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METHOD FOR CALCULATION OF DISPERSIONS OF DEPENDENT VARIABLES OF A STOCHASTIC MODEL OF QUASI-STATIONARY OPERATING MODES OF THE MAIN WATER PIPELINE

The **subject** of research in the article is a stochastic model of quasi-stationary modes of operation of water supply and distribution systems, which adequately describes the actual operating modes of the water supply system for a given time interval and can be used as a basic model for setting and solving problems of optimal stochastic control of the development and operation of supply and distribution systems. water. The **goal** of the work is to develop a method for calculating estimates of the dispersions of dependent variables depending on the given values of mathematical expectations and dispersions of independent variables for a stochastic model of quasi-stationary modes of operation of the main water conduit as a subsystem of the water supply and distribution system. To achieve this goal, it is necessary to solve the following **tasks**: to build a deterministic equivalent of a stochastic model of quasi-stationary modes of operation of the main water conduit; calculate estimates of mathematical expectations of dependent variables; calculate estimates of variances of dependent variables. To calculate estimates of the variances of dependent variables depending on the variances of independent variables, we will use the **method** of statistical linearization. To obtain estimates of mathematical expectations of dependent variables, the deterministic equivalent of the stochastic model of quasi-stationary modes of operation of the main water conduit is solved by the modified Newton method. The following **results** are obtained: a method for calculating estimates of the dispersions of dependent variables for a stochastic model of quasi-stationary modes of operation of the main water conduit. **Conclusions**: the paper proposes an approximate method for calculating the statistical properties of dependent variables depending on the statistical properties of the parameters and independent variables of the stochastic model of quasi-stationary modes of operation of the main water conduit. The proposed method is based on the construction of a deterministic equivalent of a stochastic model of quasi-stationary modes of operation of the main water conduit and its use for calculating estimates of variances of dependent variables depending on the given values of mathematical expectations and variances of independent variables. Compared with the simulation method, the proposed approximate method does not require significant time and computational resources. The use of the proposed method was demonstrated by an example.

Keywords: stochastic model; dispersion; main water conduit; quasi-stationary regime; deterministic equivalent.

Introduction

At present, considerable experience has been accumulated in mathematical modeling and optimization of water transport and distribution regimes in water supply systems [1 – 12]. In works [1, 10] quasi-stationary operation modes of water supply systems and their application for modeling and optimization are considered; in publications [3 – 6, 12] issues of zoning of water supply systems for the purpose of energy and resource saving are studied; in works [8, 11] stochastic models of water supply networks and gas pipelines with leaks are proposed. The stochastic model of quasi-stationary operation modes of water supply and distribution systems developed in studies [7 – 9] adequately describes the actual operation modes of the water supply system at a given time interval. Also, this model can be used as a basic model for formulating and solving problems of optimal stochastic control of development and functioning of water supply and distribution systems. This is an important and urgent task for water utilities, as optimization is aimed primarily at reducing energy costs and water losses. For effective application of this model, it is necessary to obtain estimates of mathematical expectations and variances of dependent variables. The aim of the article is to develop a method for calculating estimates of dispersions of dependent variables for given values of mathematical expectations and dispersions of independent variables for stochastic model of quasi-stationary modes of operation of the main water pipeline.

A simple, but extremely time-consuming method of calculating statistical properties of dependent variables from statistical properties of parameters and independent variables of a stochastic model of quasi-stationary operation modes of water supply and distribution systems is the method of simulation (computer) modeling. At that, it is assumed that model equations should be solvable for any realizations of random variables included in this system. In order to obtain unbiased, efficient, and consistent estimates of the model's dependent variables, the number of experiments N must be at least

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1000 to 10 000. This approach guarantees, if, obtaining unbiased, efficient, and consistent estimates, but requires considerable time and computational resources.

To reduce time costs and computing resources, we use an approximate method for calculating the statistical properties of the dependent variables in accordance with the statistical properties of the parameters and independent variables of the stochastic model of quasi-stationary modes of operation of the main water pipeline (MP) [10 – 13]. The proposed method is based on the construction of a deterministic equivalent of the stochastic model of quasi-stationary operation modes of MP and its use to calculate estimates of mathematical expectations (ME) and dispersions of dependent variables for given values of ME and dispersions of independent variables. The approximated method involves solving the following problems: building a deterministic equivalent of a stochastic model of quasi-stationary modes of operation of a main water pipeline; calculating estimates of mathematical expectations of dependent variables; calculating estimates of dispersions of dependent variables.

Stochastic model of quasi-stationary modes of operation of the main water pipeline

Let us consider a stochastic model of quasi-stationary modes of MP operation, the structure of which can be represented as an interconnected sequence of multi-process pumping stations (PSs) with clean water reservoirs (CWR) at their inlets, which are connected by multi-line sections of main pipelines, and CWRs at MP outlets [14 – 17].

To represent MP structure in the form of an orgraph *G (V, E)*, where *V* is the set of vertices, *E* is the set of arcs (*e=Card (E), v=Card (V)*, the real MP is added by zero vertex and dummy chords connecting zero vertex with all MP inputs and outputs. The set E of MP graph arcs can be represented as $E = L \cup M \cup K \cup R$, where L is the set of MP graph arcs corresponding to sections with pump units (PU); M is the set of MP graph arcs corresponding to passive sections ($M = M_1 \cup M_2$, M_1, M_2 correspond to the set of branches and real chords of the graph tree); *K* is the set of fictitious MP sections ($K = I \cup N$, where *I* is the set of dummy arcs corresponding to MP inputs, *N* is the set of fictitious arcs corresponding to MP outputs); *R* is the set of MP graph arcs corresponding to regulating gate valves (GV). For mathematical formulation of the problem the following MP coding is performed: MP graph tree is selected so that fictitious sections of MP become chords, sections corresponding to HU and GV become branches. At the same time, the real sections partially become chords and partially become branches of the tree. We divide each set *M, N, I* into two, which correspond to tree branches M_1, N_1, I_1 and chords M_2, N_2, I_2 ($N_1 \neq \varnothing$) [7]. Since each node corresponding to the input or output of MP is given a head or flow value, each of the sets I_1, I_2, N_2 is split into two, depending on whether these arcs are given the flow I_{11}, I_{21}, N_{21} or the head I_{12}, I_{22}, N_{22}. The branch of the tree that corresponds to one of the outputs from the CWR is assigned number 1, the other branches from 2 to *v*–1, the chords of real sections from *v* to $v+\eta_2-1$, the fictitious ones with given nodal rates from $v+\eta_2$ to $v+\eta_2+\xi_1-1$, where η_2 is the number of chords of real sections; the chords with other outputs from CWR from $v+\eta_2+\xi_1$ to e, where ξ_1 is the number of outputs with given nodal rates, the number of MP (from CWR) (chords) inputs. $\xi_2 = e-(v+\eta_2+\xi_1)$ the number of MP (from CWR) (chords) inputs. ($Card(I_1)=1$; $Card(I_2)=\xi_2 = e-(v+\eta_2+\xi_1)$; $Card(N_2)=\xi_1$; $Card(M_1)=v$; $Card(M_2)=\eta_2$).

The control interval [*0,T*] is divided into *K* subintervals $[0, k\cdot\Delta t]$, $(k = 1, 2, ..., K)$. Then the stochastic mathematical model of quasi-stationary modes of MP operation for each subinterval $[k, k+1]$ at a given time interval $[0,T]$ will look like this:

$$\underset{\omega}{M}\left(h_r(q_r(\omega,k)) + \sum_{i\in L} b_{1ri} h_{NAi}\ (q_i(\omega,k)) + \sum_{i\in R} b_{1ri} h_{RZi}\ (q_i(\omega,k)) + \sum_{i\in M_1} b_{1ri} h_i\ (q_i(\omega,k)) \right) = 0, \quad (r = v,\dots,v+\eta_2-1), \tag{1}$$

$$\underset{\omega}{M}\left(h_r^c(q_r(\omega,k)) - H_1(\omega,k) + \sum_{i\in L} b_{1ri} h_{NAi}(q_i(\omega,k)) + \sum_{i\in R} b_{1ri} h_{RZi}\ (q_i(\omega,k)) + \sum_{i\in M_1} \left(b_{1ri} h_i(q_i(\omega,k)) + h_i^g \right) \right) = 0,$$
$$(r = v+\eta_2,\dots,v+\eta_2+\xi_1-1), \tag{2}$$

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$$\underset{\omega}{M}\left(h_r(q_r(\omega,k))+h_r^g-H_z(\omega,k)+H_1(\omega,k)+\sum_{i\in L}b_{1ri}h_{NAi}\ (q_i(\omega,k))+\sum_{i\in R}b_{1ri}h_{RZi}\ (q_i(\omega,k))+\sum_{i\in M_1}\left(b_{1ri}h_{\ i}\ (q_i(\omega,k))+h_i^g\right)\right)=0, \quad (3)$$

$(r=v+\eta_2+\xi_1,...,e;\quad z=1,...,Z),$

$$\underset{\omega}{M}\left(\sum_{r=v}^{v+\eta_2-1}b_{1ri}q_r(\omega,k)+\sum_{r=v+\eta_2}^{e}b_{1ri}q_r(\omega,k)-q_i(\omega,k)\right)=0\,,\quad \left(i=1,\ldots,v-1\right), \quad (4)$$

$$P\left(q_i\left(\omega,k\right)>0\right)\geq\alpha,\quad \alpha\cong1,\quad i\in L. \quad (5)$$

$$h_i(q_i(\omega,k))=sgn\,q_i(\omega,k)S_i(\omega)q_i^2(\omega,k),\quad i\in M\ , \quad (6)$$

$$h_{NAi}(q_i(\omega,k))=a_{0i}(\omega)\left(\frac{n_{1i}}{n_{0i}}\right)^2+a_{1i}(\omega)q_i(\omega,k)\frac{n_{1i}}{n_{0i}}+a_{2i}(\omega)q_i^2(\omega,k),\ i\in L, \quad (7)$$

$$\eta_{NAi}(q_i(\omega,k))=1-\frac{1-d_{0i}(\omega)-d_{1i}(\omega)q_i(\omega,k)-d_{2i}(\omega)q_i^{\,2}(\omega,k)}{\left(n_{0i}\,/\,n_{1i}\right)^{0,36}},\quad i\in L, \quad (8)$$

$$N_{NAi}(q_i(\omega,k))=c_{0i}(\omega)\left(\frac{n_{1i}}{n_{0i}}\right)^3+c_{1i}(\omega)\left(\frac{n_{1i}}{n_{0i}}\right)^2q_i(\omega,k)+c_{2i}(\omega)\left(\frac{n_{1i}}{n_{0i}}\right)\ q_i^{\,2}(\omega,k),\quad i\in L, \quad (9)$$

$$N_{NAi}(q_i(\omega,k))=\frac{9,81\cdot h_{NAi}(q_i(\omega,k))\cdot q_i(\omega,k)}{0,9\cdot\eta_{NAi}(q_i(\omega,k))},\quad i\in L, \quad (10)$$

$$h_{RZi}(q_i(\omega,k))\ =\ \frac{q_i(\omega,k)C_i(\omega)}{E_i^2},\quad i\in R, \quad (11)$$

$$H_z(\omega,k)=H_z(\omega,k-1)+c_z(q_{z\,vh}\ (\omega,k)-q_{z\,vih}(\omega,k)),\ (z=1,...,Z)\,, \quad (12)$$

$$P\left(H_z^{\min}\leq H_z(\omega,k)\right)\geq\beta_1,\quad \beta_1\cong1,\qquad (z=1,...,Z)\,, \quad (13)$$

$$P\left(H_z(\omega,k)\leq H_z^{\max}\right)\geq\beta_2,\quad \beta_2\cong1,\qquad (z=1,...,Z)\,, \quad (14)$$

$$P\left(h_r^c\ (q_r(\omega,k))\geq h_r^+\right)\geq\gamma,\quad \gamma\cong1,\ (r=v+\eta_2,...,v+\eta_2+\xi_1-1)\,, \quad (15)$$

where random variables characterize: $q_i\,(\omega,k)$ water flow rate at the *i*-th pipeline section at the *k*-th time interval; $h_{NAi}(q_i(\omega,k))$ – head of the *i*-th PU at the *k*-th time interval; $h_r^c\,(q_r(\omega,k))$ – free head at the *r*-th MP node at the *k*-th time interval $\left(r=v+\eta_2,...,v+\eta_2+\xi_1-1\right)$; h_r^+ – minimum allowable head at the *r*-th MP node. $S_i(\omega)$ – evaluation of hydraulic resistance of the *i*-th pipeline section ($i\in M$); $h_{RZi}(q_i(\omega,k))$ – evaluation of head drop on the *i*-th HV at the *k*-th time interval; $\eta_{NAi}(q_i(\omega,k))$ – evaluation of efficiency of the *i*-th PU at the *k*-th time interval; $a_{0i}(\omega),a_{1i}(\omega),a_{2i}(\omega),d_{0i}(\omega),$ $d_{1i}(\omega),d_{2i}(\omega)$ – evaluation of PU parameters ($i\in L$); $C_i(\omega)$ – evaluation of HV parameters ($i\in R$); E_i – degree of HV opening ($E\epsilon(0,1]$); h_i^g – geodetic mark of the *i*-th pipeline section ($i\in M$); b_{1ri} – element of cyclomatic matrix; $N_{NAi}(q_i(\omega,k))$ – estimation of PU power at the *k*-th time interval; $H_z(\omega,k)$ – estimation of water level in *z*-th CWR at the *k*-th time interval; $H_1(\omega,k)$ – estimation of water level in CWR that corresponds to tree branch number 1 at the *k*-th time interval; $H_z^{\min},H_z^{\max}$ – given limits of water level change in *z*-th CWR; $q_{z\,vh}\,(\omega,k),q_{z\,vih}(\omega,k)$ – volume water supply in *z*-th CWR and volume water withdrawal from *z*-th CWR at the *k*-th time interval; c_z – normalizing multiplier for *z*-th CWR.

The stochastic model (1) – (15) of quasi-stationary MP operation modes allows the calculation of parameters and the state of the quasi-stationary MP operation mode at a given time interval [*0,T*]. It is assumed that all random variables included in the model have normal distribution with known statistical characteristics – ME and variances. Boundary conditions are also set as random variables having normal distribution and are also set by their parameters – ME of pressures or water discharge at MP inlets and outlets and their dispersions.

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At the same time, a boundary condition in the form of ME head must be set at one of MP inlets or outlets.

Such setting of boundary conditions allows to calculate parameters and state of stochastic model of quasi-stationary mode of MP operation. At the made assumptions about normality of distribution law, the solution of system of equations of mathematical model (1) – (15) is reduced to calculation of statistical characteristics (ME and dispersions) of dependent variables according to statistical properties of parameters and independent variables of model (1) – (15).

Deterministic equivalent of stochastic model of quasi-stationary modes of MP

To construct a deterministic equivalent of the stochastic model (1) – (15) to calculate values of ME estimates of dependent variables from given values of ME estimates of independent variables, we will use an approximate method by replacing ME of nonlinear implicit functions of random arguments (1) – (4) by values of these functions from ME of their arguments.

The deterministic equivalent of the stochastic model of quasi-stationary modes of MP is as follows:

$$f_r=\bar{h}_r(\bar{q}_r(k))+\sum_{i\in L}b_{1ri}\bar{h}_{NAi}\ (\bar{q}_i(k))+\sum_{i\in R}b_{1ri}\bar{h}_{RZi}\ (\bar{q}_i(k))+\sum_{i\in M_1}b_{1ri}\bar{h}_i\ (\bar{q}_i(k))=0,\quad (r=v,\dots,v+\eta_2-1), \tag{16}$$

$$\begin{gathered}f_r=\bar{h}_r^c(\bar{q}_r(k))-\bar{H}_1(k)+\sum_{i\in L}b_{1ri}\bar{h}_{NAi}\ (\bar{q}_i(k))+\sum_{i\in R}b_{1ri}\bar{h}_{RZi}\ (\bar{q}_i(k))+\sum_{i\in M_1}b_{1ri}\left(\bar{h}_i(\bar{q}_i(k))+h_i^g\right)=0,\\ (r=v+\eta_2,\dots,v+\eta_2+\xi_1-1),\end{gathered} \tag{17}$$

$$\begin{gathered}f_r=\bar{h}_r(\bar{q}_r(k))+h_r^g-\bar{H}_z(k)+\bar{H}_1(k)+\sum_{i\in L}b_{1ri}\bar{h}_{NAi}\ (\bar{q}_i(k))+\sum_{i\in R}b_{1ri}\bar{h}_{RZi}\ (\bar{q}_i(k))+\sum_{i\in M_1}b_{1ri}\left(\bar{h}_i(\bar{q}_i(k))+h_i^g\right)=0,\\ (r=v+\eta_2+\xi_1,\dots,e;\quad z=1,\dots,Z),\end{gathered} \tag{18}$$

$$\bar{q}_i(k)=\sum_{r=v}^{v+\eta_2-1}b_{1ri}\bar{q}_r(k)+\sum_{r=v+\eta_2}^{e}b_{1ri}\bar{q}_r(k),\quad (i=1,\dots,v-1), \tag{19}$$

$$\bar{q}_i(k)>0,\quad i\in L. \tag{20}$$

$$\bar{h}_{NAi}(\bar{q}_i(k))=\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2+\bar{a}_{1i}\bar{q}_i(k)\frac{n_{1i}}{n_{0i}}+\bar{a}_{2i}\bar{q}_i^2(k),\quad i\in L, \tag{21}$$

$$\bar{\eta}_{NAi}(\bar{q}_i(k))=1-\frac{1-\bar{d}_{0i}-\bar{d}_{1i}\bar{q}_i(k)-\bar{d}_{2i}\bar{q}_i^{\,2}(k)}{\left(n_{0i}/n_{1i}\right)^{0,36}},\quad i\in L, \tag{22}$$

$$\bar{N}_{NAi}(\bar{q}_i(k))=\bar{c}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^3+\bar{c}_{1i}\left(\frac{n_{1i}}{n_{0i}}\right)^2\bar{q}_i(k)+\bar{c}_{2i}\left(\frac{n_{1i}}{n_{0i}}\right)\bar{q}_i^{\,2}(k),\quad i\in L, \tag{23}$$

$$\bar{N}_{NAi}(\bar{q}_i(k))=\frac{9,81\cdot\bar{h}_{NAi}(\bar{q}_i(k))\cdot\bar{q}_i(k)}{0,9\cdot\bar{\eta}_{NAi}(\bar{q}_i(k))},\quad i\in L, \tag{24}$$

$$\bar{h}_{RZi}(\bar{q}_i(k))\ =\ \frac{\bar{q}_i(k)\bar{C}_i}{E_i^2(k)},\quad i\in R, \tag{25}$$

$$\bar{h}_i(\bar{q}_i(k))=sgn\,\bar{q}_i(k)\bar{S}_i\bar{q}_i^{\,2}(k),\quad i\in M, \tag{26}$$

$$\bar{H}_z(k)=\bar{H}_z(k-1)+c_z(\bar{q}_{zvh}\ (k)-\bar{q}_{zvih}(k)),\quad (z=1,\dots,Z), \tag{27}$$

$$H_z^{\min}\le\bar{H}_z(k), \tag{28}$$

$$\bar{H}_z(k)\le H_z^{\max}, \tag{29}$$

$$\bar{h}_r^c(\bar{q}_r(k))\ge h_r^+. \tag{30}$$

The deterministic equivalent of the stochastic model of quasi-stationary modes of MP operation is an interrelated system of nonlinear algebraic equations (16) – (18), systems of coupling equations (19), (21) – (27) and systems of one-sided inequalities (20), (28) – (30).

The peculiarity of the system of equations (16) – (19) is that it contains $\eta_2+\xi_2$ equations with $e-(\xi_1-1)=e-\xi_1+1$

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unknowns. In this case, the number of equations is less than the number of unknowns by an amount equal to the total number of inputs and outputs MP $\xi_1+\xi_2$. To meet the necessary conditions for solvability of the system of equations (16) – (19) (coincidence of the number of equations with the number of unknowns) it is augmented with values of boundary conditions, which are treated as independent variables. As independent variables of model (16) – (19) we will use ME boundary conditions: pressures $\bar{H}_r (r \in I_{12} \cup I_{22} \cup N_{22})$, set on tree branch number 1 (I_{12}) and on MP inputs I_{22} and outputs N_{22} MP, which correspond to fictitious chords; flows $\bar{q}_i (i \in I_{21} \cup N_{21})$, set on MP inputs I_{21} and outputs N_{21} MP. As shown in [7], the obtained system of nonlinear algebraic equations (16) – (19) coincides with the system of equations of steady-state flow distribution and has a single solution if conditions (19) – (30) are satisfied.

Statistical linearization method

As a result of solution of the interrelated systems of nonlinear algebraic equations (16) – (18), systems of connection equations (19), (21) – (27) and systems of one-sided inequalities (20), (28) – (30) we got estimates of ME of dependent variables. To calculate the variance of dependent variables depending on the variance of independent variables, we will use the method of statistical linearization [11]. The system of equations (16) – (18) will be treated as a system of implicitly given functions of dependent variables from independent variables. The entire set of variables will be divided into two subsets: independent and dependent variables. We will use ME boundary conditions as independent variables of model (16) – (18): pressures $\bar{H}_r (r \in I_{12} \cup I_{22} \cup N_{22})$, set on branch of the tree number1 (I_{12}) and at the inputs I_{22} and outputs N_{22} of MP, which correspond to fictitious chords; flows $\bar{q}_i (i \in I_{21} \cup N_{21})$, set at the inputs I_{21} and outputs N_{21} of MP. The dependent variables are the flow rates in the real chords and at MP inputs and outputs with the given heads $q_r (r \in M_2 \cup I_{22} \cup N_{22})$; and the flow rates at MP inputs and outputs with the given flows $H_r (r \in I_{21} \cup N_{21})$.

For further calculations, we present the system (16) – (18) in the form:

$$f_r(\omega) = \operatorname{sgn} q_r(\omega) S_r(\omega) q_r^2(\omega) + \sum_{i\in L} b_{1ri}\left(\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2 + \bar{a}_{1i} q_i(\omega)\frac{n_{1i}}{n_{0i}} + \bar{a}_{2i} q_i^2(\omega)\right) + \sum_{i\in R} b_{1ri}\frac{q_i(\omega)\bar{C}_i}{E_i^2} + \sum_{i\in M_1} b_{1ri}\, sgn\, q_i(\omega) S_i q_i^2(\omega) = 0, \quad (r \in M_2), \tag{31}$$

$$f_r(\omega) = H_r(\omega) - \bar{H}_1 + \sum_{i\in L} b_{1ri}\left(\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2 + \bar{a}_{1i} q_i(\omega)\frac{n_{1i}}{n_{0i}} + \bar{a}_{2i} q_i^2(\omega)\right) + \sum_{i\in R} b_{1ri}\frac{q_i(\omega)\bar{C}_i}{E_i^2} + \sum_{i\in M_1} b_{1ri}\left(sgn\, q_i(\omega)\bar{S}_i q_i^2(\omega) + h_i^g\right) = 0, \quad (r \in N_{21}), \tag{32}$$

$$f_r(\omega) = \bar{H}_1 - H_r(\omega) + \sum_{i\in L} b_{1ri}\left(\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2 + \bar{a}_{1i} q_i(\omega)\frac{n_{1i}}{n_{0i}} + \bar{a}_{2i} q_i^2(\omega)\right) + \sum_{i\in R} b_{1ri}\frac{q_i(\omega)\bar{C}_i}{E_i^2} + \sum_{i\in M_1} b_{1ri}\left(sgn\, q_i(\omega)\bar{S}_i q_i^2(\omega) + h_i^g\right) = 0, \quad (r \in I_{21}), \tag{33}$$

$$f_r(\omega) = \operatorname{sgn} q_r(\omega)\bar{S}_r q_r^2(\omega) + h_r^g - H_r(\omega) + \bar{H}_1 + \sum_{i\in M_1} b_{1ri}\left(sgn\, q_i(\omega)\bar{S}_i q_i^2(\omega) + h_i^g\right) + \sum_{i\in L} b_{1ri}\left(\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2 + \bar{a}_{1i} q_i(\omega)\frac{n_{1i}}{n_{0i}} + \bar{a}_{2i} q_i^2(\omega)\right) + \sum_{i\in R} b_{1ri}\frac{q_i(\omega)\bar{C}_i}{E_i^2} = 0, \quad (r \in I_{22} \cup N_{22}), \tag{34}$$

$$q_i(\omega) = \sum_{r\in M_2 \cup I_{22} \cup N_{22}} b_{1ri} q_r(\omega) + \sum_{r\in I_{21}\cup N_{21}} b_{1ri} q_r(\omega), \quad (i \in M_1 \cup I_{12}). \tag{35}$$

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The resulting system of implicit functions can be represented as:

$$f\left(q_r(\omega), r\in M_2\bigcup I_{22}\bigcup N_{22}; H_1(\omega); H_j(\omega), j\in I_{22}\bigcup N_{22}; q_j(\omega), j\in I_{21}\bigcup N_{21}\right)=0, \quad (36)$$

$$f\left(H_r(\omega), r\in I_{21}\bigcup N_{21}; H_1(\omega); H_j(\omega), j\in I_{22}\bigcup N_{22}; q_j(\omega), j\in I_{21}\bigcup N_{21}\right)=0. \quad (37)$$

Let us decompose functions (36), (37) into Taylor series approximately the point $\bar{H}_j(j\in I_{12}\bigcup I_{22}\bigcup N_{22})$, $\bar{q}_j(j\in I_{21}\bigcup N_{21})$, which corresponds to the steady-state flux distribution mode, and limit ourselves to the linear terms of the expansion, and then we obtain:

$$q_r(\omega)=q_r\left(\bar{H}_1,\bar{H}_j,(j\in I_{22}\bigcup N_{22});\bar{q}_j(j\in I_{21}\bigcup N_{21})\right)+\sum_{j\in I_{12}\bigcup I_{22}\bigcup N_{22}}\left(\frac{\partial q_r(\omega)}{\partial H_j(\omega)}\right)^0\delta H_j(\omega)+\sum_{j\in I_{21}\bigcup N_{21}}\left(\frac{\partial q_r(\omega)}{\partial q_j(\omega)}\right)^0\delta q_j(\omega), \quad (38)$$

$$r\in I_{22}\bigcup N_{22},$$

$$H_r(\omega)=H_r\left(\bar{H}_1,\bar{H}_j,(j\in I_{22}\bigcup N_{22});\bar{q}_j(j\in I_{21}\bigcup N_{21})\right)+\sum_{j\in I_{12}\bigcup I_{22}\bigcup N_{22}}\left(\frac{\partial H_r(\omega)}{\partial H_j(\omega)}\right)^0\delta H_j(\omega)+\sum_{j\in I_{21}\bigcup N_{21}}\left(\frac{\partial H_r(\omega)}{\partial q_j(\omega)}\right)^0\delta q_j(\omega), \quad (39)$$

$$r\in I_{21}\bigcup N_{21},$$

where the derivatives are calculated at the decomposition point, and

$$\delta H_j(\omega)=H_j(\omega)-\bar{H}_j,\ \ \delta q_j(\omega)=q_j(\omega)-\bar{q}_j. \quad (40)$$

For (36):

$$\underset{\omega}{M}\left[q_r(\omega)-\bar{q}_r\left(\bar{H}_1,\bar{H}_j,(j\in I_{22}\bigcup N_{22});\bar{q}_j(j\in I_{21}\bigcup N_{21})\right)\right]^2=$$
$$=\underset{\omega}{M}\left[\sum_{j\in I_{12}\bigcup I_{22}\bigcup N_{22}}\left(\frac{\partial q_r(\omega)}{\partial H_j(\omega)}\right)^0\delta H_j(\omega)+\sum_{j\in I_{21}\bigcup N_{21}}\left(\frac{\partial q_r(\omega)}{\partial q_j(\omega)}\right)^0\delta q_j(\omega)\right]^2,\quad r\in I_{22}\bigcup N_{22}, \quad (41)$$

$$\sigma_{q_r}^2=\underset{\omega}{M}\left[\sum_{j\in I_{12}\bigcup I_{22}\bigcup N_{22}}\left(\left(\frac{\partial q_r(\omega)}{\partial H_j(\omega)}\right)^0\right)^2\sigma_{\delta H_j}^2+\sum_{j\in I_{21}\bigcup N_{21}}\left(\left(\frac{\partial q_r(\omega)}{\partial q_j(\omega)}\right)^0\right)^2\sigma_{\delta q_j}^2+2\sum_{\substack{i\in I_{12}\bigcup I_{22}\bigcup N_{22}\\ j\in I_{21}\bigcup N_{21}}}\left(\frac{\partial q_r(\omega)}{\partial H_i(\omega)}\right)^0\left(\frac{\partial q_r(\omega)}{\partial q_j(\omega)}\right)^0 k_{ij}\sigma_{\delta H_i}\sigma_{\delta q_j}+\right.$$
$$\left.+2\sum_{\substack{i,j\in I_{12}\bigcup I_{22}\bigcup N_{22}\\ i<j}}\left(\frac{\partial q_r(\omega)}{\partial H_i(\omega)}\right)^0\left(\frac{\partial q_r(\omega)}{\partial H_j(\omega)}\right)^0 k_{ij}\sigma_{\delta H_i}\sigma_{\delta H_j}+2\sum_{\substack{i,j\in I_{21}\bigcup N_{21}\\ i<j}}\left(\frac{\partial q_r(\omega)}{\partial q_i(\omega)}\right)^0\left(\frac{\partial q_r(\omega)}{\partial q_j(\omega)}\right)^0 k_{ij}\sigma_{\delta q_i}\sigma_{\delta q_j}\right],\quad r\in I_{22}\bigcup N_{22}. \quad (42)$$

Taking into account that $\underset{\omega}{M}\left(\delta H_j(\omega)\right)=\underset{\omega}{M}\left(\delta q_j(\omega)\right)=0,\ \sigma_{\delta q_j}^2=\sigma_{q_j}^2,\ \sigma_{\delta H_j}^2=\sigma_{H_j}^2$, and $H_1; H_j, j\in I_{22}\bigcup N_{22}; q_j, j\in I_{21}\bigcup N_{21}$ are independent quantities, so all their correlation coefficients $k_{ij}=0,\ \ i\neq j,$ we obtain estimates of the variances of the dependent variables:

$$\sigma_{q_r}^2=\sum_{j\in I_{12}\bigcup I_{22}\bigcup N_{22}}\left(\left(\frac{\partial q_r(\omega)}{\partial H_j(\omega)}\right)^0\right)^2\sigma_{H_j}^2+\sum_{j\in I_{21}\bigcup N_{21}}\left(\left(\frac{\partial q_r(\omega)}{\partial q_j(\omega)}\right)^0\right)^2\sigma_{q_j}^2,\quad r\in I_{22}\bigcup N_{22}. \quad (43)$$

For (37)

$$\underset{\omega}{M}\left[H_r(\omega)-H_r\left(\bar{H}_1,\bar{H}_j,(j\in I_{22}\bigcup N_{22});\bar{q}_j(j\in I_{21}\bigcup N_{21})\right)\right]^2=$$
$$=\underset{\omega}{M}\left[\sum_{j\in I_{12}\bigcup I_{22}\bigcup N_{22}}\left(\frac{\partial H_r(\omega)}{\partial H_j(\omega)}\right)^0\delta H_j(\omega)+\sum_{j\in I_{21}\bigcup N_{21}}\left(\frac{\partial H_r(\omega)}{\partial q_j(\omega)}\right)^0\delta q_j(\omega)\right]^2,\quad r\in I_{21}\bigcup N_{21}, \quad (44)$$

ISSN 2522-9818 (print)
ISSN 2524-2296 (online) Innovative technologies and scientific solutions for industries. 2022. No. 4 (22)

$$\sigma_{H_r}^2 = \underset{\omega}{M}\left[\sum_{j\in I_{12}\cup I_{22}\cup N_{22}}\left(\left(\frac{\partial H_r(\omega)}{\partial H_j(\omega)}\right)^0\right)^2\sigma_{\delta H_j}^2 + \sum_{j\in I_{21}\cup N_{21}}\left(\left(\frac{\partial H_r(\omega)}{\partial q_j(\omega)}\right)^0\right)^2\sigma_{\delta q_j}^2 + \right.$$
$$+2\sum_{\substack{i\in I_{12}\cup I_{22}\cup N_{22}\\ j\in I_{21}\cup N_{21}}}\left(\frac{\partial H_r(\omega)}{\partial H_i(\omega)}\right)^0\left(\frac{\partial H_r(\omega)}{\partial q_j(\omega)}\right)^0 k_{ij}\sigma_{\delta H_i}\sigma_{\delta q_j} + 2\sum_{\substack{i,j\in I_{12}\cup I_{22}\cup N_{22}\\ i<j}}\left(\frac{\partial H_r(\omega)}{\partial H_i(\omega)}\right)^0\left(\frac{\partial H_r(\omega)}{\partial H_j(\omega)}\right)^0 k_{ij}\sigma_{\delta H_i}\sigma_{\delta H_j} + \quad (45)$$
$$\left.+2\sum_{\substack{i,j\in I_{21}\cup N_{21}\\ i<j}}\left(\frac{\partial H_r(\omega)}{\partial q_i(\omega)}\right)^0\left(\frac{\partial H_r(\omega)}{\partial q_j(\omega)}\right)^0 k_{ij}\sigma_{\delta q_i}\sigma_{\delta q_j}\right],\quad r\in I_{21}\cup N_{21}.$$

Taking into account that $M\left(\delta H_j(\omega)\right) = M\left(\delta q_j(\omega)\right) = 0,\ \sigma_{\delta q_j}^2 = \sigma_{q_j}^2,\ \sigma_{\delta H_j}^2 = \sigma_{H_j}^2$, and $H_1; H_j, j\in I_{22}\cup N_{22}; q_j, j\in I_{21}\cup N_{21}$ are independent quantities, so all their correlation coefficients $k_{ij} = 0,\ i\neq j$, we obtain estimates of the variances of the dependent variables:

$$\sigma_{H_r}^2 = \sum_{j\in I_{12}\cup I_{22}\cup N_{22}}\left(\left(\frac{\partial H_r(\omega)}{\partial H_j(\omega)}\right)^0\right)^2\sigma_{H_j}^2 + \sum_{j\in I_{21}\cup N_{21}}\left(\left(\frac{\partial H_r(\omega)}{\partial q_j(\omega)}\right)^0\right)^2\sigma_{q_j}^2,\ r\in I_{21}\cup N_{21}. \quad (46)$$

To obtain derivatives $\partial q_r/\partial H_j$, $j\in I_{12}\cup I_{22}\cup N_{22}$ and $\partial q_r/\partial q_j$, $j\in I_{21}\cup N_{21}$ we will consider the system of equations (31)–(34) as a system of implicit functions:

$$\begin{aligned} &f_1\left(q_r(\omega), r\in M_2\cup I_{22}\cup N_{22}; q_t(\omega), t\in I_{21}\cup N_{21}; H_1(\omega); H_r(\omega), r\in I_{22}\cup N_{22}\right) = 0,\\ &\ldots\\ &f_{n1}\left(q_r(\omega), r\in M_2\cup I_{22}\cup N_{22}; q_t(\omega), t\in I_{21}\cup N_{21}; H_1(\omega); H_r(\omega), r\in I_{22}\cup N_{22}\right) = 0.\end{aligned} \quad (47)$$

For clarity, let us introduce the following notations:

$$Y_i = q_r(\omega),\ r\in M_2\cup I_{22}\cup N_{22},\ i\in 1,..,n1, \quad (48)$$

$$X_i = q_t(\omega),\ t\in I_{21}\cup N_{21},\ i\in 1,..,n2, \quad (49)$$

$$X_i = H_1(\omega),\ i\in 1+n2, \quad (50)$$

$$X_i = H_r(\omega),\ r\in I_{22}\cup N_{22},\ i\in n2+2,...,n4, \quad (51)$$

$$n1 = Card\left(M_2\cup I_{22}\cup N_{22}\right),\ n2 = Card\left(I_{21}\cup N_{21}\right), \quad (52)$$

$$n3 = Card\left(I_{22}\cup N_{22}\right),\ n4 = n2+n3+1. \quad (53)$$

Then the system of implicit functions (47) can be expressed as:

$$\begin{cases} f_1\left(Y_1,...,Y_{n1}, X_1,...,X_{n4}\right) = 0\\ f_2\left(Y_1,...,Y_{n1}, X_1,...,X_{n4}\right) = 0\\ \ldots\\ f_{n1}\left(Y_1,...,Y_{n1}, X_1,...,X_{n4}\right) = 0\end{cases} \quad (54)$$

Partial derivatives are calculated by the formula:

$$\frac{\partial Y_j}{\partial X_k} = -\frac{\dfrac{D\left(f_1,...,f_{n1}\right)}{D\left(X_1,...,X_{n4}\right)}}{\dfrac{D\left(f_1,...,f_{n1}\right)}{D\left(Y_1,...,Y_{n1}\right)}}, \quad (55)$$

where $\dfrac{D\left(f_1,...,f_{n1}\right)}{D\left(Y_1,...,Y_{n1}\right)}$ – Jacobian of functions $f_1,...,f_{n1}$ on variables $Y_1,...,Y_{n1}$.

The elements of the Jacobi matrix are calculated by the formulas:

$$\frac{\partial f_r(\omega)}{\partial q_r(\omega)} = 2\,\text{sgn}\,q_r(\omega)\bar{S}_r q_r(\omega) + \sum_{i\in M_1}2b_{1ri}^2 sgn\,q_i(\omega)\bar{S}_i q_i(\omega) + \sum_{i\in L}b_{1ri}^2\left(\bar{a}_{1i}\frac{n_{1i}}{n_{0i}} + 2\bar{a}_{2i}q_i(\omega)\right) + \sum_{i\in R}b_{1ri}^2\frac{\bar{C}_i}{E_i^2},\quad (r\in M_2), \quad (56)$$

ISSN 2522-9818 (print)
ISSN 2524-2296 (online)
Сучасний стан наукових досліджень та технологій в промисловості. 2022. № 4 (22)

$$\frac{\partial f_r(\omega)}{\partial q_k(\omega)} = \sum_{i\in M_1} 2b_{1ri}b_{1ki}sgn\,q_i(\omega)\bar{S}_i q_i(\omega) + \sum_{i\in L} b_{1ri}b_{1ki}\left(\bar{a}_{1i}\frac{n_{1i}}{n_{0i}} + 2\bar{a}_{2i}q_i(\omega)\right) + \sum_{i\in R} b_{1ri}b_{1ki}\frac{\bar{C}_i}{E_i^2}, \tag{57}$$

$(r\in M_2\cup I_{22}\cup N_{22})$, $(k\in I_{22}\cup N_{22})$,

$$\frac{\partial f_r(\omega)}{\partial q_t(\omega)} = \sum_{i\in M_1} 2b_{1ri}b_{1ti}sgn\,q_i(\omega)\bar{S}_i q_i(\omega) + \sum_{i\in L} b_{1ri}b_{1ti}\left(\bar{a}_{1i}\frac{n_{1i}}{n_{0i}} + 2\bar{a}_{2i}q_i(\omega)\right) + \sum_{i\in R} b_{1ri}b_{1ti}\frac{\bar{C}_i}{E_i^2}, \tag{58}$$

$(r\in M_2\cup I_{22}\cup N_{22})$, $(t\in I_{21}\cup N_{21})$,

$$\partial f_r(\omega)/\partial H_1(\omega) = 0, \quad (r\in M_2), \tag{59}$$

$$\partial f_r(\omega)/\partial H_1(\omega) = 1, \quad (r\in I_{22}), \tag{60}$$

$$\partial f_r(\omega)/\partial H_1(\omega) = -1, \quad (r\in N_{22}), \tag{61}$$

$$\partial f_r(\omega)/\partial H_k(\omega) = 0, \quad (r\in M_2), \quad (k\in I_{22}\cup N_{22}), \tag{62}$$

$$\partial f_r(\omega)/\partial H_r(\omega) = -1, \quad (r\in I_{22}), \tag{63}$$

$$\partial f_r(\omega)/\partial H_r(\omega) = 1, \quad (r\in N_{22}). \tag{64}$$

To obtain the derivatives $\frac{\partial H_r(\omega)}{\partial H_1(\omega)}$, $\frac{\partial H_r(\omega)}{\partial H_j(\omega)}$, $j\in I_{22}\cup N_{22}$; $\frac{\partial H_r(\omega)}{\partial q_j(\omega)}$, $j\in I_{21}\cup N_{21}$ we present equations (32), (33) in the following form:

$$H_r(\omega) = H_1(\omega) - \sum_{i\in L} b_{1ri}\left(\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2 + \bar{a}_{1i}q_i(\omega)\frac{n_{1i}}{n_{0i}} + \bar{a}_{2i}q_i^2(\omega)\right) + \sum_{i\in R} b_{1ri}\frac{q_i(\omega)\bar{C}_i}{E_i^2} + \sum_{i\in M_1} b_{1ri}\left(sgn\,q(\omega)_i\,\bar{S}_i q_i^2(\omega) + h_i^g\right) = 0, \quad (r\in N_{21}), \tag{65}$$

$$H_r(\omega) = H_1(\omega) + \sum_{i\in L} b_{1ri}\left(\bar{a}_{0i}\left(\frac{n_{1i}}{n_{0i}}\right)^2 + \bar{a}_{1i}q_i(\omega)\frac{n_{1i}}{n_{0i}} + \bar{a}_{2i}q_i^2(\omega)\right) + \sum_{i\in R} b_{1ri}\frac{q_i(\omega)\bar{C}_i}{E_i^2} + \sum_{i\in M_1} b_{1ri}\left(sgn\,q_i(\omega)\bar{S}_i q_i^2(\omega) + h_i^g\right) = 0, \quad (r\in I_{21}). \tag{66}$$

Then the derivatives of the corresponding variables will be:

$$\frac{\partial H_r(\omega)}{\partial H_1(\omega)} = 1 \pm \sum_{i\in L} b_{1ri}\left(\bar{a}_{1i}\frac{\partial q_i(\omega)}{\partial H_1(\omega)}\frac{n_{1i}}{n_{0i}} + 2\bar{a}_{2i}q_i(\omega)\frac{\partial q_i(\omega)}{\partial H_1(\omega)}\right) + \sum_{i\in R} b_{1ri}\frac{\bar{C}_i}{E_i^2}\frac{\partial q_i(\omega)}{\partial H_1(\omega)} + \sum_{i\in M_1} b_{1ri}2sgn\,q_i(\omega)\bar{S}_i q_i(\omega)\frac{\partial q_i(\omega)}{\partial H_1(\omega)}, \tag{67}$$

$$\frac{\partial H_r(\omega)}{\partial H_j(\omega)} = \pm\sum_{i\in L} b_{1ri}\left(\bar{a}_{1i}\frac{\partial q_i(\omega)}{\partial H_j(\omega)}\frac{n_{1i}}{n_{0i}} + 2\bar{a}_{2i}q_i(\omega)\frac{\partial q_i(\omega)}{\partial H_j(\omega)}\right) + \sum_{i\in R} b_{1ri}\frac{\bar{C}_i}{E_i^2}\frac{\partial q_i(\omega)}{\partial H_j(\omega)} + \sum_{i\in M_1} b_{1ri}2sgn\,q_i(\omega)\bar{S}_i q_i(\omega)\frac{\partial q_i(\omega)}{\partial H_j(\omega)}, \quad j\in I_{22}\cup N_{22}, \tag{68}$$

$$\frac{\partial H_r(\omega)}{\partial q_j(\omega)} = \pm\sum_{i\in L} b_{1ri}\left(\bar{a}_{1i}\frac{\partial q_i(\omega)}{\partial q_j(\omega)}\frac{n_{1i}}{n_{0i}} + 2\bar{a}_{2i}q_i(\omega)\frac{\partial q_i(\omega)}{\partial q_j(\omega)}\right) + \sum_{i\in R} b_{1ri}\frac{\bar{C}_i}{E_i^2}\frac{\partial q_i(\omega)}{\partial q_j(\omega)} + \sum_{i\in M_1} b_{1ri}2sgn\,q_i(\omega)\bar{S}_i q_i(\omega)\frac{\partial q_i(\omega)}{\partial q_j(\omega)}, \quad j\in I_{22}\cup N_{22}. \tag{69}$$

Let us use formula (35) to obtain partial derivatives $\frac{\partial q_i(\omega)}{\partial H_1(\omega)}$, $\frac{\partial q_i(\omega)}{\partial H_j(\omega)}$, $j\in I_{22}\cup N_{22}$; $\frac{\partial q_i(\omega)}{\partial q_j(\omega)}$, $j\in I_{21}\cup N_{21}$:

$$\frac{\partial q_i(\omega)}{\partial H_1(\omega)} = \sum_{r\in M_2\cup I_{22}\cup N_{22}} b_{1ri}\frac{\partial q_r(\omega)}{\partial H_1(\omega)}, \quad i\in M_1, \tag{70}$$

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ISSN 2524-2296 (online) Innovative technologies and scientific solutions for industries. 2022. No. 4 (22)

$$\frac{\partial q_i(\omega)}{\partial H_j(\omega)} = \sum_{r\in M_2\cup I_{22}\cup N_{22}} b_{1ri}\frac{\partial q_r(\omega)}{\partial H_j(\omega)}, \quad (71)$$

$i \in M_1$, $j \in I_{22} \cup N_{22}$.

The procedure for calculating derivatives $\frac{\partial q_r(\omega)}{\partial H_1(\omega)}$, $\frac{\partial q_r(\omega)}{\partial H_j(\omega)}$, $j \in I_{22} \cup N_{22}$; $\frac{\partial q_i(\omega)}{\partial q_j(\omega)}$, $j \in I_{21} \cup N_{21}$, was considered above.

Study results

Let us apply the proposed method for an MP consisting of an interconnected sequence of multi-processing NSs with CWRs at their inputs, interconnected by multi-line sections of trunk pipelines, and CWRs at MP outputs [18 – 20].

An example of calculation of dispersions of dependent variables in accordance with the statistical properties of parameters and independent variables of the stochastic model of quasi-stationary modes of MP operation is given in tables 1 and 2.

Table 1. *Estimates of dispersion of free pressures in nodes that correspond to outputs with given flow rates ($\bar{q}_0$)*

Nod	Input data		Calculation results	
	$\bar{q}_0$, м^3/с	$\sigma_{\bar{q}_0}$	$\bar{h}^c$, м	$\sigma_{\bar{h}^c}$
1	0,00348989	0,00058	26,89	2,05
2	0,00142236	0,00024	63,22	4,80
3	0,00142236	0,00024	59,7	13,29
4	0,2024136	0,03374	15,25	13,84
5	1,11676467	0,18613	47,41	13,87
6	0,00069798	0,00012	34,66	10,14
7	0,06979779	0,01163	53,75	2,80
8	0,06979779	0,01163	43,26	4,49
9	0,00348989	0,00058	42,72	5,06
10	0,00139596	0,00023	51,56	10,14
11	0,8724724	0,14541	11,1	13,56
12	0,00139596	0,00023	42,85	10,15
13	0,8724724	0,14541	2,31	13,56
14	0,27919117	0,04653	46,6	13,87

Table 2. *Estimates of expenditure dispersion, which correspond to chords with active sources*

Chord	H, м	σ_H	q, м^3/с	σ_q
1	2	0,333	0,3316	0,0635
2	2	0,333	0,3765	0,064
3	2	0,333	0,3968	0,0638
4	2	0,333	0,4082	0,0632
5	2	0,333	1,4671	0,1734
6	2	0,333	1,2815	0,1633
7	2	0,333	1,1874	0,1472
8	2	0,333	0,6868	0,0809
9	2	0,333	0,6868	0,0809
10	2	0,333	1,2522	0,1707
11	2	0,333	1,2913	0,1899
12	2	0,333	1,6573	0,2051
13	2	0,333	1,6894	0,2403
14	2	0,333	–0,704	0,1173
15	2	0,333	–1,268	0,2114
16	2	0,333	–2,525	0,4208
17	2	0,333	–0,806	0,1343
18	1,45	0,2116	0,2259	0,0377
19	1,45	0,2116	–4,14	0,5721

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МЕТОД РОЗРАХУНКУ ДИСПЕРСІЙ ЗАЛЕЖНИХ ЗМІННИХ СТОХАСТИЧНОЇ МОДЕЛІ КВАЗІСТАЦІОНАРНИХ РЕЖИМІВ РОБОТИ МАГІСТРАЛЬНОГО ВОДОВОДУ

Предметом дослідження є стохастична модель квазістаціонарних режимів роботи систем подання та розподілення води. Зазначена модель адекватно описує фактичні режими роботи системи водопостачання на заданому часовому інтервалі. Також вона може використовуватися як базова модель для постановки та вирішення завдань оптимального стохастичного управління розвитком та функціонуванням систем подання та розподілення води. **Мета роботи** – розроблення методу розрахунку оцінок дисперсій залежних змінних за умови заданих значень математичних сподівань та дисперсій незалежних змінних для стохастичної моделі квазістаціонарних режимів роботи магістрального водоводу як підсистеми системи подання та розподілення води. Для досягнення цієї мети необхідно виконати такі **завдання:** побудувати детермінований еквівалент стохастичної моделі квазістаціонарних режимів роботи магістрального водоводу; розрахувати оцінки математичних сподівань залежних змінних; розрахувати оцінки дисперсій залежних змінних. Для розрахунку оцінок дисперсій залежних змінних, відповідно до дисперсій незалежних змінних, використовується **метод** статистичної лінеаризації. Для отримання оцінок математичних сподівань залежних змінних детермінований еквівалент стохастичної моделі квазістаціонарних

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режимів роботи магістрального водоводу розв'язується модифікованим методом Ньютона. **Результатом** роботи є метод розрахунку оцінок дисперсій залежних змінних для стохастичної моделі квазістаціонарних режимів роботи магістрального водоводу. **Висновки**: у роботі запропоновано наближений метод розрахунку статистичних властивостей залежних змінних відповідно до статистичних властивостей параметрів та незалежних змінних стохастичної моделі квазістаціонарних режимів роботи магістрального водоводу. Наближений метод оснований на побудові детермінованого еквівалента стохастичної моделі квазістаціонарних режимів роботи магістрального водоводу та його використанні для розрахунку оцінок математичних сподівань і дисперсій залежних змінних за умови заданих значень математичних сподівань та дисперсій незалежних змінних. На відміну від методу імітаційного моделювання, запропонований метод не потребує значних часових витрат та обчислювальних ресурсів. Застосування наближеного методу показано на прикладі.

Ключові слова: стохастична модель; дисперсія; магістральний водовід; квазістаціонарний режим; детермінований еквівалент.

МЕТОД РАСЧЕТА ДИСПЕРСИЙ ЗАВИСИМЫХ ПЕРЕМЕННЫХ СТОХАСТИЧЕСКОЙ МОДЕЛИ КВАЗИСТАЦИОНАРНЫХ РЕЖИМОВ РАБОТЫ МАГИСТРАЛЬНОГО ВОДОВОДА

Предметом исследования в статье является стохастическая модель квазистационарных режимов работы систем подачи и распределения воды. Эта модель адекватно описывает фактические режимы работы системы водоснабжения на заданном интервале времени и может использоваться в качестве базовой модели для постановки и решения задач оптимального стохастического управления развитием и функционированием систем подачи и распределения воды. **Целью** работы является разработка метода расчета оценок дисперсий зависимых переменных при заданных значениях математических ожиданий и дисперсиях независимых переменных для стохастической модели квазистационарных режимов работы магистрального водовода как подсистемы системы подачи и распределения воды. Для достижения этой цели необходимо решить следующие **задачи**: построить детерминированный эквивалент стохастической модели квазистационарных режимов работы магистрального водовода; рассчитать оценки математических ожиданий зависимых переменных; рассчитать оценки дисперсий зависимых переменных. Для расчета оценок дисперсий зависимых переменных, в зависимости от дисперсий независимых переменных, будем использовать **метод** статистической линеаризации. Для получения оценок математических ожиданий зависимых переменных детерминированный эквивалент стохастической модели квазистационарных режимов работы магистрального водовода решается модифицированным методом Ньютона. **Результатом** работы является метод расчета оценок дисперсий зависимых переменных для стохастической модели квазистационарных режимов работы магистрального водовода. **Выводы**: в работе предложен приближенный метод расчета статистических свойств зависимых переменных в соответствии со статистическими свойствами параметров и независимыми переменными стохастической модели квазистационарных режимов работы магистрального водовода. Приближенный метод основан на построении детерминированного эквивалента стохастической модели квазистационарных режимов работы магистрального водовода и его использовании для расчета оценок дисперсий зависимых переменных при заданных значениях математических ожиданий и дисперсиях независимых переменных. В отличие от метода имитационного моделирования, предложенный метод не требует значительных временных затрат и вычислительных ресурсов. Использование приближенного метода показано на примере.

Ключевые слова: стохастическая модель; дисперсия; магистральный водовод; квазистационарный режим; детерминированный эквивалент.

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