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OPTIMIZATION OF TRANSPORTATION ROUTES IN A CLOSED LOGISTICS SYSTEM

The **subject** of research in the article is the network of closed logistics. The **goal** of the work is the creation of mathematical models and methods for solving problems of optimizing transportation routes in closed-loop logistics systems, taking into account many topological and functional limitations. The following **tasks** are solved in the article: the development of a systemological model for the problem of optimizing transportation routes in a closed logistics system; development of a mathematical model of the problem of structural and topological optimization of a three-level centralized network of closed micro-logistics routes for global transportation; development of a mathematical model for the optimization of ring transportation routes in a closed logistics system for local transportation; development of methods for optimizing ring transport routes in a closed logistics system. The following **methods** are used: methods of system theory, graph theory, methods of the theory of utility, optimization and research of operations. The following **results** were obtained: the decomposition of the problem of optimizing transportation routes in closed-loop logistics systems at the macro and micro levels was performed; a systemological model of the problem has been developed, which reflects the whole complex of tasks that are solved at various stages of the life cycle of logistics systems. To improve the efficiency of closed-loop logistics systems, a joint solution to the problems of network optimization for the transportation of direct product flows and return flows has been proposed. A mathematical model of the problem of structural and topological optimization of a three-level centralized network of closed micro-logistics routes for global transportation has been developed. To solve such problems, methods using directed enumeration of options are recognized effective; a mathematical model of the optimization problem of the network of closed micro-logistics routes for local transportation with the simultaneous transportation of the contents of the forward and reverse flows was developed; To solve the problem of optimizing closed-loop micro-logistics route networks at the local transportation level while simultaneously transporting the contents of the forward and reverse flows, modifications of the Clark-Wright methods are proposed, as well as a method based on coordinate-wise optimization and insertion schemes; experimental studies of the proposed modifications of the methods made it possible to obtain estimates of their temporal complexity. **Conclusions**: Practical use of the proposed mathematical models and modifications of methods for optimizing closed-loop logistics systems by jointly solving problems for direct and reverse flows will reduce the cost of implementing transport companies. The obtained estimates of the time complexity of the optimization methods will make it possible to predict the costs of computing and time resources in their practical use for solving optimization problems of closed-loop logistics networks.

Keywords: logistics network; direct and reverse flow networks; closed logistics; network optimization; systemological model.

Introduction

Profitability and environmental friendliness of business processes and other anthropogenic processes is largely determined by the quality of the corresponding logistics. Traditionally, the logistics processes covered the main stages of economic activity, starting from the development of sources of raw materials and ending with the delivery of products and services to the final consumer [1]. Traditionally, the logistics processes covered the main stages of economic activity, starting from the development of sources of raw materials and ending with the delivery of products and services to the final consumer. One of the most important planning problems in supply chain management (SCM) is supply chain network design (SCND) [2]. A solution to optimize the supply chain of SC (supply chain) needs to be sustainable and viable in complex and uncertain external environment. In the light of current trends in environmental safety, environmental (green) logistics has emerged and is developing rapidly in recent decades. [3–4]. It covers management activities in the logistics cycle from the rational use of natural raw materials to waste management activities. The methodology of environmental logistics is aimed at reducing the level of risks of economic losses caused by environmental degradation, as well as improving the environmental and economic efficiency of enterprises [5]. One of the directions of ecological logistics is reverse logistics, in which the ordering and systematization of reverse commodity-material,

information, cash flows is carried out [6-7]. Traditionally, direct and reverse logistics tasks are regarded as conditionally independent, which does not allow obtaining globally effective solutions to the logistics problem as a whole. The joint solution of direct and reverse logistics problems gives rise to many new tasks that require formalization and development of effective methods for solving them.

Analysis of the problem and existing methods for solving it

Macro- and micro-logistic systems are distinguished in logistics [8]. Macrological systems form flows and processes within the framework of the interaction of several independent property objects, usually not related to territorial distribution. Macro logistics tasks are solved for international, transcontinental companies or intermediary organizations. Micrologistics solve the issues of interaction of elements of one or several enterprises assembled into a group of common economic interests or an individual enterprise [9]. Transport micro-logistics of a relatively large enterprise is divided into two conventional systems: global and local transportation. In the general case, the task of global transportation is the task of optimal placement of intermediate cargo points (warehouses, hubs, etc.) to minimize the cost of cargo transportation [10].

A typical example of the global transportation problem can be the structural and topological synthesis of

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ISSN 2522-9818 (print)
Сучасний стан наукових досліджень та технологій в промисловості. 2019. № 4 (10)   *ISSN 2524-2296 (online)*

a three-level centralized transport and storage system at the regional level in the following statement [11]. Given the location of the supply center, a set of consumers of products $I = \{ i : i = \overline{1,n} \}$, for each of which a location on the transport network of the region is determined, the volume of products ordered by each of them and the transport tariff for its delivery. It is necessary to determine the optimal number and location of distribution centers (DC), as well as a subset of consumers served by each DC $I_j = \{ i \}, j = \overline{1,l^o}$. In practice, there are two main varieties of the problem: if there are restrictions on the places of possible placement of the DC and without restrictions on the places of their possible placement. An example of restrictions is the requirement for the location of the DC only at the base or in close proximity to consumers.

The objective function of this task is the reduced costs of delivering products to consumers.

$$C = \sum_{j=1}^{l} c_j + \sum_{i=1}^{n} \sum_{j=}^{l} q_i r_i d_{ij} v_{ij} \rightarrow \min_{r,d,v}, \qquad (1)$$

where n – number of product consumers; l – number of DC; q_i – volume of freight traffic to the i-th consumer; r_i – transport tariff for the i-th consumer; d_{ij} – distance between the i-th consumer and the j-th DC; c_j – the costs given for the j-th DC; v_{ij} – boolean variable: $v_{ij} = 1$, if the i-th customer is served by the j-th DC, $v_{ij} = 0$ – otherwise.

To solve such problems with objective functions of the form (1) – (3), methods for the synthesis of geographically distributed objects with radial-node structures that use the idea of directional enumeration in the direction of increasing the number of nodes are quite effective. [11–13]. Their temporal complexity is of order $O[n^2]$, and the error in solving problems with the number of consumers $n \leq 50$ is less than 5%.

The vast majority of practical problems of optimizing local transportation are formulated as variants of the traveling salesman task (ST) in terms of graph theory, and among the methods for solving them, exact, heuristic and metaheuristic are distinguished [14]. The most effective accurate algorithms for solving the problem of one or several salesmen without additional restrictions are based on the branch and bound method [15–16]. Heuristic methods make it possible to find approximate solutions of ST taking into account many practically important limitations. They use various search schemes in the vicinity of some basic solution. Among them, the iterative two-phase method (algorithm) is one of the most effective, including the steps of clustering and building routes using, for example, the Clark-Wright algorithm. Metaheuristic methods (algorithms) are based on modeling self-organization processes and, thanks to the use of special procedures (mutation, multi-start); allow finding approximations of optimal solutions in an acceptable time.

In the process of optimizing logistics structures, environmental requirements for production, storage, transportation and processing of waste products are increasingly being taken into account. This requires solving, together with traditional tasks, the problems of optimizing the return flows from consumers to the places of production or disposal of waste. Such tasks are solved within the framework of green (reverse) logistics [17, 18].

Competition conditions orient modern companies towards the rapid development of new types of products, which leads to changes in flows in all supply chains. With relatively insignificant changes in flows, the problems of adapting existing structures are solved, which requires decision-making under conditions of uncertainty, changing demand and other data [2, 19]. With significant changes in flows, a decrease in the efficiency of existing structures is observed, leading to the need for their reengineering. With this in mind, technologies for reengineering logistics structures should provide for a joint solution to the problems of optimizing closed (direct and reverse) route networks in terms of a variety of functional, cost, environmental indicators and restrictions. This determines the urgency of the tasks of optimizing transportation routes in closed logistics systems, taking into account the restrictions arising from the interaction of oncoming flows.

The purpose of this article is to develop methods for solving the problems of optimizing transportation routes in a closed logistics system, taking into account many topological and functional limitations. To achieve the goal, the article solves the following tasks:

- development of a systemological model for the problem of optimizing transportation routes in a closed logistics system;

- development of a mathematical model of the structural and topological optimization problem for a three-level centralized network of closed micro-logistics routes for global transportation;

- development of a mathematical model for the optimization of ring transportation routes in a closed logistics system for local transportation;

- development of methods for optimizing ring transport routes in a closed logistics system.

Results of the study

Closed logistics systems are objects consisting of many geographically distributed elements (suppliers, processors, terminals, consumers), interconnected by route networks. The structure of each of these systems can be represented as a tuple $< E, R >$ (where E – a set of elements; R – a set of connections (routes) between elements). Based on the set of acceptable locations for the terminals of the system, each of the structure $< E, R >$ options can have different topologies G^*. Given this, each of the topological implementations of the system $s = < E, R, G >$, $G = < G_E, G_R, G_A >$ (where $G \in G^*$ – topological

implementation of the structure $< E, R >$; G_E – elements' topology; G_R – connections' topology; G_A – routes' topology) will correspond its own set of functional and cost properties defined by some mapping [20, 21]: $\varphi : (E, R, G) \rightarrow P$.

Subsets of elements E', relationships between elements R' and topologies G' on which it can be implemented can be determined as a result of the analysis of the purpose of creating the system, the conditions of its functioning and its desired properties P'. On this basis, subsets of elements, links, and topologies $E^o \subseteq E'$, $R^o \subseteq R'$, $G^o \subseteq G'$, are determined that ensure the obtaining of required properties P' (allowable cost values, flow delivery time, etc.) with minimal costs when meeting technological $R_T \subseteq R_T^*$ and ecological $R_E \subseteq R_E^*$ restrictions

$$s^o = \arg\max_{s \in S^*} Q(s), \quad R_T(s) \subseteq R_T^*, \quad R_E(s) \subseteq R_E^*, \qquad (2)$$

where s – system building option; S^* – a set of acceptable options for building a system; $Q(s)$ – evaluation of the effectiveness of the s-th system design option.

According to the incomplete certainty of the requirements for the properties of the logistic system, it is proposed to use the membership function of the fuzzy set "the best option for building the system" as an assessment of efficiency $P(s)$ (function of general utility) (2) [22, 23]. In this case, the fuzzy set "the best option for constructing a technological system" can be represented as a set of ordered pairs:

$$\text{«The\_best\_option\_for\_building\_a\_system»} = \{< s, P(s) >\}, \quad (3)$$

where $s \in S$ – system building option; $P(s)$ – degree of affiliation of the option $s \in S$ to a fuzzy set "the best option for building the logistic system".

To quantify the effectiveness, we use the most universal additive-multiplicative model based on the Kolmogorov-Gabor polynomial:

$$P(s) = \sum_{i=1}^{m} \lambda_i \xi_i(s) + \sum_{i=1}^{m} \sum_{j=i}^{m} \lambda_{ij} \xi_i(s) \xi_j(s) + $$
$$+ \sum_{i=1}^{m} \sum_{j=i}^{m} \sum_{k=j}^{m} \lambda_{ijk} \xi_i(s) \xi_j(s) \xi_k(s) ... \qquad (4)$$

where m – the number of particular criteria characterizing the properties of the system; $\lambda_i, \lambda_{ij}, \lambda_{ijk}$ – weighting factors of particular criteria and their products; $\lambda_i \geq 0$, $\lambda_{ij} \geq 0$, $\lambda_{ijk} \geq 0$, $\xi_l(s)$ – utility function of particular criterion $k_l(x)$, $l = i, j, k$.

It is proposed to use a modified gluing function as a utility function of particular criteria, which has advantages in the accuracy of approximation of expert estimates and computation time in comparison with the Gauss, Harrington and logistic functions [22]:

$$\xi(s) = \begin{cases} \bar{a}(b_1+1)\left(1 - \dfrac{b_1}{b_1 + \dfrac{\bar{k}(s)}{\bar{k}_a}}\right), & 0 \leq \bar{k}(s) \leq \bar{k}_a; \\[4ex] \bar{a} + (1-\bar{a})(b_2+1)\left(1 - \dfrac{b_2}{b_2 + \dfrac{\bar{k}(s) - \bar{k}_a}{1 - \bar{k}_a}}\right), & \bar{k}_a < \bar{k}(s) \leq 1, \end{cases} \qquad (5)$$

where $\bar{k}_a, \bar{a}$ – normalized coordinates of the gluing point, $0 \leq \bar{k}_a \leq 1$, $0 \leq \bar{a} \leq 1$; b_1, b_2 – coefficients that determine the form of the dependence on the initial and final segments of the function.

The problem of optimizing transportation routes in closed logistics systems as a meta-task consists of a set of tasks that require the development of new mathematical models and methods for solving them. The two-level scheme of its decomposition includes complexes of tasks at the macro and micro levels, and each of the tasks can be considered as a converter of input data to output [21]:

$$Task_i^l : In_i^l \rightarrow Out_i^l, \quad MetaTask = \{Task^l\},$$
$$Task^l = \{Task_i^l\}, \quad l = \overline{1,2}, \quad i = \overline{1,6}, \qquad (6)$$

where In_i^l, Out_i^l – input and output data of the i-th task of the l-th level.

Most of the macro-level tasks are optimization tasks and are characterized by limitations reflecting the specifics of the main stages of the system life cycle: $Task_1^l$ – formation of requirements for the system and statement of its optimization problem; $Task_2^l$ – system design; $Task_3^l$ – system development planning; $Task_4^l$ – modernization of the system; $Task_5^l$ – system reengineering. At the micro level, the tasks of optimizing the logistics system that arise at the stages of its pre-project research, design and operation are solved: $Task_1^2$ – selection of construction principles; $Task_2^2$ – structure optimization; $Task_3^2$ – topology optimization; $Task_4^2$ – optimization of the functioning technology; $Task_5^2$ – optimization of parameters of elements and relationships; $Task_6^2$ – performance evaluation and choice of solutions.

The task of structural and topological synthesis of a centralized three-level system of closed logistics is characterized by the requirement to take into account the reverse flows from customers to production or processing centers (utilization). Let n be the number of points in the closed logistics system for which location coordinates are given, as well as their types (end-user, production, processing or terminal). It is necessary to synthesize an effective three-tier structure, in which the production and processing centers are located on the first level, the terminals on the second and the final consumers on the third. The costs of the network with direct and reverse

flows consist of the amount of the reduced costs for the delivery of the direct stream from the production center to the terminals C_{PT}, for processing in the terminals to the centers C_{T1}, C_{T2}, for the delivery of the direct stream from the terminals to the final consumers C_{TS}, for the delivery of the reverse stream from the final consumers to the terminals C_{ST}, reverse flow delivery from terminals to the processing center C_{TP} in the form of [20]:

$$C = C_{PT} + C_{T1} + C_{TS} + C_{ST} + C_{T2} + C_{TP}. \qquad (7)$$

The mathematical model of this problem with the objective function (7) for the case of territorial combination of production and processing points can be represented as:

$$C = \sum_{i=1}^{n} c_i x_{ii} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij} \to \min_{x}, \qquad (8)$$

$$X = [\, x_{ij}\,] = \begin{cases} x_{ij} \in \{0,1\}, \ i,j = \overline{1,n}; \\ \sum_{i=j}^{n} x_{ij} \geq 1, \ \forall j = \overline{1,n}; \\ \sum_{j=1}^{n}\sum_{i=j}^{n} x_{ij} = 2(\,n-1\,) + \sum_{i=1}^{n} x_{ii}, \end{cases} \qquad (9)$$

where c_i – the given costs for a terminal located on the basis of the i-th structural element; x_{ij} – element of a matrix describing the relationships of network elements; c_{ij} – the cost of delivering the flow from point i to point j.

The constraint system in task (8) – (9) describes the entire set of admissible three-level centralized structures with forward and reverse flows.

The basic task of local transportation in a closed micro-logistics system is the optimization of ring routes. Various formulations of such a problem are variants of the TS problem. Their characteristic feature is that the direct flow (of goods, materials, components) can be delivered to the points of the logistics network and the reverse flow (containers, defective goods, waste) can be collected from the points of the network by the same vehicle. This feature requires changes in mathematical models and methods for solving problems of optimizing ring route networks.

The basic problem of optimizing a single ring route in a closed micro-logistics system is considered in the following statement. Asked: a lot of system elements dispersed throughout the territory (delivery and pick-up points) $I = \{i\}, i = \overline{1, n}$, their characteristics (matrix of distances or fares $C = [\, c_{ij}\,]$, $i,j = \overline{1, n+1}$, correspondence matrix $W = [\, w_{ij}\,]$, $i,j = \overline{1, n+1}$, where $n+1$ – the number of the point in which the base is located) and the characteristics of the vehicle used (speed, cost of operation, carrying capacity, transported volume). It is necessary to determine the route of detour

points $R = [\, r_{ij}\,]$, satisfying constraints and extremizing selected performance criteria. The criteria used are indicators of minimum financial costs, mileage, and time to deliver the contents of the streams. The restrictions are weight, volume of transported cargo, carrying capacity, vehicle capacity.

Taking this into account, the mathematical model of the basic task of optimizing the ring route in a closed-loop logistics system by the criterion of minimum costs can be represented as:

$$\sum_{i=1}^{n+1}\sum_{j=1}^{n+1} c_{ij} r_{ij} \to \min_{R} \qquad (10)$$

$$\sum_{i=1}^{n+1} r_{ij} = 1, j = \overline{1, n}, \quad \sum_{j=1}^{n+1} r_{ij} = 1, i = \overline{1, n}, \qquad (11)$$

$$u_i - u_j + n\, r_{ij} \leq n-1, \quad i, j = \overline{2, n+1}, i \neq j, \quad (12)$$

$$z_0 = \sum_{j=1}^{n} w_{n+1,j}, z_k = z_{k-1} + \Delta z_k,$$

$$\Delta z_k = w_{n+1,k} - w_{k,n+1}, z_k \leq z^* \ \forall k = 1, n, \quad (13)$$

where c_{ij} – cost of delivery of goods from point i to point j; r_{ij} – boolean variable ($r_{ij} = 1$, if the route passes directly from point i to point j; $r_{ij} = 0$, otherwise); u_i – auxiliary variables, $u_i \geq 0$, $i = \overline{1, n+1}$; z_0 – loading (filling) a vehicle at the base; z_k – loading (filling) of the vehicle at the point k, $k = \overline{1,n}$; $w_{n+1,k}$ – weight (volume) of cargo delivered from the base to the point k, $k = \overline{1,n}$; $w_{k,n+1}$ – weight (volume) of cargo delivered from the point k to the base, $k = \overline{1,n}$; Δz_k – changing the loading of the vehicle at the point k, $k = \overline{1,n}$; z^* – carrying capacity (capacity) of the vehicle.

Constraints (11) – (12) correspond to the constraints of the classical TS. They provide the connection of each route point with only two other route points, the requirement to start and end the route at the point $n+1$ and ensure its isolation. New restrictions $z_k \leq z^* \ \forall k = 1, n$ introduced in (13) exclude overloading of the vehicle delivering the contents of the direct and reverse flows. For the classic transportation task, loading a vehicle in the database with the contents of the direct stream is $z^{(P)} = \sum_{j=1}^{n} w_{n+1,j}$, $z^{(P)} \leq z^*$. There is no reverse flow $z^{(O)} = \sum_{j=1}^{n} w_{j,n+1} = 0$. For the task of collecting reverse flow content $z^{(P)} = \sum_{j=1}^{n} w_{n+1,j} = 0$, and the weight (volume) of the reverse flow is $z^{(O)} = \sum_{j=1}^{n} w_{j,n+1} \leq z^*$.

In the task of optimizing the set of ring transportation routes in a closed micro-logistics system, it is necessary to determine the required number of vehicles

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ISSN 2522-9818 (print)
ISSN 2524-2296 (online)   *Innovative technologies and scientific solutions for industries. 2019. No. 4 (10)*

p and detour routes $R = [\,r_{ij}\,]$ that satisfy the restrictions and extremize the selected performance criteria. As a criterion, indicators of minimum financial costs, mileage, time and the required number of vehicles for the delivery of the contents of the streams are used. The restrictions are weight, volume of transported cargo, length, time of the route.

It is proposed to use an estimate of the delivery time of the contents of a stream along routes τ as an indicator of the system's responsiveness. Depending on the specific formulation of the problem, the maximum $\tau = \max\limits_{1 \le l \le p} \tau_l$ or

weighted average time $\tau = \sum\limits_{l=1}^{p} \tau_l w_l$, where τ_l – maximum delivery time of cargo along the l-th route; p – number of routes; w_l, $l = \overline{1,\ p}$ – specific gravity of the l-th route.

With this in mind, the mathematical model of one route (10) – (13) is supplemented by a restriction for the formation of p subsets of points, on the basis of each of which a minimum cost route will be formed:

$$\sum_{i=1}^{n} r_{i,\,n+1} = \sum_{j=1}^{n} r_{n+1,\,j} = p \ . \quad (14)$$

The problem to be solved with a fixed location of the base can be considered as one of the TS modifications - the task of an undetermined number of salesmen. For a fixed location of the base and a given number of substructures (depending on the type and detail of restrictions), the formulated problem can be considered, for example, as the task p of a salesman with a central base.

When determining the search strategy for the optimal number of routes, we will take into account that increasing their number increases their total length and reduces the load on the vehicles used. Thus, increasing the number of routes increases the cost of delivering the contents of the forward and reverse streams. When solving the problem, it is necessary to determine the minimum number of routes p that ensure the fulfillment of restrictions of the form (11)–(14).

Taking this into account, the scheme of the proposed method for solving the problem can be represented as follows. To determine the minimum number of nodes $p = p_{min}$, enforcing restrictions (13): $p_{min} = \lceil z_0 / z^* \rceil$. To synthesize a radial-ring structure by solving the problem $p = p_{min}$ salesmen with a central base.

To determine the characteristics of the resulting option, verify that all system restrictions are met (11)–

(13). To increase the number of routes, if necessary $p := p+1$. To optimize the network of p routes, to verify compliance with restrictions, to compare with the option for $p-1$ routes and to choose the best option. Continue the decision until all restrictions are met.

The solution of the salesmen p task with a central base, depending on available resources, can be obtained using two main approaches: by reducing it to the TS and by forming p subsets of points with the subsequent solution of the TS for each of them.

The essence of the first approach is to split a single-ring structure into a multi-ring. For this purpose, it is necessary to build a network for one salesman. To do this, we introduce a fictitious item $p-1$ with numbers $n+2, n+3, n+p$. We expand the matrix by adding a row $p-1$ and a column $p-1$ to it. Unlike [24], we will make the cost of transportation $C = [c_{ij}]$ between the points entered in this network equal:

$$c_{i,\,n+j} = c_{i,\,n+1}, \ i = \overline{1,\,n+1} \ \ i = \overline{1,\,n+1}, \ j = \overline{2,\,p} \ ,$$
$$c_{n+i,\,j} = c_{n+1,\,j}, \ \ i = \overline{2,\,p} \ , \ \ j = \overline{1,\,n+1} \ , \ \ c_{n+i,\,n+j} = \infty \ ,$$
$$i, j = \overline{2,\,p} \ . \quad (15)$$

After solving the TS with an extended cost matrix $C = [c_{ij}]$ to obtain p routes, we combine all items with numbers large than n, in one item with a number $n+1$ (13).

In the framework of the second approach, modifications of the Clark-Wright method [14–15] and a method using the idea of coordinate-wise optimization are proposed.

The essence of the modification of the Clark-Wright method consists in additional verification for each of the points of the obtained ring routes of restrictions (13) on vehicle loading. The proposed modification of the method has a relatively low time complexity; however, taking into account the constraints of the form (13) in it increases the error of the obtained solutions. Table 1 shows the results of an experimental study of the modification of the Clark-Wright method for solving sets of 10 problems with dimensions from 4 to 100 points (fig. 1).

Based on the results of the study, a polynomial time complexity estimate is obtained for a modification of the Clark-Wright method $t[n] = 0,0004n^2 + 0,094n + 7,25$.

Table 1. *The results of an experimental study of the time complexity of the modification of the Clark-Wright method*

Task dimension, n	Minimal decision time $t(n)$, ms	Maximum decision time $t(n)$, ms	Most frequent decision times $t(n)$, ms	Average decision time $t(n)$, ms
4	5	7	5	6,0
9	7	11	8	9,0
10	7	12	8	9,5
25	8	12	11	10,0
50	10	15	12	12,5
75	11	20	15	17,0
100	14	27	16	20,5

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ISSN 2522-9818 (print)
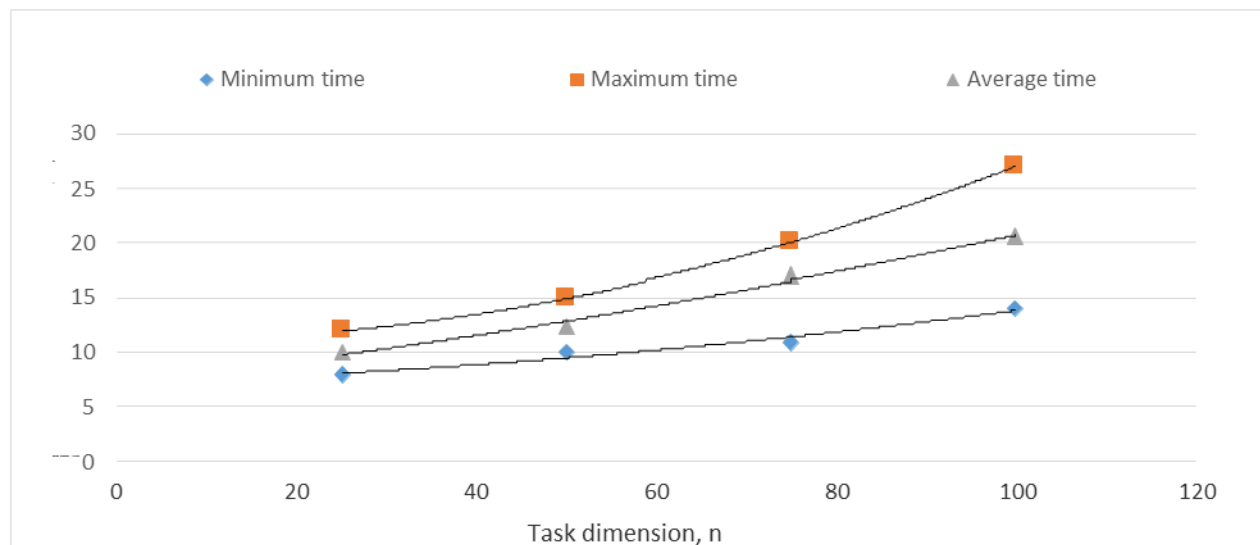*Сучасний стан наукових досліджень та технологій в промисловості. 2019. № 4 (10)* *ISSN 2524-2296 (online)*

Fig. 1. Dependences of the time for solving the problem by the modified Clark-Wright method on its dimension

The essence of modifying the method based on coordinate-wise optimization is as follows. Fictitious points p which play the role of auxiliary points are located on the territory of the transportation. Subsets of elements included in one route are formed according to these points. The separation of points along the routes is carried out on a territorial basis within the p sectors relative to the base. As a criterion for assigning an object to a k -th route, it is proposed to use the minimum of the function, which for a symmetric matrix of costs has the form:

$$ k^o = \arg \min_{1 \le k \le p} \{ c_{n+1,k} + c_{k,i} + c_{i,n+1} \} . \qquad (16) $$

Criterion (16) allows minimizing the cost of the cycle "base - route point - base" for the point included in the route.

The results of an experimental study of the modification of the method based on coordinate wise optimization are given in table 2 (fig. 2).

Table 2. *Results of an experimental study of the time complexity of modifying a method based on a coordinate-wise optimization scheme*

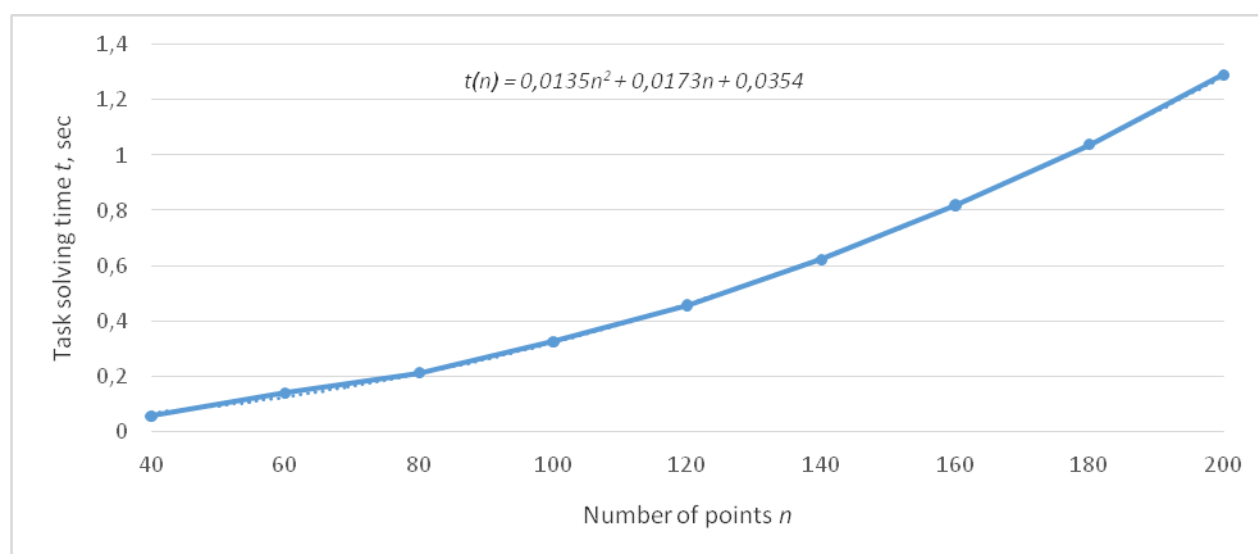| n	40	60	80	100	120	140	160	180	200
$t(n)$, c	0,055	0,138	0,211	0,324	0,456	0,621	0,818	1,038	1,291

Fig. 2. Dependences of the time for solving the problem modified on the basis of the coordinate-wise optimization scheme on its dimension

According to the results of solving 100 tasks, the average value of the relative error of the method using the insertion scheme was $\varepsilon = 0,024$, and its maximum value was $\varepsilon_{max} = 0,168$. Moreover, the relative error of 84% of

the solutions did not exceed 5%. The method has polynomial time complexity $O[n^2]$. The accuracy of the method can be improved by applying the multistart procedure, which involves multiple solutions to the problem with the choice of different places for the initial placement of fictitious auxiliary points.

Conclusions

In the process of analyzing the problem of optimizing transportation routes in the closed logistics system, it was decomposed at the macro and micro levels. A systemological model of the problem has been developed, which reflects the whole complex of tasks that are solved at various stages of the life cycle of logistics systems. To improve the efficiency of closed logistics systems, a joint solution to the problems of network optimization for the transportation of direct product flows and return flows of containers, waste, defective products has been proposed. A mathematical model of the problem of structural and topological optimization of a three-level centralized network of closed micro-logistics routes for

global transportation has been developed. To solve such problems, methods using directed enumeration of options are recognized as effective. A mathematical model of the optimization problem for the network of closed micro-logistics routes for local transportation with the simultaneous transportation of the contents of the forward and reverse flows has been developed. To solve it, modifications of the Clark-Wright methods are proposed, as well as a method based on coordinate-wise optimization and insertion schemes. Based on the results of experimental studies of the proposed modifications of the methods, estimates of their time complexity are obtained. The obtained estimates will make it possible to predict the costs of computing and time resources in the practical use of methods for solving optimization problems of closed logistics networks.

The practical use of the proposed mathematical models and modifications of optimization methods for closed-loop logistics systems through the joint solution of problems for direct and reverse flows will reduce the cost of implementing transport activities of companies.

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ОПТИМІЗАЦІЯ МАРШРУТІВ ПЕРЕВЕЗЕНЬ У СИСТЕМІ ЗАМКНЕНОЇ ЛОГІСТИКИ

Предметом дослідження в статті є мережі замкненої логістики. **Мета** роботи – створення математичних моделей і методів вирішення завдань оптимізації маршрутів перевезень у системах замкненої логістики з урахуванням множини топологічних і функціональних обмежень. У статті вирішуються наступні **завдання**: розробка системологічної моделі проблеми оптимізації маршрутів перевезень у системі замкненої логістики; розробка математичної моделі задачі структурно-топологічної оптимізації трирівневої централізованої мережі маршрутів замкненої мікрологістики для глобальних перевезень; розробка математичної моделі задачі оптимізації кільцевих маршрутів перевезень у системі замкненої логістики для локальних перевезень; розробка методів оптимізації кільцевих маршрутів перевезень у системі замкнутої логістики. Використовуються такі **методи**: методи теорії систем, теорія графів, методи теорії корисності, оптимізації та дослідження операцій. Отримані такі **результати**: виконана декомпозиція проблеми оптимізації маршрутів перевезень у системах замкненої логістики на макро- і мікрорівні; розроблена системологічна модель проблеми, що відображає весь комплекс завдань, що вирішуються на різних етапах життєвого циклу логістичних систем; для підвищення ефективності систем замкненої логістики запропоновано спільне розв'язання задач оптимізації мереж для транспортування прямих потоків продукції та зворотних потоків; розроблена математична модель задачі структурно-топологічної оптимізації трирівневої централізованої мережі маршрутів замкненої мікрологістики для глобальних перевезень. Для вирішення таких завдань ефективними визнані методи, які використовують спрямований перебір варіантів; розроблена математична модель задачі оптимізації мережі маршрутів замкненої мікрологістики для локальних перевезень з одночасним транспортуванням вмісту прямого та зворотного потоків; для розв'язання задачі оптимізації мереж маршрутів замкненої мікрологістики на рівні локальних перевезень з одночасним транспортуванням вмісту прямого та зворотного потоків запропоновані модифікації методів Кларка-Райта, а також методу на основі схем покоординатної оптимізації та вставки; експериментальні дослідження запропонованих модифікацій методів дозволили отримати оцінки їх часової складності. **Висновки**: Практичне використання запропонованих математичних моделей і модифікацій методів оптимізації систем замкненої логістики за рахунок спільного розв'язання задач для прямих і зворотних потоків дозволить скорочувати витрати на реалізацію транспортної діяльності компаній. Отримані оцінки часової

складності методів оптимізації дозволять прогнозувати витрати обчислювальних і часових ресурсів при їх практичному використанні для розв'язання задач оптимізації мереж замкненої логістики.

Ключові слова: логістична мережа; мережі прямих і зворотних потоків; замкнена логістика; оптимізація мереж; системологічна модель.

ОПТИМИЗАЦИЯ МАРШРУТОВ ПЕРЕВОЗОК В СИСТЕМЕ ЗАМКНУТОЙ ЛОГИСТИКИ

Предметом исследования в статье являются сети замкнутой логистики. **Цель** работы – создание математических моделей и методов решения задач оптимизации маршрутов перевозок в системах замкнутой логистики с учетом множества топологических и функциональных ограничений. В статье решаются следующие **задачи**: разработка системологической модели проблемы оптимизации маршрутов перевозок в системе замкнутой логистики; разработка математической модели задачи структурно-топологической оптимизации трехуровневой централизованной сети маршрутов замкнутой микрологистики для глобальных перевозок; разработка математической модели задачи оптимизации кольцевых маршрутов перевозок в системе замкнутой логистики для локальных перевозок; разработка методов оптимизации кольцевых маршрутов перевозок в системе замкнутой логистики. Используются следующие **методы**: методы теории систем, теория графов, методы теории полезности, оптимизации и исследования операций. Получены следующие **результаты**: выполнена декомпозиция проблемы оптимизации маршрутов перевозок в системах замкнутой логистики на макро- и микроуровне; разработана системологическая модель проблемы, отражающая весь комплекс задач, решаемых на различных этапах жизненного цикла логистических систем; для повышения эффективности систем замкнутой логистики предложено совместное решение задач оптимизации сетей для транспортировки прямых потоков продукции и обратных потоков; разработана математическая модель задачи структурно-топологической оптимизации трехуровневой централизованной сети маршрутов замкнутой микрологистики для глобальных перевозок. Для решения таких задач эффективными признаны методы, использующие направленный перебор вариантов; разработана математическая модель задачи оптимизации сети маршрутов замкнутой микрологистики для локальных перевозок с одновременной транспортировкой содержимого прямого и обратного потоков; для решения задачи оптимизации сетей маршрутов замкнутой микрологистики на уровне локальных перевозок с одновременной транспортировкой содержимого прямого и обратного потоков предложены модификации методов Кларка-Райта, а также метода на основе схем покоординатной оптимизации и вставки; экспериментальные исследования предложенных модификаций методов позволили получить оценки их временной сложности. **Выводы**: Практическое использование предложенных математических моделей и модификаций методов оптимизации систем замкнутой логистики за счет совместного решения задач для прямых и обратных потоков позволит сокращать затраты на реализацию транспортной деятельности компаний. Полученные оценки временной сложности методов оптимизации позволят прогнозировать затраты вычислительных и временных ресурсов при их практическом использовании для решения задач оптимизации сетей замкнутой логистики.

Ключевые слова: логистическая сеть; сети прямых и обратных потоков; замкнутая логистика; оптимизация сетей; системологическая модель.

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Безкоровайний В. В., Куропатенко О. В., Гобов Д. А. Оптимізація маршрутів перевезень у системі замкненої логістики. *Сучасний стан наукових досліджень та технологій в промисловості*. 2019. № 4 (10). С. 24–32. DOI: https://doi.org/10.30837/2522-9818.2019.10.024.

Beskorovainyi, V., Kuropatenko, O., Gobov, D. (2019), "Optimization of transportation routes in a closed logistics system", *Innovative Technologies and Scientific Solutions for Industries*, No. 4 (10), P. 24–32. DOI: https://doi.org/10.30837/2522-9818.2019.10.024.