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COMPUTER SIMULATIONS OF CONTROLLABILITY PROCESSES FOR ROBOTIC WHEELED PLATFORMS TAKING INTO ACCOUNT RESTRICTIONS OF JERK MOTIONS

The computer simulations are considered as the required tool to design the suitable autonomous control systems optimal in different senses and especially in providing the restrictions of the jerk motions for the robotic wheeled platforms. The **subject** matter of this research is the development of the theory and methods for computer simulations of the controllability processes of the robotic wheeled platforms. The **goal** of this research is to consider the jerks of the wheeled platforms, and the jerks are reduced to the limitation of the acceleration time derivative of the mass center of the wheeled platform, so that this derivative is considered as the quantitative estimation of the jerks. The incorrectness in the Hadamard's sense for direct defining the jerks by differentiations of the phase coordinates in the case of computer simulations using the numerical methods is discussed. **Tasks** of this research are in developing the generalized approaches for mathematical modelling and computer simulations and in theoretical receiving of the properties inherent for the wheeled platforms and suitable for verification the computer simulations results, as well as in making the calculations to have the quantitative results about the controllability processes for the particular case of the electromechanical four-wheeled platform under the straight motion with the mode of speeding-up from the state of the rest. **Methods** of this research are based on the Lagrange's equations of second kind, as well as on the electromechanical analogies, and on final representing the mathematical models in the form of the system of the first ordered ordinary differential equations with the initial conditions for further numerical solving. The computer simulations are accomplished by using the Scilab free open source software. **Results** of this research are in the proposed suitable way for computing the jerks by the phase coordinates without its differentiations allow excluding the incorrectness in the Hadamard's sense, as well as in representing the controllability processes for the electromechanical wheeled platform, including the results for the velocities, the accelerations and the jerks which are necessary to illustrate the controllability processes for the robotic wheeled platforms. By comparison with the theoretically established inherent properties of the wheeled platforms it is shown the correctness of the results of the computer simulations. **Conclusions** about this research are that the developed approaches for computer simulations of the controllability processes for the robotic wheeled platforms allow considering influence of the control on the different characteristics including the velocity, the acceleration, as well as the jerk motions which are required for designing the controls optimal in different senses.

Keywords: Robotics; Wheeled Platforms; Jerk; Controllability; Mathematical Model; Computer Simulations.

Introduction

Implementation of the robotic wheeled platform for excluding human from the executed processes is one of the main modern trends of the technological development, and the robotic wheeled platforms are widely used now for industrial, transportation, military, police, house-holding and different other purposes. Developing of the improved control is significantly required to have the necessary properties providing the suitable operating for the robotic wheeled platforms relevant to its operation purposes. It is clearly understood that developing and designing the improved control is mostly based on using the opportunities of improved mathematical modelling and computer simulations of the controllability processes for the wheeled platforms, and this research is exactly directed to developing the approaches for computer simulations of the controllability processes of the robotic wheeled platforms which is critically required for designing the control systems optimal in different senses and relevant for the operation purposes. Thus, the theme of this research is of current interest, because of full agreement with the modern global trend in technological development by means wide implementing the robotic systems including the wheeled platforms for making the different operations supporting by the relevant control systems properties.

Analysis of last achievements and publications

The existed wide interest to the problems about control of the robotic wheeled platforms are due to the wide range of the operation purposes for these platforms, and it can be illustrated by the follows researches [1-3]. Such interest to the control problems is due to the different circumstances [4–6]. First of all, the structural and algorithmic designs of the control systems are principally predefined by the operation purposes of the controlled systems, so wide range of the operational purposes for the robotics wheeled platforms inherent for the modern industries will naturally require the different kinds of the control systems that must be designed. Besides, trends in implementing the robotic systems including based on the wheeled platform cover permanently the new areas of purposes, and it will require developing the new kinds of the control systems for the robotic wheeled platform relevantly to theirs new purposes. The understandable differences between controls required for the robotic wheeled platforms used in the industrial technological processes and for the police purposes is the illustration in necessities in developing the different kinds of the control systems for the robotic wheeled platforms relevantly to their purposes. Secondly, the operational environment can be significantly different even for the robotic wheeled platforms with the identical purposes, and theirs control systems must be relevant to the particular operational environment, so the wide range of the possible operational environments leads to necessities of different kinds control systems for the

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robotic wheeled platforms [7]. Really, the different environments have the different and even unpredictable external influences on the robotic wheeled platforms, and the control systems must provide compensations of these influences to have the exactly required current states of the platform. The understandable differences between the wheeled platform on the soft soil and the wheeled platform on the on the rigid road is the illustration of influencing the environment on the requirements to the control systems. Thus, the wide range of requirements to the control systems for the robotic wheeled platforms lead the permanent interest to the problems form the areas about the structural and algorithmic designs of such control systems, and each separate research deals with some particular problem from this area like controlling the velocity, the acceleration, the path, the jerks and others.

It is understandable that the mathematical models are used to substantiate the structural and algorithmic designs of the control systems including for the robotic wheeled platforms, and computer simulations give the significant opportunities to have the extended imaginations about the designed systems even without having the physical prototypes. Due to these circumstances, the computer simulations of the controllability processes are widely used for designing the structural and algorithmic structures of the control systems for the robotic wheeled platforms, and the relevant methodologies of such simulations must be developed, so developing of the methodologies for mathematical modelling and computer simulations of the robotic wheeled platforms are in current interest at present [8-10]. Interest to methodologies of mathematical modelling and the computer simulations of the robotic wheeled platforms are permanently due to a lot of existed particularities inherent for the different kinds of the wheeled platforms which must be considered in the mathematical models and must be imagined during the computer simulations. These particularities can include the difficultly predicted interactions between the wheels and the soils and the roads, the aerodynamics effects during different velocities, the different kinds of damping in the driving mechanisms. Besides, the perfect mathematical models and computer simulations are significantly required for providing the improved indirect measurements and for designing the intelligent control systems including for the autonomous wheeled platforms.

Highlight of the earlier unresolved parts of the general problem. Aim of the study

It seems that the problems of the robotic wheeled platforms control are fully researched due to a lot of existed publications [1-3], but the most of the existed publications deals with the separate particularities of the general problem about control of the robotic wheeled platforms [4–6], so this general problem is actually not fully researched now. Besides, permanent implementing the new kinds of the robotic wheeled platforms requires the developing the relevant control systems [7], so it seems that the problem about control of the robotic wheeled platforms is and will be permanently in current interest too. Nevertheless, the problem about control the robotic wheeled platform is widely researched now, especially in the particulars of controlling the velocities, the accelerations and the paths including theirs planning. At the same time, optimization the control is not fully researched, especially in considering the different optimum criteria like the used power, the life time, and the jerks. Control of robotic wheeled platforms taking into account the motions' jerks restrictions is one of the not fully developed problem at present, although it really has significant importance for delicate cargo transportation [11], as well as for providing the suitable operation conditions for the sensitive on-board measure devices and systems. It is naturally that considering of the different optimal criteria of the designed control for the robotic wheeled platforms requires the approaches for mathematical modelling and computer simulations relevant with these optimal criteria. Taking into account all these, the subject matter of this research is to develop the theory and methods for computer simulations of the controllability processes of the robotic wheeled platforms, but the goal of this research is in considering the jerks of the wheeled platforms.

To realize this subject matter and goal of this research it is planned to solve the follows tasks:

- developing the generalized approaches for mathematical modelling and computer simulations and theoretical receiving of the properties inherent for the wheeled platforms and suitable for verification the computer simulations results;
- making the calculations to have the quantitative results about the controllability processes for the particular case of the electromechanical four-wheeled platform under the straight motion with the mode of speeding-up from the state of the rest.

To solve the noted tasks it will be used the Lagrange's equations of second kind with the electromechanical analogies, and it will be used final representing of the mathematical models in the form of the system of the first ordered ordinary differential equations with the initial conditions for further numerical solving. The computer simulations will be accomplished by using the Scilab free open source software.

Materials and methods

The general idea of this research is based on using only the computer simulations to have opportunities of imaging the controllability processes for the wheeled platforms without building any physical prototypes. To have the full imagines about the researched controllability processes it is necessary to have the improved generalised approaches providing the relevant mathematical modelling and computer simulations of these processes for the robotic wheeled platforms.

To build the mathematical model of the wheeled platform representing the controllability processes taking into account the different controls' criteria we will assume that the state of the wheeled platform can be defined by the finite dimensional vector $\mathbf{x}$ which will be named as the state vector. It is naturally to imagine that the state vector is depended on the t time

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$$\mathbf{x} = \mathbf{x}(t). \tag{1}$$

Besides, we will assume that the state of control influencing on the considered wheeled platform can be defined by the finite dimensional vector $\mathbf{u}$ which will be named further as the vector of the control or just the control. It is naturally to imagine the vector of the control as depended on the time t too:

$$\mathbf{u} = \mathbf{u}(t). \tag{2}$$

Taking into account the assumptions (1) and (2), we can reduce the mathematical modelling of the wheeled platform to building the following mapping:

$$\mathbf{u}(t) \to \mathbf{x}(t). \tag{3}$$

The mapping (3) shows that some control will lead to some state of the wheeled platform. To research the controllability processes of transition between the different given states of the wheeled platform it is reasonable to define the mapping (3) by means the system of the ordinary differential equations with the initial conditions as it is well-known in the control theory. So, we will assume that the mapping (3) can be represented in the following view:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}; \mathbf{u}), \ \ \mathbf{x}(t_0) = \mathbf{x}_0, \tag{4}$$

where $\mathbf{f}(t, \mathbf{x}; \mathbf{u})$ is the vector defining the velocity of changing the state of the wheeled platform depending on the time t, the current state $\mathbf{x}$ and on the control $\mathbf{u}$; t_0 is some given time moment, and $\mathbf{x}_0$ is the vector defining the state of the researched wheeled platform at the initial time moment $t = t_0$.

The mathematical model (4) of the wheeled platform allows having the full imaginations about changing the state (1) of this platform at the time moments $t \geq t_0$ relevant to some given control (2), and this given control (2) actually characterises the driving power of the wheeled platform. It is possible in general case at least due to numerical solving the Cauchy problem (4), and it is the standard mathematical task, and a lot of computer technologies are existed to solve this task, including the Scilab free open source software suitable for engineering and scientific calculations. At the same time, to design the control systems relevant to different criteria for the robotic wheeled platforms it is necessary to have imagines about time changing of the correspondent parameters defining the controls' criteria. In the case of the controlling the robotic wheeled platforms with the restrictions on the jerk motions formulated for the higher ordered accelerations we will have the following parameters:

$$\mathbf{j} = \boldsymbol{\phi}\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}, \frac{d^2\mathbf{x}}{dt^2}, \ldots, \right), \tag{5}$$

where $\mathbf{j}$ is the vector consisted of the different kinds of the jerks used for defining the control's criteria; $\boldsymbol{\phi}(\ldots)$ is some function relevant to the considered jerks used for defining the control's criteria.

It seems that it is not difficult to calculate the parameters (5) for the given state vector (1), but such calculating has in fact some principal difficulties due to involving the derivatives. Really, computing the derivatives required to estimate the criteria's parameters (5) is the principal problem because of it we will have no the exact analytical solutions for the state vector (1), but we will have only the approximate solution for this vector (1), and besides, this approximate solution will be represented in the discrete form of the values corresponded to the time moments relevant to the integrating steps. It seems, that it is possible to use the finite differences technique to find the derivatives of the discrete defined functions, but the discrete definitions of the functions will be predefined by the integrating procedure, so to have opportunities of using the finite differences technique the we must have the relevant integrating steps to have the suitable representation of the state vector (1) as the result of numerical solving the Cauchy problem (4). At the same time, it is well-known, differentiating of the approximate functions is the incorrect problem in the Hadamard's sense, because of the small changing of the function will lead to the big and even infinite changing of their derivatives, so using the finite differences technique to find the derivatives of the state vector (1) by using its approximate solution in any case will lead to the incorrect results, even if we will have the reliable grid of the data. We had experience of viewing such incorrectness in the previous researches [10, 11], so ours previous researches has no results about the higher derivatives of the state vector (1). At the same time, the higher derivatives of the state vector (1) are significantly required to estimate the some kinds of the quality of the control, including the jerks for example which are defined as the time derivative of the acceleration. Thus, it is significantly required to have the reliable general approach for evaluating the derivatives of the state vector (1) involved in the definition (5) of the parameters defining the quality of the control. The main principle of the reliable general approach for evaluating the derivatives of the state vector (1) involved in the definition (5) is in excluding the differentiations of the approximate defined state vector (1) by using the differential equations (4):

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}, \ \frac{d^2\mathbf{x}}{dt^2} = \frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{f} + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}, \ldots, \tag{6}$$

where $\mathbf{f} = \mathbf{f}(t, \mathbf{x}; \mathbf{u})$.

The relations (6) based on the mathematical model (4) allow having the correct evaluations for the derivatives of the state vector (1) represented by the approximate solution of the Cauchy problem (4), because of excluding the differentiation of the approximately represented functions with the incorrectness in the Hadamard's sense. Really, the partial derivatives $\partial \mathbf{f}/\partial t$, $\partial \mathbf{f}/\partial \mathbf{x}$ and $\partial \mathbf{f}/\partial \mathbf{u}$ can be find correctly due to the exactly defined mathematical model (4), but the derivative $d\mathbf{u}/dt$ can be find correctly because of existing the exact data about the control. It is necessary to note, that evaluating the third

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and higher derivatives using the scheme (6) will lead to the more cumbersome relations, but all these relations can be realized correctly.

Using the mathematical modelling (4) especially effective is combining with the computer simulations providing automatically the required numerical solving of the Cauchy problem (4) and the follows calculations using the relations (6) for evaluating the researched parameters (5). It is naturally that using such computer simulations requires the reliable substantiations of correctness in using the relevant especially developed software or the proposed computer models developed by the existed tools of the used software similar to the Scilab, Matlab, and others. Such substantiations are often reduced to considering the especially formulated tasks with the known solutions and to comparing the results of the computer simulations with these known solutions. To substantiate the correctness of the computer simulations based on using the mathematical model (4) it is proposed to use the test task corresponded to the following case:

$$t_0 = 0,\ \mathbf{x}_0 = \mathbf{0},\ \mathbf{u}(t) = \mathbf{u}_C\,, \tag{7}$$

where $\mathbf{0}$ is the zero vector with the relevant dimension; $\mathbf{u}_C$ is some given constant vector.

The well-known fundamental property inherent for the wheeled platforms is in existing of the maximal velocity corresponded to the equilibrium between the driving and resistance powers, so taking into account the relations (7), we will have the follows:

$$\lim_{\substack{t\to\infty,\\ u=\mathbf{u}_C}} \frac{d\mathbf{x}}{dt} = \mathbf{v}_C \Rightarrow \lim_{\substack{t\to\infty,\\ u=\mathbf{u}_C}} \mathbf{x}(t) = \mathbf{v}_C t\,, \tag{8}$$

where $\mathbf{v}_C$ is some constant vector corresponded to the $\mathbf{u}_C$ is some given constant vector defining the control.

The mathematical model (4) and the properties (8) allow to formulate the equations for evaluating the $\mathbf{v}_C$ vector corresponded to the given constant vector $\mathbf{u}_C$ defining the control:

$$\lim_{t\to\infty} \mathbf{f}\left(t, \mathbf{v}_C t, \mathbf{u}_C\right) = \mathbf{v}_C\,. \tag{9}$$

Thus, the relation (9) is the consequence of the mathematical model (4), and the results of the computer modelling the robotic wheeled platforms and their controllability processes must be in fully agreement with the relation (9). The relation (9) represents the most general property of the wheeled platforms and it can be sufficient for substantiating the correctness of the computer simulations. It is necessary to note, that the considered particular case (7) corresponds to the case of the transition function of the robotic wheeled platform imagined as the automation object, and it is in agreement with the purposes of this research in developing the approaches for computer simulations of the controllability processes.

Representing the mathematical models of the robotic wheeled platforms in the generalized view (4) is in agreement with the common approaches of the mathematical modelling of the discrete systems, but it is necessary to have the approaches to build the mathematical model (4) representing the particular robotic wheeled platform. It is really difficult to give the really generalized approaches for building the mathematical model (4) for the particular robotic wheeled platform, but it is naturally to use the well-known approaches of analytical mechanics and electromechanical analogies, because of the robotic wheeled platforms are usually the electromechanical systems. We will assume further the robotic wheeled platforms as the holonomic systems, because of such schematisation is suitable for considering some operational modes including the straight motion on their rigid road [10, 11]. This assumption allows using the principal idea of the analytical mechanics about the freedoms degrees in defining the state of the wheeled platforms by means the independent parameters:

$$q_k = q_k(t), k = 1, 2, \ldots, n\,, \tag{10}$$

where q_k is the generalised coordinate; n is the number of the freedom degrees for the considered wheeled platforms.

It should be noted that the generalized coordinates (10) can represent not only the mechanical values like the linear and curvilinear displacements as well as the angles of the rotations, but also the electrical values like the electric charges or the voltages in depending on the used electromechanical analogies for modelling the electromechanical systems. In any case, it is well-known in the analytical mechanics that the most general representation of the differential equations representing the holonomic electromechanical systems is based on using the electromechanical analogies and the Lagrange's equations of second kind:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_k} - \frac{\partial \mathcal{L}}{\partial q_k} = -\frac{\partial \mathcal{R}}{\partial \dot{q}_k} - \frac{\partial \mathcal{H}}{\partial \dot{q}_k} + Q_k\left(t, q_1, q_2, \ldots, q_n, \dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n; \mathbf{u}\right),\ k = 1, 2, \ldots, n\,, \tag{11}$$

where $\mathcal{L}$ is the Lagrange's function defined as the difference between the kinetic and potential energies; $\mathcal{R}$ is the Raleigh's function defining the damping; $\mathcal{H}$ is the function defining the gyroscopic effects; Q_k is the generalized force relevant to the generalized coordinate q_k.

In the Lagrange's equations of second kind represented in the form (11) it is assumed that the generalized coordinates Q_k represent the driving power of the considered wheeled platform. The Lagrange's equations of second kind (11) represent the system of the second ordered ordinary differential equations, so these equations must be considered with the relevant initial conditions:

$$q_k(t_0) = q_k^{(0)}, \dot{q}_k(t_0) = \dot{q}_k^{(0)},\ k = 1, 2, \ldots, n\,, \tag{12}$$

where $q_k^{(0)}$ and $\dot{q}_k^{(0)}$ are the given initial values of the generalized coordinates and the generalized velocities.

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The generalized coordinates (10) allow finding different parameters representing the different characteristics of the controlled wheeled platforms. To consider the controllability processes for the robotic wheeled platforms under the restrictions on the motions' jerks it is required considering the quantitative evaluations of these jerks which will be considered further as the time derivative of the acceleration of the mass center of the wheeled platform. In this research, we will assume that the mass center has the straight motion, so for the wheeled platform considering as the holonomic system with the generalized coordinates (10) we will have the following:

$$s = s\left(q_1, q_2, \ldots, q_n\right), \tag{13}$$

where s is the natural coordinate of the mass center of the wheeled platform.

Due to assumption about the straight motion of the mass center, and due to the assumed quantitative evaluation of the jerks as the time derivative of the acceleration, the relation (13) allows having the follows representation of the jerk for the considered robotic wheeled platform:

$$j(t) = \frac{d^3 s}{dt^3} \Rightarrow j(t) = \sum_{k=1}^{n}\left(\frac{\partial^3 s}{\partial q_k{}^3}\left(\frac{dq_k}{dt}\right)^3 + 3\frac{\partial^2 s}{\partial q_k{}^2}\frac{dq_k}{dt}\frac{d^2 q_k}{dt^2} + \frac{\partial s}{\partial q_k}\frac{d^3 q_k}{dt^3}\right), \tag{14}$$

where $j(t)$ is the quantitative evaluation of the jerk.

Relation (14) shows that the jerks of the translational motions of the wheeled platforms are depended on the generalized velocities, generalized accelerations and the time derivatives of the generalized accelerations, as well as on the building of the wheeled platform.

It is necessary to understand that the Lagrange's equations of second kind (11) with the initial conditions (12) have not direct form (4), but it is possible to represent the differential equations (11) and the initial conditions (12) in the form (4) by means introducing the relevant state vector (1) in the following view:

$$\mathbf{x}^T = \left(x_1 \quad x_2 \quad \ldots \quad x_n \quad x_{n+1} \quad x_{n+2} \quad \ldots \quad x_{2n}\right),$$

$$\begin{aligned} &x_1 = q_1, x_2 = q_2, \ldots, x_n = q_n, \\ &x_{n+1} = \dot{q}_1, x_{n+2} = \dot{q}_2, \ldots, x_{2n} = \dot{q}_n \end{aligned}. \tag{15}$$

Although, it is possible principally to consider the mathematical model representing the wheeled platform in the view (12), (13) of the system of the second ordered differential equations, but the most of numerical method for solving the Cauchy problems are foresaw for the systems of the first ordered differential equations (4), so reducing the problem (12), (13) to the equivalent generalized form (4) is principally required for computer simulations, and the Lagrange's equations of second kind are just the approach for building of the mathematical models of the robotic wheeled platforms. Thus, we have the generalised approaches to build the mathematical models of the controllability processes of the robotic wheeled platforms based on the Lagrange's equations of second kind (11) and the electromechanical analogies well-known in the analytical mechanics. Besides, reducing the relevant Lagrange's equations of second kind (11) to the equivalent system (4) of the first ordered differential equations as well as using the differential equations to evaluate the derivatives (6) of the state vector all allow providing the computer simulations of the controllability processes for the robotic wheeled platforms taking into account the restrictions of the motion's jerks, and due to the relation (9) we have the reliable criteria for substantiating the correctness of the results of such computer simulations.

Study results and their discussion

To illustrate the possibilities of the proposed generalized approaches and to have the quantitative estimations about the controllability processes in the robotic wheeled platforms we will consider the particular case of the four-wheeled electromechanical robotic platform, because this case covers the wide class of the robotic wheeled platforms typical for the different applications. Schematisation of the considered four-wheeled electromechanical robotic platform is presented on the fig. 1. It is assumed, that the considered wheeled platform has the straight translational motion of the housing (pos. 1 on fig. 1a) defining by the s natural coordinate, and this motion is due to rotations of two driving wheels (pos. 2 on fig. 1a) defining by the rotation angle φ and occurred by the drive direct current electric motors joined with this wheels (pos. 3 on fig. 1a); the housing (pos. 1 on fig. 1a) of the wheeled platform is supported on the road by means two driven wheels (pos. 4 on the fig. 1a). The viscous damping on the housing will be represented be the force F_{v} depending on the velocity of the translational motion, and the rolling friction will be represented by the constant rolling friction couples M_{rf} on the wheels. The equivalent scheme of the drive electric motors (fig. 1b) is reduced to the electrical inductance, the electrical resistance, to the supplied voltage U and to the inducted voltage depending on the rotor angle velocity and counteracting to the supplied voltage. The state of the electrical parts of the driving direst current electric motors is defined by the electric charge e (fig. 1b). To have opportunities to show the correctness of the results of the computer simulations we will consider the case corresponded to the relations (7)–(9), so we will consider the particular case of the initial state corresponded to the state of the rest and the case of the constant supplied voltage:

$$U(t) = U_C, \tag{16}$$

where U_C is some given constant.

To make the reliable computer simulations of the controllability processes for the considered particular

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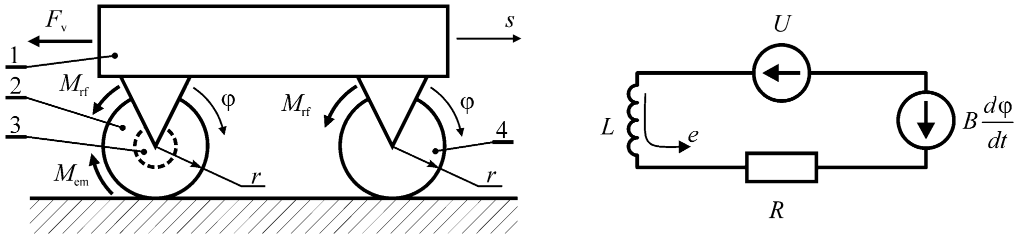
robotic wheeled platform (fig. 1) it is firstly necessary to build the relevant mathematical model, and to do this we will use the Lagrange's equations of second kind (11) with the electromechanical analogies as it was foresaw in the proposed generalised approaches. We will have the following evident relation between the translational coordinate of the housing and the rotation angles of all wheels (fig. 1a):

$$s = r\varphi, \tag{17}$$

where r is the radius of the wheels (fig. 1a).



a) b)

1 – housing; 2 – driving wheels; 3 – drive electric motors; 4 – driven supporting wheels

Fig. 1. Schematisation of the mechanic (a) and the electric motors (b) of the considered wheeled platform

Due to the relation (17) we have the holomomic system with two freedoms' degrees ($n = 2$), and we will use the follows generalized coordinates:

$$q_1 = \varphi,\ q_2 = e\,. \tag{18}$$

So, taking into account the generalised coordinates (18) and the relation (17) the Lagrange's function $\mathcal{L}$, the Raleigh's function $\mathcal{R}$, the function $\mathcal{H}$ defining the gyroscopic effects, and the generalized forces Q_k for the considered electromechanical wheeled platform (fig. 1) will have the following views:

$$\mathcal{L} = \tfrac{1}{2}J\dot{q}_1^2 + \tfrac{1}{2}2L\dot{q}_2^2,\ \mathcal{R} = 4M_{\text{rf}}\dot{q}_1 + \tfrac{1}{2}\beta r^2\dot{q}_1^2 + \tfrac{1}{2}2R\dot{q}_2^2,\ \mathcal{H} = 0,\ Q_1 = 2M_{\text{em}},\ Q_2 = 2\left(U_C - B\dot{q}_1\right). \tag{19}$$

where J is the inertia moment of the wheeled platform relatively the rotation of the wheels; β is the parameter defining the linear viscous damping proportional to the velocity of the housing (pos. 1 on fig. 1a); L is the inductance, R is the resistance of the equivalent electric circuit and B is the electromechanical parameter of the driving direct current electric motor.

We will assume that the driving couple from the direct current electric motor is proportional to the electric current in the drive's electric circuit

$$M_{\text{em}} = B\dot{q}_2\,. \tag{20}$$

The Lagrange's equations of second kinds (11) and the relations (19), (20) as well as the assumptions about the state of the rest at the initial time moment will lead to the follows differential equations:

$$\begin{aligned} J\ddot{q}_1 &= -\beta r^2\dot{q}_1 + 2B\dot{q}_2 - 4M_{\text{rf}}, \\ 2L\ddot{q}_2 &= -2B\dot{q}_1 - 2R\dot{q}_2 + 2U_C \end{aligned}, \tag{21}$$

$$q_1(0) = 0,\ \dot{q}_1(0) = 0,\ q_2(0) = 0,\ \dot{q}_2(0) = 0\,. \tag{22}$$

We have the mathematical model (21), (22) representing the considered robotic wheeled platform (fig. 1), but to make computer simulations of the controllability processes we must represent this mathematical model (21), (22) in the form (4) of the system of the first ordered differential equations. To do this, we will use the new variables (15) corresponded to the case of the $n = 2$. Besides, we will use the control vector (2) as the one dimensions vector with one element represented by the constant voltage (16). Taking into account all these circumstances, we will have the follows vectors representing the problem (21), (22) in the suitable form (4):

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix},\ \mathbf{u} = \left(U_C\right),\ \mathbf{f}(t,\mathbf{x},\mathbf{u}) = \begin{pmatrix} x_3 \\ x_4 \\ -\dfrac{\beta r^2}{J}x_3 + \dfrac{2B}{J}x_4 - \dfrac{4M_{\text{rf}}}{J} \\ -\dfrac{B}{L}x_3 - \dfrac{R}{L}x_4 + \dfrac{U_C}{L} \end{pmatrix},\ t_0 = 0,\ \mathbf{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{23}$$

The generalized relations (13) and (14) representing the natural coordinate and the jerk, as well as the relations for the velocity and accelerations of the straight motion and for the electric current in the drive electric motors of the considered wheeled platform taking into account the relations (17), (18) (15) will have the following view:

$$s = rx_1, v = rx_3, a = r\frac{dx_3}{dt}, j = r\frac{d^2x_3}{dt^2}, I = x_4, \tag{24}$$

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where s is the natural coordinate, v is the velocity, a is the acceleration, j is the jerk of the straight motion of the mass center and I is the electric current in the drive electric motors of the considered wheeled platform.

The general relations (6) allow excluding the differentiation of the state vector from the formulas (24) in the considered particular caes will have the following view:

$$\frac{dx_3}{dt} = -\frac{\beta r^2}{J}x_3 + \frac{2B}{J}x_4 - \frac{4M_{\text{rf}}}{J}, \quad \frac{d^2x_3}{dt^2} = -\frac{\beta r^2}{J}\left(-\frac{\beta r^2}{J}x_3 + \frac{2B}{J}x_4 - \frac{4M_{\text{rf}}}{J}\right) + \frac{2B}{J}\left(-\frac{B}{L}x_3 - \frac{R}{L}x_4 + \frac{U_C}{L}\right). \tag{25}$$

Thus, we have the mathematical model of the researched controllability processes for the considered robotic wheeled platform (fig. 1) in the suitable view (4), (23) for the computer simulations, and we can define wished additional parameters characterizing the quality of the control by means of the correct formulas (24), (25).

To have substantiations about reliability the results of computer simulation we will use the relation (9). In the considered example (fig. 1) the steady state (8) corresponded to the equilibrium between the driving and the damping powers for the considered wheeled platform (fig. 1) will be corresponded to the motion of the wheeled platform with the constant velocity and to the constant electric current in the electric motors circuits:

$$\mathbf{v}_C^T = \begin{pmatrix} v_C & I_C & 0 & 0 \end{pmatrix}, \tag{26}$$

where v_C is the maximum velocity possible for the considered wheeled platform and I_C is the relevant constant current in the drive electric motors' circuits under the U_C supplied voltage.

The relation (9) for the vector (26), the relations (23) defining the problem (4) will lead to the following:

$$-\beta r v_C + 2BI_C = 4M_{\text{rf}}, \quad \frac{B}{r}v_C + RI_C = U_C. \tag{27}$$

Relations (27) can be considered as the system of two linear algebraic equations for finding the established constant velocity v_C of the wheeled platform and the relevant electric current I_C in the drive electric motors' circuit:

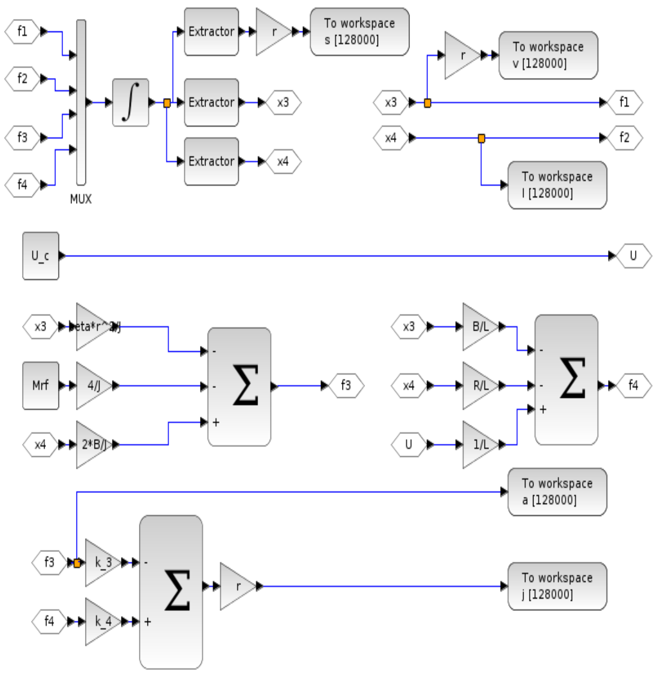
$$v_C = r\frac{2BU_C - 4M_{\text{rf}}R}{\beta r^2 R + 2B^2}, \quad I_C = \frac{\beta r^2 U_C + 4BM_{\text{rf}}}{\beta r^2 R + 2B^2}. \tag{28}$$

Exactly the values (28) will be used for comparing with the results of computer simulations to show correctness of these results.

We will consider further the computer simulations of the controllability processes for the considered wheeled platform (fig.1) with the follows numerical parameters:

$$J = 80\,\text{kg}\cdot\text{m}^2, r = 0{,}2\,\text{m}, \beta = 75\,\text{kg/s},$$
$$M_{\text{rf}} = 50\,\text{N}\cdot\text{m}, L = 2{,}6\,\text{mH}, R = 1{,}18\,\Omega,. \tag{29}$$
$$B = 4\,\text{N}\cdot\text{m/A}$$

To direct accomplish such computer simulations of the controllability processes for the considered robotic wheeled platforms (fig. 1) based on the mathematical model (4), (23), on the relations (24), (25) and (28), and on the input data (29) we will use the Scilab free open source software as shown on the fig. 2. The results of the computer simulations are presented at fig. 3 and fig. 4.



a)

```
J=80; r=0.2; beta=75; Mrf=50; L=2.6E-3; R=1.18; B=4; U_c=60;
k_3=beta*r^2/J; k_4=2*B/J
loadXcosLibs();
importXcosDiagram("/home/yurii/MyScilab/SP2022_F/SP2022_F4.zcos");
typeof(scs_m); scs_m.props.context;
scicos_simulate(scs_m);
Vc=r*(2*B*U_c-4*Mrf*R)/(beta*r^2*R+2*B^2);
Ic=(beta*r^2*U_c+4*B*Mrf)/(beta*r^2*R+2*B^2);
subplot(3,2,1); plot(s.time,s.values,'black');
subplot(3,2,2); plot(v.time,v.values,'black');
subplot(3,2,2); plot([0 20],[Vc Vc],'black');
subplot(3,2,3); plot(a.time,a.values,'black');
subplot(3,2,4); plot(j.time,j.values,'black');
subplot(3,2,5); plot(I.time,I.values,'black');
subplot(3,2,5); plot([0 20],[Ic Ic],'black');
U_c=50; scicos_simulate(scs_m);
Vc=r*(2*B*U_c-4*Mrf*R)/(beta*r^2*R+2*B^2);
Ic=(beta*r^2*U_c+4*B*Mrf)/(beta*r^2*R+2*B^2);
subplot(3,2,1); plot(s.time,s.values,'red');
subplot(3,2,2); plot(v.time,v.values,'red');
subplot(3,2,2); plot([0 20],[Vc Vc],'red');
subplot(3,2,3); plot(a.time,a.values,'red');
subplot(3,2,4); plot(j.time,j.values,'red');
subplot(3,2,5); plot(I.time,I.values,'red');
subplot(3,2,5); plot([0 20],[Ic Ic],'red');
U_c=40; scicos_simulate(scs_m);
Vc=r*(2*B*U_c-4*Mrf*R)/(beta*r^2*R+2*B^2);
Ic=(beta*r^2*U_c+4*B*Mrf)/(beta*r^2*R+2*B^2);
subplot(3,2,1); plot(s.time,s.values,'blue');
subplot(3,2,2); plot(v.time,v.values,'blue');
subplot(3,2,2); plot([0 20],[Vc Vc],'blue');
subplot(3,2,3); plot(a.time,a.values,'blue');
subplot(3,2,4); plot(j.time,j.values,'blue');
subplot(3,2,5); plot(I.time,I.values,'blue');
subplot(3,2,5); plot([0 20],[Ic Ic],'blue');
```

b)

Fig. 2. Graphical representing of the computer model (a) and the script (b) for automation on the basis of the program, control of the researches in the Scilab free open source software

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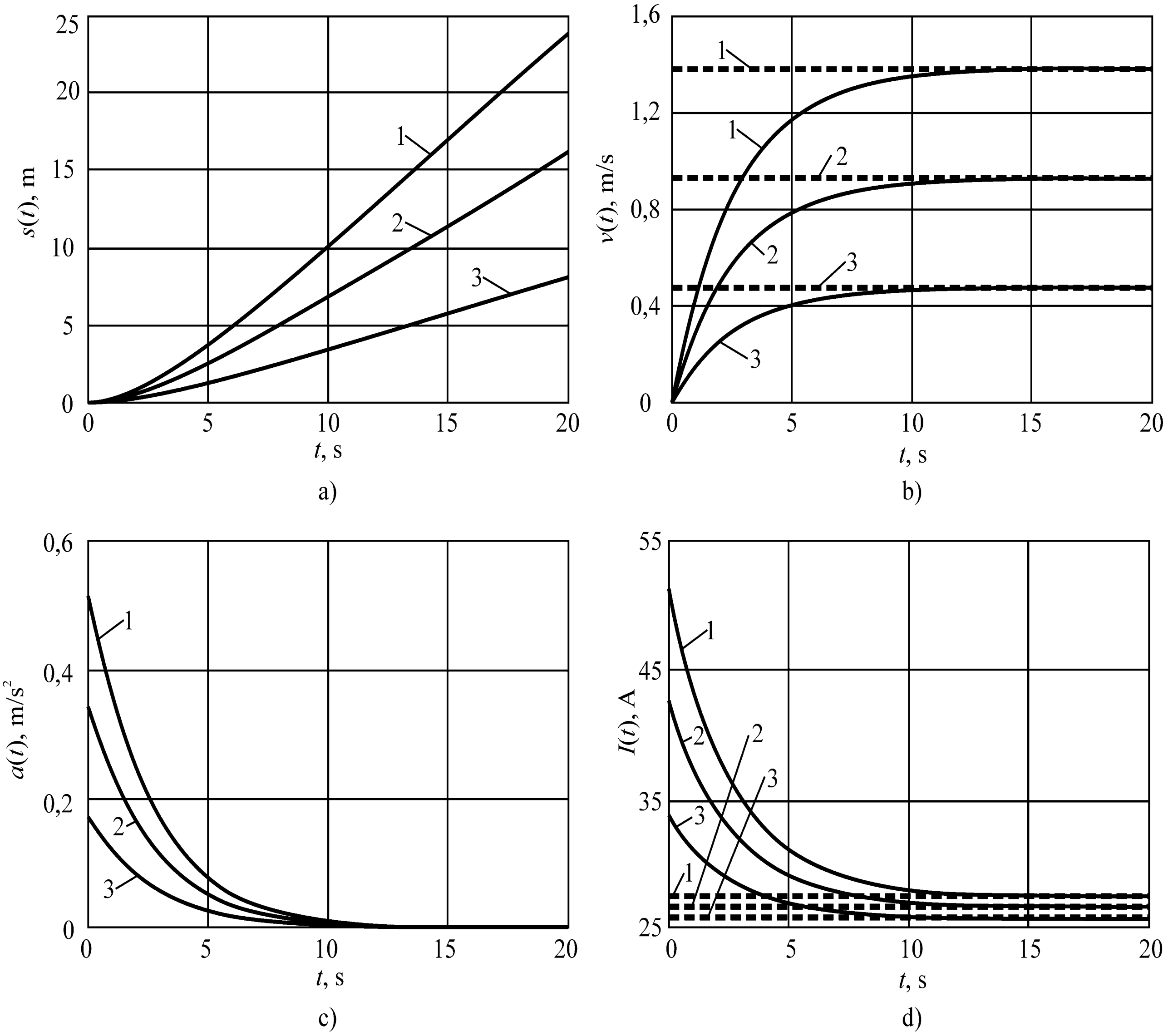
The results of the computer simulations (fig. 3) are the full agreement with the fundamental property of the wheeled platforms generally represented by the relations (7)–(9) which had the particular view (26)–(28) for the considered wheeled platform (fig. 1). This full agreement shows correctness of the result of the computer simulations using the computer models and script (fig. 2) especially developed using the Scilab tools. The results for the jerks (fig. 4) show that the maximum value of the jerk is at the initial time moment, and this is in full agreement with the physical sense of the considered problem about speeding-up the wheeled platform from the state of the rest. Considering the results for the different values of the voltage (16) supplied on the drive electric motors (fig. 3 and fig. 4) allows having imagination about the controllability of the considered wheeled platform and it allows having the imagination about this wheeled platform as the automation object. We can see (fig. 4) the significant values of the jerks and these results are due to the impact supplying of the voltage on the drive electric motors of the wheeled platform. It is possible to decrease the jerk by decreasing the value of the supplied voltage, but such smaller values of the jerks will be significant yet, so to significantly decrease the jerk it is necessary to provide the smooth time dependent of the voltage supplied on the drive electric motors. The significant values of the electric current in the winding on the drive electric motors at the initial time moment (fig. 3d) are due to the impact supplying of the voltage on the drive electric motors too. All these results (fig. 3 and fig. 4) of the computer simulations lead to the conclusion that optimisation of control by means choosing the suitable smooth voltage supplied on the drive electric motors will allow increasing the quality of operation of the considered wheeled platform.



1 – $U_C = 60\text{V}$; 2 – $U_C = 50\text{V}$; 3 – $U_C = 40\text{V}$

Fig. 3. Results of computer simulations (solid) of the translational coordinate (a), the velocity (b), the acceleration (c) and for the electric current (d) in the drive electric motors for the robotic wheeled platform, as well as the theoretically predicted results (dash) for the steady state

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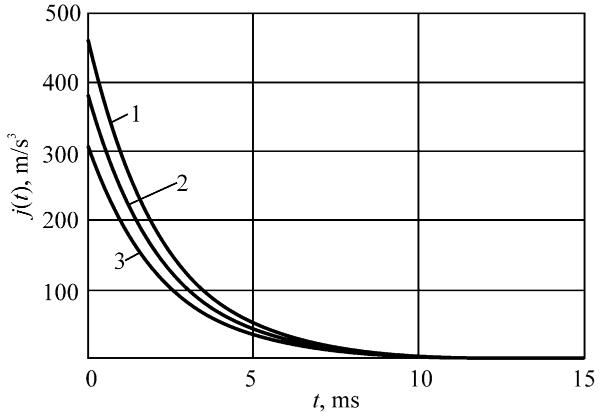

1 – $U_C = 60\text{V}$; 2 – $U_C = 50\text{V}$; 3 – $U_C = 40\text{V}$

Fig. 4. Results of computer simulations of the jerks for the robotic wheeled platform

Conclusion and perspectives of further development

Successful accomplishing of this research allows formulating the conclusions important for developing the theory and methods for computer simulations of the controllability processes of the robotic wheeled platforms.

It is developed the generalized approaches for computer simulations of the controllability processes of the robotic wheeled platforms, and these approaches have the advantages comparing with the existed due to possibilities of correct estimating not only the coordinates, the velocity, the acceleration, but as well as the jerk motions which are required for designing the control optimal in the different senses. These approaches are based on the mathematical modeling of the robotic wheeled platforms by using the Lagrange's equations of second kind with the electromechanical analogies, so the wide kinds of the robotic electromechanical wheeled platforms can be considered. The computer simulations of the controllability processes for the robotic wheeled platforms are reduced to numerical solving of the differential equations and the initial conditions of the relevant mathematical models represented in the suitable form of the system of first ordered ordinary differential equations with the initial conditions. On the example of the Scilab free open source software it is shown that the universal computer systems developed for the scientific and engineering calculations can be effectively used for computer simulations of the controllability processes for the robotic wheeled platforms.

It is proposed to use the differential equations of the mathematical model to compute the second and higher derivatives of the state vector necessary for evaluating the motions' jerks of the wheeled platforms using the numerical approximate solutions. This proposed approach allows excluding the incorrectness in the Hadamard's sense due to necessities of differentiating of the approximate functions. To have the reliable substantiations of correctness of the results of computer simulations of the controllability processes for the robotic wheeled platform it is proposed to use the property inherent for the wheeled platforms in existing of the maximal steady velocity relevant to the equilibrium between the driving and damping powers.

In the considered particular case of the four-wheeled electromechanical robotic platform typical for the different applications it is shown, that the results of the computer simulations are in full agreement with the fundamental property inherent for the wheeled platforms, so it shows correctness of the result of the computer simulations using the computer models and the script especially developed using the Scilab tools. It is shown that the maximum value of the jerk is at the initial time moment, and this is in full agreement with the physical sense of the considered problem about speeding-up the wheeled platform from the state of the rest. It is shown also, that considering the results for the different values of the voltage supplied on the drive electric motors allows having the imaginations about the controllability of the wheeled platforms and it allows having the imagination about this wheeled platform as the automation object. The significant values of the jerks are due to the impact supplying of the voltage on the drive electric motors of the wheeled platform. It is shown that optimization of control by means choosing the suitable smooth voltage supplied on the drive electric motors will allow increasing the quality of operation of the considered wheeled platform. For the further it is recommended to research the controllability processes for other parameters like the supplied power and others.

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КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ ПРОЦЕСІВ КЕРОВАНОСТІ ДЛЯ РОБОТИЗОВАНИХ КОЛІСНИХ ПЛАТФОРМ З ВРАХУВАННЯМ ОБМЕЖЕНЬ РИВКІВ РУХІВ

Комп'ютерне моделювання розглядається як необхідний інструмент для розробки відповідних автономних систем управління, оптимальних у різних сенсах, і, особливо, щодо забезпечення обмежень ривків рухів для роботизованих колісних платформ. **Предметом** дослідження є розробка теорії та методики комп'ютерного моделювання процесів керованості роботизованих колісних платформ. **Метою** даного дослідження є врахування ривків колісних платформ, які визначаються похідними за часом прискорення центру мас колісної платформи, так що ця похідна розглядається як кількісна оцінка ривків. Обговорюється некоректність у розумінні Адамара визначення ривків шляхом прямого диференціювання фазових координат у разі комп'ютерного моделювання за допомогою обчислювальних методів. **Завдання** даного дослідження полягають у розробці узагальнених підходів щодо математичного та комп’ютерного моделювання та в теоретичному отриманні властивостей колісних платформ, що придатні для перевірки результатів комп’ютерного моделювання, а також у проведенні розрахунків для отримання кількісних результатів щодо процесів керованості для конкретного випадку електромеханічної чотириколісної платформи при прямолінійному русі в режимі розганяння зі стану спокою. **Методи** цього дослідження базуються на рівняннях Лагранжа другого роду, а також на електромеханічних аналогіях та на остаточному представленні математичних моделей у вигляді системи звичайних диференціальних рівнянь першого порядку з початковими умовами для подальшого обчислювального розв’язування. Комп'ютерне моделювання виконується за допомогою вільного програмного забезпечення Scilab з відкритим кодом. **Результати** цього дослідження полягають у запропонованому коректному способі обчислення ривків через фазові координати без їхнього диференціювання, що дозволяє виключити некоректність у розумінні Адамара, а також представити процеси керованості для електромеханічної колісної платформи, включаючи результати для швидкості, прискорення та ривків, які необхідні для ілюстрації процесів керованості роботизованих колісних платформ. Шляхом порівняння з теоретично встановленими властивостями колісних платформ показано правильність результатів комп'ютерного моделювання. **Висновки** щодо цього дослідження полягають у тому, що розроблені підходи щодо комп’ютерного моделювання процесів керованості роботизованих колісних платформ дозволяють врахувати вплив керування на різні характеристики, включаючи швидкість, прискорення, а також ривки рухів, які необхідні для проектування управління, оптимального в різних сенсах.

Ключові слова: робототехніка; колісні платформи; ривок; керованість; математична модель; комп'ютерне моделювання.

КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССОВ УПРАВЛЯЕМОСТИ РОБОТОТИЗИРОВАННЫХ КОЛЕСНЫХ ПЛАТФОРМ С УЧЕТОМ ОГРАНИЧЕНИЙ РЫВКОВ ДВИЖЕНИЙ

Компьютерное моделирование рассматривается как необходимый инструмент для проектирования подходящих автономных систем управления, оптимальных в различных смыслах и, особенно, в обеспечении ограничения рывков движений роботизированных колесных платформ. **Предметом** исследования является разработка теории и методов компьютерного моделирования процессов управляемости роботизированных колесных платформ. **Целью** данного исследования является рассмотрение рывков колесных платформ, причем рывки определяются производными по времени ускорения центра масс колесной платформы, так что эта производная рассматривается как количественная оценка рывков. Обсуждается некорректность по Адамару вычисления рывков прямым дифференцированием фазовых координат в случае компьютерного моделирования с использованием численных методов. **Задачи** настоящего исследования заключаются в разработке обобщенных подходов к математическому и компьютерному моделированию; в теоретическом получении свойств, присущих колесным платформам и пригодных для верификации результатов компьютерного моделирования; в проведении расчетов для получения количественных результатов о процессах управляемости для частного случая электромеханической четырехколесной платформы при прямолинейном движении в режиме разгона из состояния покоя. Методы исследования основаны на уравнениях Лагранжа второго рода, а также на электромеханических аналогиях и окончательном представлении математических моделей в виде системы обыкновенных дифференциальных уравнений первого порядка с начальными условиями для дальнейшего численного решения. Компьютерное моделирование выполняется с использованием бесплатного программного обеспечения Scilab с открытым исходным кодом. **Результаты** исследования заключаются в том, что предложен корректный способ вычисления рывков по фазовым координатам без их дифференцирования, что позволяет исключить некорректность в смысле Адамара, а также в изучении процессов управляемости электромеханической колесной платформы, в том числе результатов для скоростей, ускорений и рывков, необходимых для иллюстрации процессов управляемости роботизированных колесных платформ. Путем сравнения с теоретически установленными собственными свойствами колесных платформ показана правильность результатов компьютерного моделирования. **Выводы** по данному исследованию заключаются в том, что разработанные подходы к компьютерному моделированию процессов управляемости роботизированных колесных платформ позволяют учитывать влияние управления на различные характеристики, включая скорость, ускорение, а также рывки движений, необходимые для проектирования оптимального в разных смыслах управления.

Ключевые слова: робототехника; колесные платформы; рывок; управляемость; математическая модель; компьютерное моделирование.

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