T}; \bar{u}_a^{(e)}(\cdot))$  and  $\bar{v}(\cdot) \in V(\overline{0, T})$ . Then, for  $(T-1)$ -position  $g_a^{(e)}(T-1) = \{T-1, \bar{x}_a^{(e)}(T-1)\} \in \hat{G}(T-1)$  of player  $P$  (here  $\bar{x}_a^{(e)}(T-1) = \bar{x}_{0, T}(\cdot; \bar{x}_0, \bar{u}_a^{(e)}(\cdot), \bar{w}_a^{(e)}(\cdot), \bar{v}_a(\cdot))$ ), which corresponds to these realizations, the following relations are true

selected class of admissible adaptive management strategies, the recurrent algorithm that reduces the initial multistage problem to the realization of the finite sequence of tasks of the EIP minimax software management is suggested.

In turn, the solution of each of these tasks is reduced to the realization of the finite sequence of only one-step

optimization operations as solving the tasks of linear and convex mathematical programming, as well as discrete optimization (e.g. [3], [4]). Then it can be stated that the solution of the considered task 1 is reduced to the solution of the finite sequence of tasks of linear convex mathematical programming and discrete optimization.

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## ЗАДАЧА МІНІМАКСНОГО АДАПТИВНОГО УПРАВЛІННЯ ІННОВАЦІЙНИМИ ПРОЦЕСАМИ НА ПІДПРИЄМСТВІ З УРАХУВАННЯМ РИЗИКІВ

**Предметом** дослідження статті є дискретна динамічна система, що складається з об'єкта, динаміка якого описується векторним лінійним дискретним рекурентним співвідношенням і схильна до впливу керованих параметрів (управлінь) і неконтрольованого параметра (вектору ризиків або перешкоди). Передбачається, що фазові стани об'єкта, керуючі впливи та вектор ризиків динамічної системи, що розглядається, в кожен момент часу обмежені заданими кінцевими або опуклими багатограними множинами в відповідних скінченновимірних векторних просторах. **Ціллю** статті є моделювання адаптивного управління інноваційними процесами підприємства (ІПП) при наявності ризиків що вимагає виконання наступних **завдань**: формування моделі програмного управління ІПП при наявності ризиків; формалізація задачі оптимізації адаптивного управління ІПП та загальної схеми її вирішення у вигляді гарантованого результату на основі мінімакса (оптимізації гарантованого результату) на заданий фінальний момент часу з урахуванням наявності ризиків. При цьому під ризиками в системі управління ІПП будемо розуміти фактори, які впливають негативно або катастрофічно на результати розглянутих в ній процесів. З цією метою пропонується використовувати детермінований підхід на основі **методів** теорії оптимального управління та динамічної оптимізації. **Результатом** дослідження є рекурентний алгоритм, який зводить вихідне багатокрокове завдання до реалізації кінцевої послідовності завдань мінімаксного програмного управління ІПП. У свою чергу, рішення кожного з таких завдань зводиться до реалізації кінцевої послідовності тільки однокрокових оптимізаційних операцій в формі вирішення завдань лінійного опуклого математичного програмування та дискретної оптимізації. **Висновки**: запропонований метод дає можливість розробляти ефективні чисельні процедури, що дозволяють реалізувати комп'ютерне моделювання динаміки розглянутої задачі, сформувані адаптивне мінімаксне управління ІПП та отримати оптимальний гарантований результат. Представлені в роботі результати можуть бути використані для економіко-математичного моделювання та вирішення інших завдань оптимізації процесів прогнозування даних і управління в умовах дефіциту інформації та наявності ризиків, а також для розробки відповідних програмно-технічних комплексів для підтримки прийняття ефективних управлінських рішень на практиці.

**Ключові слова**: інноваційний процес, економіко-математична модель, ризики, динамічна модель, оптимізація, процес управління, мінімаксний адаптивний менеджмент, гарантований результат.

## ЗАДАЧА МИНИМАКСНОГО АДАПТИВНОГО УПРАВЛЕНИЯ ИННОВАЦИОННЫМИ ПРОЦЕССАМИ НА ПРЕДПРИЯТИИ С УЧЕТОМ РИСКОВ

**Предметом** исследования статьи является дискретная динамическая система, состоящая из объекта, динамика которого описывается векторным линейным дискретным рекуррентным соотношением и подвержена влиянию управляемых параметров (управлений) и неконтролируемого параметра (вектора рисков или помехи). Предполагается, что фазовые состояния объекта, управляющие воздействия и вектор рисков рассматриваемой динамической системы в каждый момент времени стеснены заданными конечными или выпуклыми многогранными множествами в соответствующих конечномерных векторных пространствах. **Целью** статьи является моделирование задачи адаптивного управления инновационными процессами предприятия (ИПП) при наличии рисков, что требует выполнение следующих **задач**: формирование модели програмного управления ИПП при наличии рисков; формализация задачи оптимизации адаптивного управления ИПП и общей схемы ее решения в виде гарантированного результата на основе минимакса (оптимизации гарантированного результата на заданный финальный момент времени с учетом наличия рисков. При этом под рисками в системе управления ИПП будем понимать факторы, которые влияют негативно или катастрофически на результаты рассматриваемых в ней процессов. С этой целью предлагается использовать детерминированный подход на основе **методов** теории оптимального управления и динамической оптимизации. **Результатом** исследования является рекуррентный алгоритм, который сводит исходную многошаговую задачу к реализации конечной последовательности задач минимаксного програмного управления ИПП. В свою очередь, решение каждой из таких задач сводится к реализации конечной последовательности только одношаговых оптимизационных операций в форме решения задач линейного выпуклого математического программирования и дискретной оптимизации. **Выводы**: предлагаемый метод дает возможность разрабатывать эффективные численные процедуры, позволяющие реализовать компьютерное моделирование динамики рассматриваемой задачи, сформировать адаптивное минимаксное управление ИПП и получить оптимальный гарантированный результат. Представленные в работе результаты могут быть использованы для экономико-математического моделирования и решения других задач оптимизации процессов прогнозирования данных и управления в условиях дефицита информации и наличия рисков, а также для разработки соответствующих программно-технических комплексов для поддержки принятия эффективных управленческих решений на практике.

**Ключевые слова**: инновационный процесс, экономико-математическая модель, риски, динамическая модель, оптимизация, процесс управления, минимаксное адаптивное управление, гарантированный результат.

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