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MATHEMATICAL MODELS FOR DETERMINING THE PARETO FRONT FOR BUILDING TECHNOLOGICAL PROCESSES OPTIONS UNDER THE CONDITIONS OF INTERVAL PRESENTATION OF LOCAL CRITERIA

The **subject** of research in the article is decision-making support processes in the tasks of optimizing technological processes (TP) at the stages of their design or reengineering. The **goal** of the work is to improve the efficiency of technologies of automated design of TP due to the development of mathematical models of the tasks of selecting subsets of effective design solutions with intervally specified characteristics of options. The following **tasks** have been solved in the article: review and analysis of the current state of the problem of supporting decision-making in the tasks of optimization of TP at the stages of their design or reengineering; decomposition of the problem of making project decisions; formalization of the task of comparing intervals for selection of Pareto fronts using comparison indices based on the generalized Hukuhari difference; development of a mathematical model of the problem for the method based on Carlin's lemma; development of a mathematical model of the problem for the method based on Hermeyer's theorem; determination of the Pareto front in the task of optimization of TP by the method of pairwise comparisons. The following **methods** were used: system approach, theories of systems, theories of usefulness, theories of decision-making, system design, optimization and operations research. **Results**. The place and connections of the problem of determining the Pareto front in the problem of making project decisions are determined. A formalized interval comparison procedure for the selection of Pareto fronts using Hukuhari total difference comparison indices. Mathematical models of the problem of selection of Pareto fronts using methods based on Carlin's lemma and Hermeyer's theorem have been developed for the case of interval publication with the value of local criteria. An example of the formation of the Pareto front in the problem of optimization of the technological process by the method of pairwise comparison according to the indicators of the duration of the technological cycle, reliability and specified costs is given. **Conclusions.** The proposed mathematical models expand the methodological bases of the automation of TP design processes. They make it possible to correctly reduce the set of alternative options for construction of TP for the final choice, taking into account the knowledge, experience of designers and factors that are difficult to formalize. The practical use of mathematical models will allow to increase the degree of automation of design or control processes, to reduce the time of decision-making in conditions of incomplete certainty of input data and to guarantee their quality by selecting them only from a subset of effective ones.

Keywords: technological processes; design automation; optimization; reengineering; multi-criteria evaluation; decision support; Pareto front.

Introduction

In the context of the transition to Industry 4.0, the quality and price of manufacturing companies' products are increasingly determined by the quality of the technological processes (TP) used to make them [1]. The effectiveness of manufacturing processes, in turn, is determined by decisions made at the stages of their design or reengineering. The process of TP design involves the iterative solution of a set of problems of their structural, parametric, topological optimization and the establishment of basic modes of operation [2–4]. The selection of the best options for the TP construction is carried out using a set of functional and cost indicators (local performance criteria). Mathematical models and methods of decision theory are used to optimize the options for the TP construction [5–7]. Due to the combinatorial nature of most TP optimization models, the number of alternative options for their construction increases sharply with the growth of the problem dimension. The overwhelming majority of the options for constructing TPs generated in the process of their design are inefficient (dominated). There is a problem of forming a subset of efficient (non-dominated) design solutions that form a Pareto front or selecting such a subset from the formed set of valid options [8–9]. The evaluation of options for the construction of TPs according to local criteria is based on the results of modeling with a certain error [10]. As a result, decisions on the choice of the best option for the construction of the TP are made under conditions of incomplete certainty of its quality indicators. For TPs that involve dozens of operations, the generated or selected Pareto front may be quite powerful, unsuitable for final expert evaluation and selection. It is necessary to further reduce the set of effective options for building a TA, taking into account the given preferences between quality indicators.

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The aforementioned, as well as the need to consider the errors in the estimates of the options determined using modeling, raises the problem of supporting design decisions on the TP, taking into account the interval representation of the values of local quality criteria. The use of interval mathematics methods to account for this kind of uncertainty requires the formalization of the interval comparison operation.

Analysis of the current state of the problem and methods of its solution

Today, the transition from traditional to additive manufacturing is relevant. Hybrid (integrated, combined) production, which combines the advantages of additive and traditional production technologies, is considered an intermediate stage. This requires a comprehensive optimization of the technological production chain and the necessary equipment. It is believed that the integration of product design, optimization of production technology, and operational management will allow companies to increase their competitiveness [11]. However, such integration significantly complicates the process of designing TP.

Digital modeling is used to determine the functional characteristics of the created TPs. It allows you to automatically evaluate the technological routes generated during the design process. It involves formalization based on modular technology, optimization, product realization modeling, and analysis of the results. Technological routes optimized in this way ensure a balanced and efficient organization of the production process [10]. However, the use of modeling allows only an approximate determination of TP characteristics, which in decision-making situations meets the challenges of incomplete data certainty.

As a result of the decomposition of the TP system optimization problem as a territorially or spatially distributed object at the lower level l a set of interrelated tasks is identified at the lower level $Tasks = \{Task_i^l\}$ [12]: $Task_1^l$ – determination of requirements for TP and their formalization; $Task_2^l$ – optimization of the structure of TP (a set of equipment and transitions between them); $Task_3^l$ – optimization of the topology of TP elements (equipment placement); $Task_4^l$ – optimization of operating modes; $Task_5^l$ – selection of types or parameters of equipment and transitions between them; $Task_6^l$ – evaluation and selection of the best option for building TP.

The process of system TP optimization as a distributed object can be represented as a logical scheme for constructing an appropriate design solution [12].

$$LS = <\text{Tasks},\ InDat,\ Res,\ DD,\ PD>, \qquad (1)$$

where $Tasks =< Task_i^l >$, $i = \overline{1,6}$ – an ordered set of TP design tasks (models);

$InDat$ – set of input data of tasks;

Res – set of task constraints;

DD – a set of local design solutions (problem solutions);

PD – mappings presented in the form of design procedures (solution methods) that correspond to each pair $< InDat_i^l, \mathrm{Re}\, s_i^l >$ a nonempty subset of local design solutions $< DD_i^l >$, $i = \overline{1,6}$.

It is known that the ordered set of tasks in (1) is completely solvable if there are design procedures for each of them PD_i^l, $i = \overline{1,6}$ and each solution is unique [12]:

$$\left| PD_i^l (< InDat_i^l, \mathrm{Re}\, s_i^l >) \right| = 1,\ i = \overline{1,6}. \qquad (2)$$

Modern technologies for designing TPs as complex objects are iterative. They involve the repeated implementation of procedures for generating and analyzing TP construction options and choosing the best among them. The essence of the decision-making in problems $Task_6^l$ is represented by the logical statement "It is necessary s^o " or formally in the form $< -, s^o >$ (where s^o is the optimal design solution belonging to the set X of acceptable solutions) [13]. In this case, the decision-making situation Sit is usually incompletely defined due to the incomplete definition of goals and/or input data. To move to the decision-making problem $< Sit, s^o >$, it is necessary to decompose the problem by solving auxiliary problems of the form "Given $< Sit, - >$, Required $< Sit, s^o >$ ", i.e. $<< Sit, - >, < Sit, s^o >>$, or "Given $< -, s^o >$, Required $< Sit, s^o >$ ", i.e. $<< -, s^o >, < Sit, s^o >>$.

Each of the tasks of the subset $Task_i^l$, $i = \overline{2,6}$ involves making decisions according to a set of local criteria $k_j(s)$, $j = \overline{1,m}$ (where m is the number of criteria). In this case, the local criteria are contradictory, have different physical content, dimensionality, and measurement interval. In the most general case, the decision-making process is a set of tasks [14]: formalizing the goal of creating a TP; determining a universal set of options for building a TP S^U; determining a subset of valid options $S \subseteq S^U$; selecting a Pareto front (a subset of effective options) $S^E \subseteq S$; ranking options $s^o \succ s_i \succ s_j \succ ... \succ s_z$; selecting the best option $s^o \in S^E$.

TP design methods involve the generation and analysis of powerful sets of feasible options S. To reduce the time for solving problems, it is proposed to generate and analyze only effective options belonging to the Pareto front $S^E \subseteq S$. The power of a subset of efficient options $Card\,(S^E)$ can range from a few percent to a few thousandths of a percent of $Card\,(S)$. A variant of a design solution is called efficient $s^E \in S^E$ if there is no variant $s \in S$ for which the inequalities are satisfied on the set of admissible ones $j = \overline{1,m}$ [13]:

$$k_j(s) \geq k_j(s^E)\text{, if } k_j(s) \to \max\text{,} \tag{3}$$

$$k_j(s) \leq k_j(s^E)\text{, if } k_j(s) \to \min \tag{4}$$

and at least one of them was strict.

Depending on the specifics of decision-making tasks, different methods and algorithms for Pareto front selection are used: pairwise comparisons, based on Carlin's lemma and Hermeyer's theorem, evolutionary search based on genetic algorithms, sector, and segment [9, 13, 14].

The method of pairwise comparisons allows you to fully isolate the Pareto front $S^E = \{s\}$ on convex sets of options S. It involves a pairwise comparison of all possible pairs of options $s, v \in S$ and therefore has a relatively high time complexity [13].

Weighting methods, in particular those based on Carlin's lemma and Hermeyer theorem, allow for the identification of incomplete fronts. In addition, the method based on Carlin's lemma, like the sector and segment methods, is designed for convex sets of valid solutions [13]. When genetic algorithms are used for multicriteria optimization of options, their effectiveness is tested by solving two problems: the ability of the algorithm to converge to the Pareto front (convergence problem) and to distribute options evenly across the Pareto front (propagation problem) [9, 15]. One of the most widely used genetic algorithms for solving the problem of forming a Pareto front on admissible sets of super-large sizes is the NSGA-II non-dominant sorting algorithm. Its features are: the ability to use binary data representation in conjunction with classical genetic operators (single-point crossing and point mutation); the ability to use decimal data representation for continuous optimization problems. The latter requires the use of specific genetic operators, such as simulated binary crossover and polynomial mutation.

To reduce the time complexity of the existing methods, procedures for preliminary selection of approximate Pareto fronts S' using sector or segment methods are used [13]. For such approximations, the requirement $S^E \subseteq S' \subseteq S$ must be fulfilled. These procedures provide for the preliminary determination of the options lying on the boundaries of the approximate set of options S' in the space of local criteria $k_j(s)$, $j = \overline{1,m}$ on the set of admissible options $S = \{s\}$. Hyperplanes are drawn through the points $< k_j^+, k_j^- >$, $j = \overline{1,m}$ lying on the boundary of the set of admissible options $S = \{s\}$ in the space of local criteria $k_j(s)$, $j = \overline{1,m}$ which divide the options into subsets that fall into the sector $S_1' \supseteq S^E$ or segment $S_2' \supseteq S^E$, respectively, and are clearly ineffective $\overline{S}^E$:

$$S = S_1' \cup \overline{S}^E, \quad S_1' \cap \overline{S}^E = \varnothing; \tag{5}$$

$$S = S_2' \cup \overline{S}^E, \quad S_2' \cap \overline{S}^E = \varnothing. \tag{6}$$

In models of TP optimization problems, you can use the interval representation of characteristics $k_j(s) = [k_j^-(s); k_j^+(s)]$, $j = \overline{1,m}$ of the variants $s \in S$. In this case, each of the characteristics will be represented not by one value, but by two values that define its boundaries. For some interval values $a \in [a^-; a^+]$ and $b \in [b^-; b^+]$ of the local criteria $k_j(s)$, $j = \overline{1,m}$ the rules for performing classical arithmetic operations are determined by the relations [16–17]:

$$[c^-; c^+] = [a^-; a^+] \circ [b^-; b^+]; \tag{7}$$

$$[a] + [b] = [a^- + b^-; a^+ + b^+]; \tag{8}$$

$$[a] - [b] = [a^- - b^+; a^+ - b^-]; \tag{9}$$

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$$[a]\cdot[b]=[\min\{a^-\cdot b^-,a^-\cdot b^+,a^+\cdot b^-,a^+\cdot b^+\};\max\{a^-\cdot b^-,a^-\cdot b^+,a^+\cdot b^-,a^+\cdot b^+\}]; \quad (10)$$

$$[a]/[b]=[a]\cdot[1/b^+;1/b^-]. \quad (11)$$

It is possible to compare the scores of design options represented by non-overlapping intervals by comparing their centers (mean values). For interval values of local criteria that intersect, the estimate of the generalized Hukuhari difference (interval difference, gH-difference) can be used [18–21].

An analysis of publications devoted to solving the problems of determining Pareto fronts in TP design decision-making technologies shows that

– existing mathematical models and methods are designed for conditions with point input data and have significantly different computational complexity and accuracy;

– features of modern decision support technologies show a tendency to increase the universalization of their mathematical support;

– it is possible to formulate requirements for effective technologies for solving the problems of forming and selecting subsets of non-dominated alternatives.

In view of this, the **aim of the article** is to increase the efficiency of TP computer-aided design technologies by developing mathematical models of the tasks selecting subsets of effective design solutions with interval-specific characteristics of options.

Study results

Mathematical model of the interval comparison task

The implementation of Pareto front detection methods involves comparing options from the set of valid ones $S=\{s\}$ for each of the local criteria $k_j(s)$, presented in interval form $k_j(s)=[k_j^-(s);k_j^+(s)]$, $j=\overline{1,m}$. The comparison of the evaluations of design options according to local criteria, represented by non-overlapping intervals, will be carried out by comparing their average values (centers). If the intervals do intersect, the choice will depend on their relative position. To make a decision in such cases, some formal indicator (additional criterion) is needed. To compare the overlapping intervals, we will use the generalized Hukuhari difference estimate as such an indicator.

Here are the values of the j-th characteristic of the options $s_i,s_l\in S$ as intervals $A=[k_j^-(s_i);k_j^+(s_i)]$ and $B=[k_j^-(s_l);k_j^+(s_l)]$ in the form $A=[\hat{a};\ \bar{a}]$ and $B=[\hat{b};\ \bar{b}]$ where $\hat{a}$, $\hat{b}$, $\bar{a}$, $\bar{b}$ – respectively, are the centers and the radiuses of the intervals A and B:

$$\hat{a}=[a^-+a^+]/2,\ \bar{a}=[a^+-a^-]/2,\ \hat{b}=[b^-+b^+]/2,\ \bar{b}=[b^+-b^-]/2. \quad (12)$$

The generalized Hukuhari difference $A\,\overset{-}{_{gH}}\,B$ and the comparison index $\gamma_{A,B}$ built on its basis for intervals $A=[\hat{a};\ \bar{a}]$ and $B=[\hat{b};\ \bar{b}]$ are determined by the relation [18–19]:

$$A\,\overset{-}{_{gH}}\,B=\left[\min\left\{a^--b^-;a^+-b^+\right\};\ \max\left\{a^--b^-;a^+-b^+\right\}\right]=\left(\hat{a}-\hat{b};\left|\bar{a}-\bar{b}\right|\right), \quad (13)$$

$$\gamma_{A,B}=\left(\bar{a}-\bar{b}\right)\Big/\left(\hat{a}-\hat{b}\right). \quad (14)$$

In optimization problems with interval parameters or comparison index variables $\gamma_{A,B}$ (14) the measure of profit or risk is important when choosing an interval A instead of B only based on the fulfillment of the inequality $\hat{a}>\hat{b}$ [20–21].

In multifactorial evaluation of design decisions, local criteria may have different directions of desired change. In maximization problems with positive average return $\hat{a}>\hat{b}$, the following situations of interval intersection are possible [18].

Situation 1.1: $a^-<b^-$. In this situation, some values of the interval $a\in A$ are worse than all values of the interval $b\in B$, and the possible loss of decision quality in the worst case is $\bar{a}-\bar{b}<0$. The ratio of the worst-case loss to the average gain is:

$$I_{1,1}(A,B)=\left(a^--b^-\right)\Big/\left(\hat{a}-\hat{b}\right)=1-\gamma_{A,B}<0. \quad (15)$$

Situation 2.1: $a^-\geq b^-$. In this situation, some values of the interval $b\in B$ are worse than all values of the interval $a\in A$.

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No losses in the worst-case scenario:

$$I_{2,1}(A,B)=\left(a^{-}-b^{-}\right)\Big/\left(\hat{a}-\hat{b}\right)=1-\gamma_{A,B}>0\,. \quad (16)$$

Situation 3.1: $a^{+}<b^{+}$. In this situation, all values of the interval $a\in A$ are worse than some values of the interval $b\in B$. Negative value of the difference $a^{+}-b^{+}<0$ reflects possible losses in the worst-case scenario. The ratio of losses to average profit in the worst case is determined by the ratio:

$$I_{3,1}(A,B)=\left(a^{+}-b^{+}\right)\Big/\left(\hat{a}-\hat{b}\right)=1+\gamma_{A,B}<0\,. \quad (17)$$

Situation 4.1: $a^{+}\geq b^{+}$. In this situation, some values of the interval $a\in A$ are better than all values of the interval $b\in B$, and there are no losses in the worst-case scenario:

$$I_{4,1}(A,B)=\left(a^{+}-b^{+}\right)\Big/\left(\hat{a}-\hat{b}\right)=1+\gamma_{A,B}>0\,. \quad (18)$$

In minimization problems, the interval A is pre-selected compared to B, if $\hat{a}<\hat{b}$. Using the value of $\gamma_{A,B}$ (14), the risk measure for such a choice can be established. A significant difference $\hat{b}-\hat{a}>0$ indicates a correct choice, but it is necessary to take into account the type of intersection of the intervals A and B [18].

Situation 1.2: $a^{-}<b^{-}$. In this situation, for each value of the interval $b\in B$ there are such values of $a\in A$, that $a<b$:

$$I_{1,2}(A,B)=\left(a^{-}-b^{-}\right)\Big/\left(\hat{a}-\hat{b}\right)=1-\gamma_{A,B}>0\,. \quad (19)$$

Situation 2.2: $a^{-}\geq b^{-}$. In this situation, some values of the interval $b\in B$ better (smaller) than all values of the interval $a\in A$. A positive difference $a^{-}-b^{-}>0$ indicates possible losses in the worst case. The ratio of the worst-case loss to the average profit is given by the ratio:

$$I_{2,2}(A,B)=\left(a^{-}-b^{-}\right)\Big/\left(\hat{a}-\hat{b}\right)=1-\gamma_{A,B}<0\,. \quad (20)$$

Situation 3.2: $a^{+}<b^{+}$. In this situation, some values of the interval $b\in B$ worse (larger) than all values of the interval $a\in A$:

$$I_{3,2}(A,B)=\left(a^{+}-b^{+}\right)\Big/\left(\hat{a}-\hat{b}\right)=1+\gamma_{A,B}>0\,. \quad (21)$$

Situation 4.2: $a^{+}\geq b^{+}$. In this situation, some values of the interval $a\in A$ are worse (larger) than all values of the interval $b\in B$. A positive difference $a^{+}-b^{+}>0$ indicates possible losses in the worst case. The ratio of worst-case losses to average return is:

$$I_{4,2}(A,B)=\left(a^{+}-b^{+}\right)\Big/\left(\hat{a}-\hat{b}\right)=1+\gamma_{A,B}<0\,. \quad (22)$$

Using comparison indices based on the estimation of the generalized Hukuhari difference (13)–(22), we obtain mathematical relations for determining Pareto fronts by the Carlin, Hermeyer, and pairwise comparison methods [13] for problems with interval values of local criteria.

Mathematical model of the problem for the method based on Carlin's lemma

A subset of feasible options S effective on a convex set S^{E} based on Carlin's lemma can be found by combining options s_j^{o} that optimize each of the local criteria $k_j(s)$, $j=\overline{1,m}$ by solving a parametric programming problem with respect to the parameters λ_j, $j=\overline{1,m}$:

$$\lambda_j\in\Lambda=\left\{\lambda_j:\lambda_j>0\ \ \forall j=\overline{1,m},\ \ \sum_{j=1}^{m}\lambda_j=1\right\}, \quad (23)$$

$$s_i^{o}=\arg\max_{s\in S}\left\{\left[P^{-}(s),P^{+}(s)\right]=\sum_{j=1}^{m}\lambda_j\left[\xi_j^{-}(s),\xi_j^{+}(s)\right]\right\}, \quad (24)$$

where λ_j, $j=\overline{1,m}$ – importance coefficients of local criteria; $P^{-}(x)$, $P^{+}(x)$ – lower and upper bounds of the global performance criterion for the option $s\in S$; $\xi_j^{-}(s)$, $\xi_j^{+}(s)$ – lower and upper bounds of the value of the utility function of the i-th local criterion, $j=\overline{1,m}$.

To compute the bounds of the utility functions of local criteria, we use the classical relations with parameters α_j [17]:

$$\xi_j^{-}(s)=\left[\frac{k_i^{-}(s)-k_j^{w}}{k_j^{b}-k_j^{w}}\right]^{\alpha_j},\quad \xi_j^{+}(s)=\left[\frac{k_i^{+}(s)-k_j^{w}}{k_j^{b}-k_j^{w}}\right]^{\alpha_j},\quad j=\overline{1,m}, \quad (25)$$

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where $k_j^-(s)$, $k_j^+(s)$ – lower and upper limits of the value of the j -th local criterion for the design solution $s \in S$; k_j^b, k_j^w – the best and worst values of the criterion on the set of valid options S .

To calculate the interval values of the total utility function $P^-(x)$, $P^+(x)$ in model (24), the operations of adding and multiplying intervals by the numbers λ_j , $j = \overline{1, m}$ are performed. Addition of intervals $A = [\hat{a};\ \bar{a}]$ and $B = [\hat{b};\ \bar{b}]$ and multiplication of an interval $A = [\hat{a};\ \bar{a}]$ by a certain number $\beta \ge 0$ are performed according to the relations [18, 21]:

$$A + B = [a^- + b^-;\ a^- + b^-] = (\hat{a} + \hat{b};\ \bar{a} + \bar{b})\ ; \tag{26}$$

$$\beta A = [\min\{\beta a^-;\ \beta a^+\};\ \max\{\beta a^-;\ \beta a^+\}] = (\beta\hat{a}; \beta\bar{a})\ . \tag{27}$$

In a reasonable time, it is possible to accurately determine the front $S^E \subseteq S$ only for small sets of valid solutions $S = \{s\}$ [13].

A mathematical model of the problem for the method based on Hermeyer's theorem

A subset of effective options S^E on an arbitrary set of admissible options S using this method can be found by combining options s_j^o that optimize each of the local criteria $k_j(s)$, $j = \overline{1, m}$, by solving a parametric programming problem with respect to the parameters λ_j , $j = \overline{1, m}$:

$$\lambda_j \in \Lambda = \{\lambda_j : \lambda_j > 0 \ \ \forall j = \overline{1, m}, \ \ \sum_{j=1}^{m} \lambda_j = 1\}\ , \tag{28}$$

$$s_j^o = \arg\max_{s \in S}\{[P^-(s), P^+(s)] = \min_j \lambda_j [\xi_j^-(s), \xi_j^+(s)]\}\ . \tag{29}$$

This method allows Pareto fronts on both convex and non-convex sets of options $S = \{s\}$. Usually, it is impossible to determine the exact front using this method for sets of admissible options of large sizes due to the high complexity of the parametric programming problem (28)–(29) [13].

Experiment

Let us consider an example of solving the problem of forming a Pareto front in the TP optimization problem using the proposed models by the method of pairwise comparisons. The basic version of this method involves comparing different pairs of options from the set of admissible options [13]. The first option is selected from the set of valid options $s \in S$, which at this stage is the basis of the Pareto front S^E. Each of the following options $v \in S$ is compared with each of the front options $s \in S^E$ (at the first step is the only option). If the current option $v \in S$ is better than every variant of the previously defined front S^E at least by one of the indicators k_j , $j = \overline{1, m}$, it is added to S^E. If some variant of the front $s \in S^E$ is worse by all indicators k_j , $j = \overline{1, m}$, than the current option $v \in S$, it is excluded from the front, and the option $v \in S$ is added to the set S^E. After reviewing all the alternative options for building a TP, a full front will be selected, containing all the effective options S^E. In this way, the set of permissible TP construction options will be divided into two non-intersecting subsets of effective ones S^E and inefficient $\bar{S}^E$ options:

$$S = S^E \cup \bar{S}^E\ ,\ S^E \cap \bar{S}^E = \varnothing\ . \tag{30}$$

This method allows you to identify complete Pareto fronts on both convex and non-convex sets of admissible options S . Based on the fact that the vast majority of TP construction options generated in the process of their design are inefficient (dominated), it is proposed to use this method already in the process of generating acceptable options $s \in S$. This will significantly reduce computer time and memory consumption in TP computer-aided design technologies.

Statement of the problem. The characteristics of the set of options for acceptable design solutions $S = \{s_i\}$, $i = \overline{1, 8}$, are evaluated according to three local criteria: the length of the technological cycle $k_1(s) \to \min$, reliability $k_2(s) \to \max$ and the

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above costs $k_3(s) \to \min$. The characteristics of the options are determined with an error $\varepsilon_j \approx 0,05$, $j = \overline{1,3}$: $k_1(s) \in [3,601;\ 5,359]$; $k_2(s) \in [0,910;\ 0,993]$; $k_3(s) \in [9,683;\ 13,927]$ (Table 1).

It is necessary to determine a subset of options that make up the Pareto front S^E on the set of options $S = \{s_i\}$ for acceptable design solutions.

According to the ratio (12) for local criteria $k_j(s)$, $j = \overline{1,3}$ we calculate the values of the centers $\hat{k}_j(s)$ and radiuses $\bar{k}_j(s)$ of the corresponding intervals (Table 1). Based on the fact that it is desirable to minimize the first and third local criteria and maximize the second, it becomes necessary to check the relative position of the corresponding intervals in all the situations considered above (15)–(22). In order to reduce the number of checks depending on the direction of the desired change in the local criteria by the relations (25) for $\alpha_j = 1$ we calculate the corresponding values of their utility (value) functions $\xi_j^-(s_i)$, $\xi_j^+(s_i)$, as well as the values of their centers and radiuses $\hat{\xi}_j(s_i)$, $\bar{\xi}_j(s_i)$, $j = \overline{1,3}$ (Table 2). Based on the fact that $\xi_j(s) \to \max$, $j = \overline{1,m}$ this will allow us to check the type of intersection of intervals only for the case of maximization – situations 1.1–4.1, i.e. (15)–(18).

Table 1. *Characteristics of acceptable design options for TP construction options*

Option	$k_1^-(s)$	$k_1^+(s)$	$\hat{k}_1(s)$	$\bar{k}_1(s)$	$k_2^-(s)$	$k_2^+(s)$	$\hat{k}_2(s)$	$\bar{k}_2(s)$	$k_3^-(s)$	$k_3^+(s)$	$\hat{k}_3(s)$	$\bar{k}_3(s)$
s_1	4,923	5,359	5,141	0,218	0,913	0,961	0,937	0,024	9,683	10,247	9,965	0,282
s_2	3,601	3,890	3,746	0,145	0,922	0,970	0,946	0,024	13,179	13,927	13,553	0,374
s_3	4,562	4,958	4,760	0,198	0,932	0,981	0,956	0,025	9,812	10,383	10,098	0,286
s_4	4,412	4,902	4,657	0,245	0,936	0,985	0,960	0,025	11,149	11,790	11,470	0,321
s_5	4,405	4,894	4,650	0,245	0,935	0,984	0,959	0,025	12,666	13,387	13,027	0,361
s_6	4,382	4,758	4,570	0,188	0,943	0,993	0,968	0,025	10,112	10,698	10,405	0,293
s_7	4,476	4,973	4,725	0,249	0,923	0,972	0,948	0,024	12,522	13,235	12,879	0,357
s_8	3,935	4,261	4,098	0,163	0,910	0,958	0,934	0,024	12,140	12,833	12,487	0,347

Table 2. *Values of utility functions of local criteria for TP construction options*

Option	$\xi_1^-(s)$	$\xi_1^+(s)$	$\hat{\xi}_1(s)$	$\bar{\xi}_1(s)$	$\xi_2^-(s)$	$\xi_2^+(s)$	$\hat{\xi}_2(s)$	$\bar{\xi}_2(s)$	$\xi_3^-(s)$	$\xi_3^+(s)$	$\hat{\xi}_3(s)$	$\bar{\xi}_3(s)$
s_1	0,248	0,000	0,124	0,124	0,036	0,614	0,325	0,289	1,000	0,867	0,934	0,066
s_2	1,000	0,836	0,918	0,082	0,139	0,723	0,431	0,292	0,176	0,000	0,088	0,088
s_3	0,453	0,228	0,341	0,113	0,264	0,855	0,560	0,295	0,970	0,835	0,902	0,067
s_4	0,539	0,260	0,399	0,139	0,310	0,904	0,607	0,297	0,655	0,504	0,579	0,076
s_5	0,543	0,265	0,404	0,139	0,299	0,892	0,595	0,296	0,297	0,127	0,212	0,085
s_6	0,556	0,342	0,449	0,107	0,402	1,000	0,701	0,299	0,899	0,761	0,830	0,069
s_7	0,502	0,220	0,361	0,141	0,161	0,747	0,454	0,293	0,331	0,163	0,247	0,084
s_8	0,810	0,625	0,717	0,093	0,001	0,578	0,290	0,289	0,421	0,258	0,339	0,082

To determine the composition of the front, we will use the method of pairwise comparisons. Taking into account that $\xi_j(s) \to \max$, $j = \overline{1,3}$, we use formulas (13)–(18) to calculate the value of the generalized Hukuhari difference and comparison indices for the utility functions of the first (Table 3), second (Table 4), and third (Table 5) local criteria.

After analyzing the value of the logical function *True* / *False* ($T/F = 1 \vee 0$) for all possible pairs $< s_i, s_j >$, we establish the ratio of strict preference $R_j(S)$ on the set of acceptable design options $S = \{s_i\}$, $i = \overline{1,8}$ for each of the local criteria $k_j(s)$, $j = \overline{1,3}$:

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$$R_1(S)=\{<s_1,s_2>,<s_1,s_3>,<s_1,s_4>,<s_1,s_6>,<s_1,s_8>,<s_5,s_6>,<s_5,s_8>,<s_7,s_8>\}, \quad (31)$$

$$R_2(S)=\{<s_1,s_2>,<s_1,s_3>,<s_1,s_4>,<s_1,s_5>,<s_1,s_6>,<s_1,s_7>,<s_1,s_8>\}, \quad (32)$$

$$R_3(S)=\{<s_1,s_2>,<s_1,s_3>,<s_1,s_4>,<s_1,s_5>,<s_1,s_7>,<s_2,s_3>,<s_5,s_7>,<s_6,s_7>,<s_6,s_8>\}. \quad 33)$$

Table 3. *Values of indices for comparing pairs of options by local criterion* $k_1(s)$

Pair	$\gamma(s_i,s_j)$	Situations	Indexes $I_{k,l}$		T / F	Pair	$\gamma(s_i,s_j)$	Situations	Indexes $I_{k,l}$		*T / F*
$s_1 \succ s_2$	-0,392	2.1, 4.1	1,392	0,608	1	$s_2 \succ s_3$	-0,053	1.1, 3.1	1,053	0,947	0
$s_1 \succ s_3$	-0,711	2.1, 4.1	1,711	0,289	1	$s_5 \succ s_6$	-0,052	2.1, 4.1	1,052	0,948	1
$s_1 \succ s_4$	-0,655	2.1, 4.1	1,655	0,345	1	$s_5 \succ s_7$	-0,055	1.1, 3.1	1,055	0,945	0
$s_1 \succ s_5$	-0,053	1.1, 3.1	1,053	0,947	0	$s_5 \succ s_8$	-0,053	2.1, 4.1	1,053	0,947	1
$s_1 \succ s_6$	-0,051	2.1, 4.1	1,051	0,949	1	$s_6 \succ s_7$	-0,053	1.1, 3.1	1,053	0,947	0
$s_1 \succ s_7$	-0,054	1.1, 3.1	1,054	0,946	0	$s_6 \succ s_8$	-0,050	1.1, 3.1	1,050	0,950	0
$s_1 \succ s_8$	-0,053	2.1, 4.1	1,053	0,947	1	$s_7 \succ s_8$	-0,054	2.1, 4.1	1,054	0,946	1

Table 4. *Values of indexes for comparing pairs of options by local criterion* $k_2(s)$

Pair	$\gamma(s_i,s_j)$	Situations	Indexes $I_{k,l}$		T / F	Pair	$\gamma(s_i,s_j)$	Situations	Indexes $I_{k,l}$		*T / F*
$s_1 \succ s_2$	0,024	2.1, 4.1	0,976	1,024	1	$s_2 \succ s_3$	0,026	1.1, 3.1	0,974	1,026	0
$s_1 \succ s_3$	0,021	2.1, 4.1	0,979	1,021	1	$s_5 \succ s_6$	0,026	1.1, 3.1	0,974	1,026	0
$s_1 \succ s_4$	0,020	2.1, 4.1	0,980	1,020	1	$s_5 \succ s_7$	0,026	1.1, 3.1	0,974	1,026	0
$s_1 \succ s_5$	0,024	2.1, 4.1	0,976	1,024	1	$s_5 \succ s_8$	0,026	1.1, 3.1	0,974	1,026	0
$s_1 \succ s_6$	0,024	2.1, 4.1	0,976	1,024	1	$s_6 \succ s_7$	0,026	1.1, 3.1	0,974	1,026	0
$s_1 \succ s_7$	0,024	2.1, 4.1	0,976	1,024	1	$s_6 \succ s_8$	0,026	1.1, 3.1	0,974	1,026	0
$s_1 \succ s_8$	0,022	2.1, 4.1	0,978	1,022	1	$s_7 \succ s_8$	0,026	1.1, 3.1	0,974	1,026	0

Table 5. *Values of indexes for comparing pairs of options by local criterion* $k_3(s)$

Pair	$\gamma(s_i,s_j)$	Situations	Indexes $I_{k,l}$		T / F	Pair	$\gamma(s_i,s_j)$	Situations	Indexes $I_{k,l}$		*T / F*
$s_1 \succ s_2$	0,078	2.1, 4.1	0,922	1,078	1	$s_2 \succ s_3$	-0,043	2.1, 4.1	1,043	0,957	1
$s_1 \succ s_3$	-0,026	2.1, 4.1	1,026	0,974	1	$s_5 \succ s_6$	-0,026	1.1, 3.1	1,026	0,974	0
$s_1 \succ s_4$	-0,026	2.1, 4.1	1,026	0,974	1	$s_5 \succ s_7$	-0,024	2.1, 4.1	1,024	0,976	1
$s_1 \succ s_5$	-0,026	2.1, 4.1	1,026	0,974	1	$s_5 \succ s_8$	-0,026	1.1, 3.1	1,026	0,974	0
$s_1 \succ s_6$	-0,030	1.1, 3.1	1,030	0,970	0	$s_6 \succ s_7$	-0,026	2.1, 4.1	1,026	0,974	1
$s_1 \succ s_7$	-0,025	2.1, 4.1	1,025	0,975	1	$s_6 \succ s_8$	-0,043	2.1, 4.1	1,043	0,957	1
$s_1 \succ s_8$	-0,024	2.1, 4.1	1,024	0,976	0	$s_7 \succ s_8$	-0,025	1.1, 3.1	1,025	0,975	0

From the intersection of the binary relations of strict preference (31)–(33), we establish the composition of a subset of inefficient options $\bar{S}^E$ and the Pareto front S^E:

$$\bar{S}^E=\{s_2,s_3,s_4\};\ S^E=\{s_1,s_5,s_6,s_7,s_8\}. \quad (34)$$

For the defined subset of inefficient options $\bar{S}^E$ and the Pareto front S^E the initial conditions are met (30):

$$S^E \cup \bar{S}^E = S = \{s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8\},$$
$$S^E \cap \bar{S}^E = \{s_1,s_5,s_6,s_7,s_8\} \cap \{\{s_2,s_3,s_4\} = \varnothing \quad (35)$$

It has been experimentally established that the accuracy and time of solving such problems by methods based on Carlin's lemma (23)–(24) and Hermeyer's theorem (28)–(29) significantly depend on the step $\Delta\lambda_j$ of changing the parameters λ_j, $j=\overline{1,m}$. Thus, during the solution of the problem with a uniform distribution of the characteristics of the options by the method of pairwise comparisons for $|S|=100\,000$ and $m=3$ the

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method based on Carlin's lemma identified 16.9%, and the method based on Hermeyer's theorem identified only 8.5% of solutions belonging to the Pareto front S^E.

Conclusion

Based on the results of the analysis of the current state of the problem of decision support in the process of TP optimization, the following has been established: the vast majority of problems are combinatorial and multicriteria; the number of alternative options for building TP increases sharply with the growth of their complexity; the vast majority of options are inefficient (dominated) and can be improved simultaneously by all local criteria; existing mathematical models, methods and algorithms are designed to solve problems with point input data; evaluation of options by In recent years, there have been publications on interval analysis, which propose a relatively simple and rational formalization of interval comparison operations in optimization problems. This has created the prerequisites for improving the efficiency of design decision support technologies by developing mathematical models of the tasks of selecting subsets of effective options, taking into account the interval representation of the values of local quality criteria.

It is proposed to compare the evaluations of design options according to local criteria, which are represented by non-overlapping intervals, by comparing their centers (average values). To compare overlapping intervals, it is proposed to use comparison indices based on the generalized Hukuhari difference as a formal indicator (additional criterion). Depending on the type of optimization criterion and the relative position of the intervals, such indices have the value of an indicator of the degree of gain or risk, when one of the intervals is selected only on the basis of comparison of their centers.

Using the comparison indices, we propose mathematical models of the problems of forming Pareto fronts based on pairwise analysis of options, Carlin's lemma, and Hermeyer's theorem. They are focused on combined technologies for ranking options using ordinal and cardinality approaches.

Given that the accuracy and time of solving such a problem by methods based on Carlin's lemma and Hermite's theorem significantly depend on the step of parameter change, it is advisable to use them only to determine approximations of large-sized fronts. Given that the vast majority of TP design options generated during their design are inefficient (dominated), it is proposed to use the method of pairwise comparisons already in the process of generating valid options. This will significantly reduce the cost of computer time and memory in automated TP design technologies.

The results obtained can be used in multifactorial decision-making technologies in design or control systems. The use of these technologies will increase the degree of automation of design or control processes, reduce decision-making time in conditions of incomplete certainty of input data, and guarantee the quality of these data by selecting them only from a subset of effective ones. The proposed mathematical models expand the methodological foundations for automating TP design processes. They make it possible to correctly reduce the set of alternative options for building TPs for the final selection, taking into account the knowledge, experience of designers and factors that are difficult to formalize.

Further research can be devoted to the development of more efficient methods of Pareto fronts extraction and multicriteria optimization for interval-specific characteristics of decision options.

References

1. Rudalija, B. (2020), "Quality management research trends in context of Industry 4.0: A short review", *Defence and Security Studies*, Vol. 1, P. 44–52. DOI: https://doi.org/10.37868/dss. v1.id149

2. Martyn, Y., Liaskovska, S., Gregus, M., Izonin, I., Velyka, O. (2021), "Optimization of Technological's Processes Industry 4.0 Parameters for Details Manufacturing via Stamping: Rules of Queuing Systems", *Procedia Computer Science*, Vol. 191, P. 290–295. DOI: https://doi.org/10.1016/j.procs.2021.07.036

3. Malea, C., Nitu, L. (2020), "Optimization of the technological process and equipment of complex profiled parts", *IOP Conf. Series: Materials Science and Engineering*", 916, 012058, P. 1–13. DOI: https://doi.org/10.1088/1757-899X/916/1/012058

4. Nevliudov, I., Chala, O., Botsman, I. (2021), "Determination of technological process modes for surface formation of substrates for functional components of microoptoelectromechanical systems", *Functional Materials*, No. 28(2), P. 381–385. DOI: https://doi.org/10.15407/fm28.02.381

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Сучасний стан наукових досліджень та технологій в промисловості. 2023. № 2 (24) ISSN 2524-2296 (online)

5. Akram, M., Alcantud, J. C. R. (2023), "Multi-criteria Decision Making Methods with Bipolar Fuzzy Sets", 214 p. DOI: https://doi.org/10.1007/978-981-99-0569-0

6. Huang, Z., Yue, H., He, Q. (2023), "Research on uncertain multiple attribute decision making based on improved possibility degree relation model and its application", *Engineering Applications of Artificial Intelligence*, Vol. 123, Part B, P. 106349. DOI: https://doi.org/10.1016/j.engappai.2023.106349

7. Wen, X. (2023), "Weighted hesitant fuzzy soft set and its application in group decision making", *Granular Computing 2023*, DOI: https://doi.org/10.1007/s41066-023-00387-w

8. Bejarano, L. A., Espitia, H. E., Montenegro, C. E. (2022), "Clustering Analysis for the Pareto Optimal Front in Multi-Objective Optimization", *Computation 2022*, Vol. 10(3), 37. DOI: https://doi.org/10.3390/computation10030037

9. Peri, D. (2020), "Direct Tracking of the Pareto Front of a Multi-Objective Optimization Problem", *Journal of Marine Science Engineering*, Vol. 8(9), 699. DOI: https://doi.org/10.3390/jmse8090699

10. Drobintsev, P., Voinov, N., Kotlyarova, L., Selin, I., Aleksandrova, O. (2020), "Optimization of Technological Processes at Production Sites Based on Digital Modeling", *Advanced Manufacturing and Automation IX. IWAMA* 2019, Vol. 634, P. 600–607. DOI: https://doi.org/10.1007/978-981-15-2341-0_75

11. Jacob, A., Steimer, S., Stricker, N., Häfner, B., Lanza, G. (2019), "Integrating product function design, production technology optimization and process equipment planning on the example of hybrid additive manufacturing", *Procedia CIRP*, Vol. 86, P. 222–227. DOI: https://doi.org/10.1016/j.procir.2020.01.013

12. Beskorovainyi, V., Imanhulova, Z. (2017), "Technology of large-scale objects system optimization", *ECONTECHMOD*, Vol. 06 (4), P. 3–8, available at: https://yadda.icm.edu.pl/baztech/element/bwmeta1.element.baztech-5d27c6b0-9c2f-4023-9186-5a7036fe9d27

13. Beskorovainyi, V., Petryshyn L., Shevchenko O. (2020), "Specific subset effective option in technology design decisions", *Applied Aspects of Information Technology*, Vol. 3., No. 1, P. 443–455. DOI: https://doi.org/10.15276/aait.01.2020.6

14. Beskorovainyi, V. (2020), "Combined method of ranking options in project decision support systems", *Innovative Technologies and Scientific Solutions for Industries*, No 4 (14), P. 13–20. DOI: https://doi.org/10.30837/ITSSI.2020.14.013

15. Shadura, O. (2019), "Modification of genetic algorithms based on the method of non-centered principal components and standard tests" ["Modyfikatsiia henetychnykh alhorytmiv na osnovi metodu netsentrovanykh holovnykh komponent ta standartni testy"], *World Science*, No. 4(44), Vol. 1, P. 4–10. DOI: https://doi.org/10.31435/rsglobal_ws/30042019/6464

16. Jiang, C., Han, X., Xie, H. (2021), "The Basic Principles of Interval Analysis", *Nonlinear Interval Optimization for Uncertain Problems*", Springer Tracts in Mechanical Engineering, Springer, Singapore, P. 25–34. DOI: https://doi.org/10.1007/978-981-15-8546-3_2

17. Beskorovainyi, V., Kolesnyk, L. (2021), "Interval model of multi-criterion task of reengineering physical structures of distributed databases", *Intelligent information systems for decision support in project and program management*: Collective monograph edited by I. Linde, European University Press., Riga: ISMA, P. 7–14. DOI: https://doi.org/10.30837/MMP.2021.007

18. Guerra, M. L., Stefanini, L. (2012), "A comparison index for interval ordering based on generalized Hukuhara difference", *Soft Computing*, No 16 (11), P. 1–25. DOI: https://doi.org/10.1007/s00500-012-0866-9

19. Stefanini, L., Guerra, M. L., Amicizia, B. (2019), "Interval Analysis and Calculus for Interval-Valued Functions of a Single Variable. Part I: Partial Orders, gH-Derivative, Monotonicity", *Axioms*, Vol. 8, No. 4, 113. DOI: https://doi.org/10.3390/axioms8040113

20. Stefanini, L., Arana-Jimenez, M. (2019), "Karush-Kuhn-Tucker conditions for interval and fuzzy optimization in several variables under total and directional generalized differentiability", *Fuzzy Sets and Systems*, Vol. 362, P. 1–34. DOI: https://doi.org/10.1016/j.fss.2018.04.009

21. Kosheleva, O., Kreinovich, V., Pham, U. (2021), "Decision-making under interval uncertainty revisited", *Asian Journal of Economics and Banking*, Vol. 5(1), P. 79–85. DOI: https://doi.org/10.1108/AJEB-07-2020-0030

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МАТЕМАТИЧНІ МОДЕЛІ ВИЗНАЧЕННЯ ПАРЕТО-ФРОНТУ ДЛЯ ВАРІАНТІВ ПОБУДОВИ ТЕХНОЛОГІЧНИХ ПРОЦЕСІВ В УМОВАХ ІНТЕРВАЛЬНОГО ПОДАННЯ ЛОКАЛЬНИХ КРИТЕРІЇВ

Предметом дослідження є процеси підтримки прийняття рішень у задачах оптимізації технологічних процесів (ТП) на етапах їх проєктування чи реінжинірингу. **Мета роботи** – підвищення ефективності технологій автоматизованого проєктування ТП з допомогою розроблення математичних моделей задач виділення підмножин ефективних проєктних рішень з інтервально заданими характеристиками варіантів. У статті розв'язуються такі **завдання:** огляд і аналіз сучасного стану проблеми підтримки прийняття рішень у задачах оптимізації ТП на етапах їх проєктування чи реінжинірингу; декомпозиція проблеми прийняття проєктних рішень; формалізація задачі порівняння інтервалів для виділення Парето-фронтів із використанням індексів порівняння на основі узагальненої різниці Хукухари; розроблення математичної моделі задачі для методу на основі леми Карліна; створення математичної моделі задачі для методу на основі теореми Гермеєра; визначення фронту Парето в задачі оптимізації ТП методом парних порівнянь. Використовуються такі **методи:** системний підхід, теорії систем, теорії корисності, теорії прийняття рішень, системного проєктування, оптимізації та дослідження операцій. **Результати.** Визначено місце та зв'язки задачі визначення Парето-фронту в проблемі прийняття проєктних рішень. Формалізовано процедуру порівняння інтервалів для виділення Парето-фронтів із використанням індексів порівняння на основі узагальненої різниці Хукухари. Розроблено математичні моделі задачі виділення Парето-фронтів методами на основі леми Карліна й теореми Гермеєра для випадку інтервального подання значень локальних критеріїв. Наведено приклад формування Парето-фронту в задачі оптимізації технологічного процесу методом парних порівнянь за показниками тривалості технологічного циклу, надійності та наведених витрат. **Висновки.** Запропоновані математичні моделі розширюють методологічні основи автоматизації процесів проєктування ТП. Вони уможливлюють коректне скорочення множини альтернативних варіантів побудови ТП для остаточного вибору з урахуванням знань, досвіду проєктувальників і факторів, що важко піддаються формалізації. Практичне використання математичних моделей підвищить ступінь автоматизації процесів проєктування чи керування, скоротить час прийняття рішень в умовах неповної визначеності вхідних даних і гарантуватиме якість цих даних завдяки їх вибору з підмножини ефективних.

Ключові слова: технологічні процеси; автоматизація проєктування; оптимізація; реінжиніринг; багатокритеріальне оцінювання; підтримка прийняття рішень; Парето-фронт.

Бібліографічні описи / Bibliographic descriptions

Безкоровайний В. В., Колесник Л. В., Dr. Chinwi Mgbere. Математичні моделі визначення Парето-фронту для варіантів побудови технологічних процесів в умовах інтервального подання локальних критеріїв. *Сучасний стан наукових досліджень та технологій в промисловості*. 2023. № 2 (24). С. 16–26. DOI: https://doi.org/10.30837/ITSSI.2023.24.016

Beskorovainyi, V., Kolesnyk, L., Dr. Chinwi Mgbere. (2023), "Mathematical models for determining the Pareto front for building technological processes options under the conditions of interval presentation of local criteria", *Innovative Technologies and Scientific Solutions for Industries*, No. 2 (24), P. 16–26. DOI: https://doi.org/10.30837/ITSSI.2023.24.016