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ANALYSIS OF THE CRITERIA SELECTION PROBLEM IN DIVERSIFICATION MODELS

The digitalization of the economy reduces the cost of doing business by automating the relevant processes, but any transformation creates new risks and economic instability. Economic instability leads to a drop in the standard of living and, as a result, negatively affects the activities of trade enterprises. Small and medium businesses are especially sensitive to any changes. The decrease in demand for most everyday goods has a painful effect on the activities of small and medium-sized businesses and leads to the emergence of new risks. These risks have a significant impact on reducing the profitability of enterprises. Therefore, it is important for each enterprise to diversify the activities of the enterprise, which includes the expansion of the product range, the reorientation of sales markets and the optimal distribution of goods between divisions of one enterprise.**The subject** of the article is multi-criteria models of a diversified portfolio that minimize the risks that arise in the era of the digital economy when managing retail chains. To formalize the problem, five models are proposed that differ in vector objective functions, both in the quantity and quality of the selected criteria. **The aim of the work** is to analyze the problem of choosing criteria in the corresponding multicriteria or vector diversification problems. The article examines the advantages of introducing an additional criterion of entropy maximization into the criteria of the classical two-criteria model of portfolio theory, which characterizes the degree of diversity of the portfolio composition. A complex combination of methods of classical portfolio theory and multicriteria optimization is applied. **The results** include a comparison of three methods for solving the following problems: criteria convolution, successive concessions, and computer simulation of the Pareto set. **Conclusions**: the results obtained will be useful for automating the risk management of retail chains. The practical value is that the obtained results of real data for the network have demonstrated the possibility of using the developed tool for automatic allocation of resources in the form of pareto-optimal portfolios in order to minimize risks.

Keywords: computer simulation; multicriteria problem; optimal portfolio problem; convolution of criteria; method of successive concessions; Pareto set; entropy.

Introduction

The transition to a digital economy and digital trade in the world has had a significant impact on the Ukrainian economy. Large-scale digital transformation was accelerated by the COVID-19 pandemic, which affected consumer behavior and changed the rules for business operations. The digitalization of the economy reduces the cost of doing business by automating relevant processes, but any transformation generates new risks and economic instability. Economic instability leads to a decline in living standards and, as a result, negatively affects the activities of trading companies, especially in the context of Ukraine's recovery [1]. Small and medium-sized businesses are very sensitive to any changes. The decline in demand for most everyday goods has a painful impact on the activities of small and medium-sized businesses and leads to the emergence of new risks. These risks have a significant impact on the decline in the profitability of enterprises. Therefore, it is important for every enterprise to diversify its activities, which involves expanding the product range, reorienting sales markets, and optimizing the distribution of goods between structural units.

The portfolio theory has high potential in many areas. Currently, different approaches to choosing optimality criteria and solution algorithms are used to determine the best strategy. Both exact algorithms for finding optimal solutions and approximate algorithms of three categories have been developed: heuristic, metaheuristic, and hyperheuristic [2]. The introduction of machine learning technologies is relevant [3, 4]. Paper [3] uses modern advances in neural network architecture for efficient convex optimization for risk-based portfolio performance: minimum variance, maximum diversification, and equal risk contribution [4]. Further development of the use of neural networks for optimal portfolio construction tasks is covered in [5-7]. For example, paper [5] presents an approach to portfolio construction strategy based on a hybrid machine learning model that combines a convolutional neural network and bidirectional long-term short-term memory with reliable input characteristics derived from the Huber location for stock forecasting and the Markovian mean-variance model for the proposed optimal portfolio construction. Paper [6] considers maximizing portfolio returns using *Reinforcement Learning*, taking into account dynamic risks corresponding to market conditions, by dynamically rebalancing the portfolio. Recurrent networks, namely *the Deep Belief-Recurrent Neural Network*, together with a hybrid algorithm called *HH-DHO*, are also used for portfolio forecasting tasks [7].

Several studies contain the results of extending methods for formalizing optimal portfolio problems [8, 9]. In [9], a method is proposed to reduce the problem to a linear minimization program subject to a linear constraint of an arbitrary positively homogeneous convex functional, the dual set of which is given by linear inequalities, which allows us to expand the class of linear problems of portfolio theory

$$F = \left(F_1(x), F_2(x), ..., F_N(x)\right), \quad (1)$$

which is defined on the admissible set X. One of the options for formalizing the set of alternatives is the Pareto set.

The Pareto set $\tilde{x}$ consists of non-dominant solutions $\tilde{x}$, for each of which there is an unacceptable solution $x^* \in X$ that satisfies the inequality

$$F_i\left(x^*\right) \le F_i\left(\tilde{x}\right), \quad (2)$$

where $i = 1, 2, ..., N$, among which at least one is strict.

It is important to distinguish between two types of formulations of multicriteria problems, namely, an individual problem and a mass problem [8]. An individual problem has fixed parameters of the vector objective function $F = \left(F\rrbracket\_1, F\_2, ..., F\_N\right)$, and a system of constraints. In the formulation of a mass problem, which has a common name, some parameters are not fixed and are given by signs. For example, the classical two-criteria Markowitz portfolio problem with a vector objective function $F = (R, D)$ is a mass problem.

Methods for solving multi-criteria (vector) problems are based on different approaches. One of them is the construction of a generalized criterion that aggregates the vector of criteria of the VOF (1). For example, the method of linear or multiplicative convolution of criteria, the majority criterion, the geometric criterion based on immersion in a metric space. Another approach is to determine the lexicographic order of the criteria. Thus, attempts are being made to move from a multi-criteria problem to a single-criteria problem or a sequence of single-criteria problems with certain constraints. The choice of a solution strategy has an impact on the solution obtained, since the previous constraints on the solution of the problem are changed and new constraints are added. Not all methods can guarantee an acceptable solution. In particular, for certain problems, the linear convolution method does not allow to obtain a Pareto set. In this regard, the problem of solving multi-criteria problems using linear convolution of criteria (LCC) is considered separately [14].

Let's consider this algorithm. The linear convolution algorithms are based on the fact that, given a positively definable VOF, the element $x \in X$ maximizing (minimizing) the linear convolution of the criteria

$$F^{\lambda}(x) = \sum_{\upsilon=1}^{N} \lambda_{\upsilon} F_{\upsilon}(x), \quad (3)$$

is pareto-optimal. Here the vector $\lambda \in \Lambda_N$, where

$$\Lambda_N = \left\{\lambda = \left(\lambda_1, ..., \lambda_N\right) : \sum_{\upsilon=1}^{N} \lambda_{\upsilon} = 1, \lambda_{\upsilon} = 1, \lambda_{\upsilon} > 0, \upsilon = 1, 2, ..., N\right\}.$$

Consider an individual problem with N criteria to be maximized and defined on the set of admissible solutions s $X = \{x\}$. Let us denote the set of alternatives to this problem by $X^*, X^* \subseteq X$. If for each element $x^* \in X^*$ there is a vector $\lambda^* \in \Lambda_N$ corresponding to the equality $F^{\lambda^*}(x^*) = \max_{x \in X} F^{\lambda^*}(x)$, then the problem of finding SA X^* is considered to be solved using a linear convolution algorithm. If the solution determined in this way is characteristic of all individual problems of the mass problem, then for each of them it is possible to find the MA using convolution algorithms. This problem is unsolvable by convolution algorithms if, for the problem under consideration, there is an individual problem from SA X^* containing such an element $x^* \in X^*$ on which the convolution extremum $F^{\lambda}(x) \forall \lambda \in \Lambda_N$ is not reached, i.e., for any $\lambda \in \Lambda_N$ the inequality $F^{\lambda}(x^*) < \max_{x \in X} F^{\lambda^*}(x)$ is clearly hold.

Another method for solving multicriteria optimization problems is the method of sequential concessions, which requires a preliminary ranking of the criteria by importance. At each step k, a single-criteria problem with an objective function of rank k is solved. New constraints are also added to the system of constraints, which ensure that the value of criteria from rank 1 to $(k-1)$ deviates by the amount of the permissible concession $\delta_1 > 0, i = 1, 2, ..., k-1$. The problem is solved when N single-criteria conditional optimization problems with criteria $F_i(x)$,

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where $i = 1, 2, ..., N$ are solved. The end result is the optimal value of the least important criterion, provided that the values of the previous criteria are guaranteed. Paper [19] analyzes the effectiveness of using the method of successive concessions to solve multicriteria problems of diversification of a centralized pharmacy network of different sizes and identifies zones of stability in the space of parameters of the method of concessions.

Solving multicriteria optimization problems is a non-trivial task due to the conceptual uncertainty of vectors that are incommensurable. The final decision is always made by the decision maker. To justify such a choice, it is necessary to assess the properties of the solutions obtained by applying different approaches.

Therefore, the purpose of this article is to analyze the problem of choosing a set of criteria and the effectiveness of solving the multicriteria problem of diversification of a retail network using different methods: sequential concessions, linear convolution and computer modeling.

Statement of the problem and results

Let's formulate the mathematical problem of diversifying a retail network in more detail and recall the basic definitions. The mathematical formulation uses the apparatus for describing multidimensional random variables. The profitability of the network is estimated as

$$R = \sum_{i=1}^{n} r_i x_i ,$$

where the vector $x^* = \begin{pmatrix} x_1^* \\ ... \\ x_n^* \end{pmatrix}$ is a share of retail chain units in the chain's asset portfolio;

r_i – profitability of the network unit, $i = \overline{1,n}$.

Risk D, estimated using the variance matrix $W = |\omega_{ij}|$, $\omega_{ij} = \mathrm{cov}(x_i, x_j)$ – covariance, $i, j = \overline{1,n}$.

$$D = \sum_{i=1}^{n} \omega_{ii} x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij} x_i x_j ,$$

if

$$0 \le x_i \le 1, \quad \sum_{i=1}^{n} x_i = 1.$$

As noted above, the classical optimal portfolio model is a two-criteria problem with a vector objective function $F = (R, D)$.

Let's apply the classical model to the formal description of the problem of optimal distribution of goods of a wholesale trade network among branches. It is necessary to determine the share of goods for each branch in order to maximize the profit R of the entire network under the condition of minimal risk D. To achieve the best possible ratio of expected profitability and risk, it is important to carry out diversification measures, the effectiveness of which requires research. The peculiarity of the problem is the presence of mutual influence between the network units.

In such a formulation, it is necessary to define the concepts of "profit" and "risk" in order to determine the factors of influence and quality criteria for evaluating possible alternatives.

Consider a chain with n outlets (branches). Let us denote:

υ_i is the expected value of the goods sold in sales prices of the i -th branch for the year (the average value of sales for each branch for m years), $i = \overline{1,n}...$;

υ_{si} is the expected value of sales of goods in purchase prices of the i -th branch for the year (average for m years), $i = \overline{1,n}...$;

υ_{zi} is the expected amount of expenses of the i -th branch for the year (average for m years), $i = \overline{1,n}$.

Then $\upsilon_{0i} = \upsilon_{si} + \upsilon_{zi}$ – cost price of goods sold.

The profitability of the i -th branch will be as follows:

$$r_i = \frac{\vartheta_i + \vartheta_{0i}}{\vartheta_{0i}} . \quad (4)$$

Let us denote the share of the distributed resource of the i -th branch by

$$x_i = \frac{\vartheta_{0i}}{\sum_{i=1}^{n} \vartheta_{0i}} , \quad (5)$$

then the profitability of the entire enterprise will be as follows

$$R = \sum_{i=1}^{n} r_i x_i . \quad (6)$$

Really,

$$R = \sum_{i=1}^{n} \frac{\vartheta_i + \vartheta_{0i}}{\vartheta_{0i}} * \frac{\vartheta_{0i}}{\sum_{i=1}^{n} \vartheta_{0i}} = \frac{\sum_{i=1}^{n} \vartheta_i - \sum_{i=1}^{n} \vartheta_{0i}}{\sum_{i=1}^{n} \vartheta_{0i}} = \frac{\vartheta + \vartheta_0}{\vartheta_0} ,$$

where ϑ – the expected cost of goods sold during the year throughout the enterprise;

ϑ_0 – the expected cost of goods sold during the year across the enterprise.

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Risks in the process of forming an assortment portfolio are calculated using the variance:

$$D = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j = \\ = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j , \quad (7)$$

where $\omega_{ij} = \text{cov}(x_i, x_j)$ is a covariance, $\omega_{ij} = \omega_{ij}$, $i, j = \overline{1,n}$.

The solution is the vector $x^* = \begin{pmatrix} x_1^* \\ ... \\ x_n^* \end{pmatrix}$. Knowing $x_i^*, i = \overline{1,n}$, we can calculate the number of distributed resources by branch. From formula (5) we have

$$x_i^* = \frac{\upsilon_{si} + \upsilon_{zi}}{\sum_{i=1}^{n} \upsilon_{si} + \sum_{i=1}^{n} \upsilon_{zi}} .$$

Hence

$$\upsilon_{si} = \left(\sum_{i=1}^{n} \upsilon_{si} + \sum_{i=1}^{n} \upsilon_{zi} \right) x_i^* - \upsilon_{zi} ,$$

where $\sum \vartheta_{si}$ – total distributed resource;

$\sum \vartheta_{zi}$ – the average value of total expenses for a certain period;

ϑ_{zi} – the average value of expenses for each division for a particular period.

The level of diversification is assessed by determining the value of entropy according to the method of K. Shannon, which characterizes the degree of diversity of the system. The introduction of entropy as the third criterion will allow to influence the level of diversification, as well as the assortment structure of the portfolio

$$E = -\sum_{i=1}^{n} x_i \ln x_i .$$

Below are five models for diversifying the portfolio of a retail chain with different composition of VOFs (2).

MODEL 1 corresponds to a two-criteria optimization problem with a vector objective function Φ_1 containing the criteria of risk D and entropy E.

It is necessary to find the vector $x^* = \begin{pmatrix} x_1^* \\ ... \\ x_n^* \end{pmatrix}$ in case of known $W = |\omega_{ij}|$, $\omega_{ij} = \text{cov}(x_i, x_j)$, that

$$\Phi_1 = (D, E) , \quad (8)$$

where

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \to \min,$$

$$E = -\sum_{i=1}^{n} x_i \ln x_i \to \max .$$

Subject to restrictions on the level of profitability r_p –, which is chosen by the expert

$$R = \sum_{i=1}^{n} r_i x_i \ge r_{p.} ,$$

and

$$0 \le x_i \le 1, \quad \sum_{i=1}^{n} x_i = 1.$$

MODEL 2 formalizes a three-criteria optimization problem with a vector objective function Φ_2 containing the criteria of network profitability R, risk D, and entropy E.

It is necessary to find the vector $x^* = \begin{pmatrix} x_1^* \\ ... \\ x_n^* \end{pmatrix}$ in case of known $W = |\omega_{ij}|$, $\omega_{ij} = \text{cov}(x_i, x_j)$ that

$$\Phi_2 = (R, D, E) , \quad (9)$$

where

$$R = \sum_{i=1}^{n} r_i x_i \to \max$$

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \to \min ,$$

$$E = -\sum_{i=1}^{n} x_i \ln x_i \to \max .$$

Subject to restrictions on the level of profitability r_p –, which is chosen by the expert

$$R = \sum_{i=1}^{n} r_i x_i \ge r_{p.} ,$$

and

$$0 \le x_i \le 1, \quad \sum_{i=1}^{n} x_i = 1 .$$

MODEL 3 is a single-criteria problem derived from the classical problem with a vector objective function $F_3 = (R, D)$ by convolution of criteria in the form R/D.

The solution of the problem is the vector $x^* = \begin{pmatrix} x_1^* \\ ... \\ x_n^* \end{pmatrix}$, if $W = |\omega_{ij}|$, $\omega_{ij} = \text{cov}(x_i, x_j)$ are known that

$$\Phi_3 = R/D \to \max , \quad (10)$$

where

$$R = \sum_{i=1}^{n} r_i x_i ,$$

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j .$$

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Subject to restrictions on the level of profitability r_p – , which is chosen by the expert

$$R = \sum_{i=1}^{n} r_i x_i \geq r_p \ ,$$

and

$$0 \leq x_i \leq 1, \quad \sum_{i=1}^{n} x_i = 1 \, .$$

MODEL 4 is a two-criteria optimization problem with a vector objective function Φ_4, which contains the convolution criterion from model 3, i.e., Φ_3, and the entropy criterion E.

It is necessary to find the vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$,

if $W = |\omega_{ij}|$, $\omega_{ij} = \text{cov}(x_i, x_j)$ are known, that

$$\phi_4 = (F_3, E) \ , \tag{11}$$

where

$$\Phi_3 = R/D \rightarrow \max \ ,$$

where

$$R = \sum_{i=1}^{n} r_i x_i \ ,$$

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \ ,$$

$$E = -\sum_{i=1}^{n} x_i \ln x_i \rightarrow \max \, .$$

Subject to restrictions on the level of profitability r_p – , which is chosen by the expert

$$R = \sum_{i=1}^{n} r_i x_i \geq r_{p.} \ ,$$

and

$$0 \leq x_i \leq 1, \quad \sum_{i=1}^{n} x_i = 1.$$

MODEL 5 is a modification of Model 2 and formalizes the problem of two-criteria optimization with a vector objective function Φ_5 containing the criteria of network profitability R and risk D.

It is necessary to find the vector $x^* = \begin{pmatrix} x_1^* \\ \dots \\ x_n^* \end{pmatrix}$ subject

to known $W = |\omega_{ij}|$, $\omega_{ij} = \text{cov}(x_i, x_j)$, that

$$\Phi_5 = (R, D) \ , \tag{12}$$

where

$$R = \sum_{i=1}^{n} r_i x_i \rightarrow \max \ ,$$

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j \rightarrow \min \, .$$

Subject to restrictions on the level of profitability r_p – , which is chosen by the expert

$$R = \sum_{i=1}^{n} r_i x_i \geq r_{p.} \ ,$$

where

$$0 \leq x_i \leq 1, \quad \sum_{i=1}^{n} x_i = 1.$$

The five models described above are mass tasks. In the process of working with real numerical indicators, corresponding individual tasks are formed.

Experiments

The experiments were carried out for individual problems of models 1–5 based on the indicators provided by the decision maker in the retail network.

To substantiate the choice of the final solution, we will perform the solution using the methods related to the construction of a common criterion and the method of concessions for a different set of criteria, as well as apply different software and analyze the results.

Numerical experiments were conducted on the same indicators using different software: 1) using the method of the reduced gradient in the "Solution Finder" service *MS Excel* and 2) using the developed software in the *Matlab* package [10].

First, let us consider the application of the solution approach using the generalized criterion of linear convolution of criteria. It is necessary to construct an optimization integral criterion with an objective function of the form

$$C = \sum_{i=1}^{N} a_i C_i \rightarrow \text{extr} \ , \tag{13}$$

where C_i – are the normalized values of the VOF components (2), i.e. $F = (C_1, C_2, \dots, C_N)$;

$\sum_{i=1}^{n} a_i = 1$, $0 \leq a_i \leq 1$ – is a constant denoting the degree of importance of each partial criterion C_i.

We build an optimization problem based on model 1 with the objective function

$$\Phi_1^{'} = -a^{D/D_{max}{}^{E/E_{max}max}} \ , \tag{14}$$

where $0 \leq a_i \leq 1$ – is a constant denoting the degree of importance of each partial criterion.

Optimization problem based on model 2 with the objective function:

$$\Phi_2^{'} = a_1^{R/R_{max2}{}^{D/D_{max3}{}^{E/E_{max}max}}} \ , \tag{15}$$

where $0 \leq a_i \leq 1$ – a constant indicating the degree of importance of each partial criterion, $\sum_{i=1}^{n} a_i = 1$.

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Optimization problem based on model 5 with an objective function:

$$\Phi_5^{'} = -a^{D/D_{max}{}^{R/R_{max}max}}. \quad (16)$$

The problem will be solved by linear convolution of the criteria (3) using the method of reduced gradient in the "Solution Finder" service of *MS Excel*.

We formulate an individual task of creating an effective investment portfolio for a trading company with five branches. Based on the data on sales and expenses of this enterprise for five years (2017–2021), the vectors of resource allocation ϑ_s and profitability r are compiled:

$$\upsilon_S = (88228{,}15; \quad 189947; \quad 170569; \quad 141857; \quad 99669),$$

$$r = (0{,}0050; \quad 0{,}0393; \quad 0{,}0123; \quad 0{,}0085; \quad 0{,}0116).$$

Covariance coefficients:

$$\omega_{ij} = \begin{pmatrix} 0{,}000663 & 0{,}0003 & 0{,}000091 & -0{,}000214 & 0{,}000152 \\ 0{,}0003 & 0{,}000011 & -0{,}00033 & -0{,}00001 & 0{,}000024 \\ 0{,}000091 & -0{,}00033 & 0{,}000151 & 0{,}000004 & -0{,}000139 \\ -0{,}000214 & -0{,}00001 & 0{,}000004 & 0{,}000043 & 0{,}000028 \\ -0{,}000152 & 0{,}000024 & -0{,}000139 & 0{,}000028 & 0{,}000135 \end{pmatrix}.$$

Standard deviation based on 2017–2021 data:

$$\sigma = (0{,}0257; \quad 0{,}003; \quad 0{,}0123; \quad 0{,}0086; \quad 0{,}0116).$$

We find the efficient portfolio using the linear convolution method under the condition $a_1 = a_2 = 0{,}5$.

$$x^* = (0{,}13; \quad 0{,}29; \quad 0{,}25; \quad 0{,}19; \quad 0{,}14),$$

$$R = \sum_{i=1}^{n} r_i x_i = 0{,}018,$$

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij} x_i x_j = 1{,}12\,E-05,$$

$$E = 0{,}67819.$$

Next, let's consider the method of sequential concessions, which is used to solve multi-criteria problems with a preliminary ranking of the criteria by importance. At each step, a single-criteria conditional optimization problem is solved. At the first step, the objective function is the first-ranked optimization criterion. The constraints coincide with the constraints of the original problem. At each subsequent step k, a single-criteria problem with an objective function of rank k is solved and new constraints are added to ensure that the values of the criteria from rank 1 to $k-1$ deviate by the amount of the permissible deviation $\delta_1 > 0, i = 1, 2, ..., k-1$.

Let's demonstrate the work of the method of sequential concessions on the example of model 2 with a vector objective function $F_2 = (R, D, E)$ under the condition of ranking the criteria: entropy > risk > profit, i.e. $E > D > R$. With the chosen ranking, we have the following sequence of single-criteria conditional optimization problems.

First step.

$$E = -\sum_{i=1}^{n} x_i \ln x_i \to \max,$$

$$\begin{cases} R = \sum_{i=1}^{n} r_i x_i \geq r_p \\ 0 \leq x_i \leq 1, \sum_{i=1}^{n} x_i = 1. \end{cases} \quad (17)$$

The value $E*$ is the optimal value according to the first rank criterion.

Second step.

The objective function is to minimize the risk. The condition of deviation $\delta_1 > 0$ of the optimal value of $E*$ by the amount of the permissible concession is added to the constraints of the original problem:

$$D = \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij} x_i x_j \to \min,$$

$$\begin{cases} \left| -\sum_{i=1}^{n} x_i \ln x_i - E^* \right| \leq \delta_1 \\ R = \sum_{i=1}^{n} r_i x_i \geq r_p \\ 0 \leq x_i \leq 1, \sum_{i=1}^{n} x_i = 1. \end{cases} \quad (18)$$

The risk value $D*$ is the optimal value according to the second-ranked criterion.

Third step.

The objective function is to maximize the profit R. To the constraints of the second step problem, we add the condition that the optimal value of $D*$ deviates by no more than the amount of acceptable deviations $\delta\_2 > 0$:

$$R = \sum_{i=1}^{n} r_i x_i \to \max,$$

$$\begin{cases} \left| -\sum_{i=1}^{n} x_i \ln x_i - E^* \right| \leq \delta_1 \\ \left| \sum_{i=1}^{n} \omega_{ij} x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij} x_i x_j - D^* \right| \leq \delta_2 \\ R = \sum_{i=1}^{n} r_i x_i \geq r_p \\ 0 \leq x_i \leq 1, \sum_{i=1}^{n} x_i = 1. \end{cases} \quad (19)$$

Eight experiments were conducted, the content of which is presented in Table 1.

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Table 1. *Content of the experiments conducted*

№ Experiment	Model	Contentofcriteria	Solutionmethod
1	*Model 1*, $\Phi'1$	$E,\ D$	*LCC* (14)
2	*Model 2*, $\Phi'2$	$E,\ D,\ R$	*LCC* (15)
3	*Model 5*, $\Phi'5$	$D,\ R$	*LCC* (16)
4	*Model 1*, $\Phi1$	$E > D$	*concession* (8)
5	*Model 2*, $\Phi2$	$E > D > R$	*concession* (9)
6	*Model 2*, $\Phi2$	$E > R > D$	*concession* (9)
7	*Model 3*, $\Phi3$	$R,\ D$	*multiplicativeconvolution* (10)
8	*Model 4*, $\Phi4$	$E > \Phi3$	*concession* (11)

The results of experiment 1 on model 1 with the solution by the method of linear convolution of criteria and computer modeling are shown in Fig. 1. The optimal solution is obtained in the form of a vector $X = (0.124; 0.201; 0.032; 0.445; 0.197)$, at which the optimal values of the criteria: $\mathrm{Min}\,D = 2.0467 \times 10^{-5}$, $\mathrm{Max}\,R = 0.015$, $\mathrm{Max}\,E = 1.3729$ are achieved.

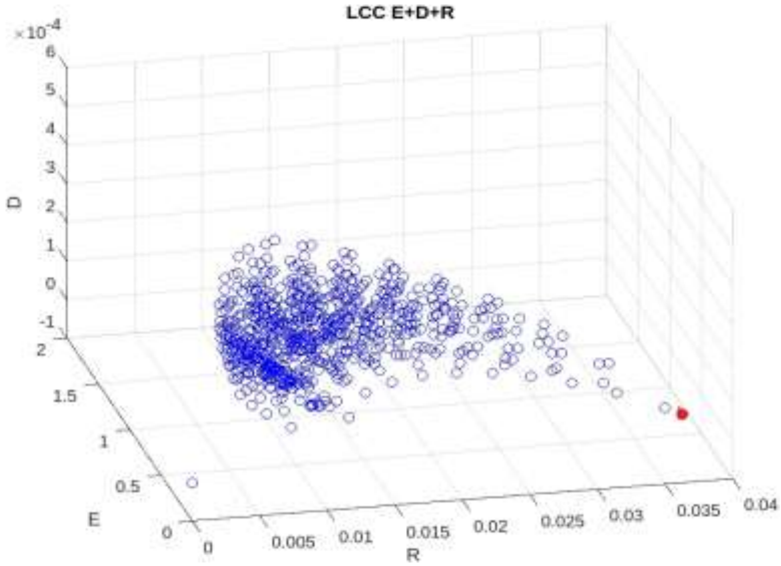

Fig. 1. Experiment 1: model 1(LCC) F1(6) entropy + risk

The second experiment consisted of solving the problem according to model 2 by the method of linear convolution of criteria. The results of computer modeling are shown in Fig. 2.

The optimal solution is obtained in the form of a vector $X = (0.001; 0.939; 0.001; 0.035; 0.023)$, at which the optimal values of the criteria: $\mathrm{Min}\,D = 0.000013$, $\mathrm{Max}\,R = 0.037512$, $\mathrm{Max}\,E = 0.2772$ are achieved.

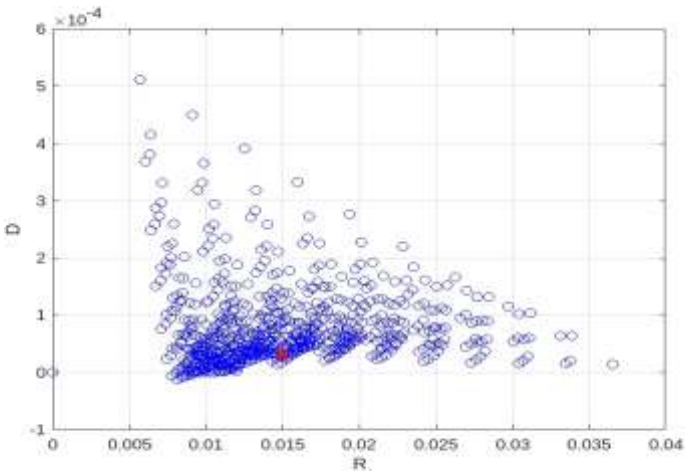

Fig. 2. Experiment 2: model 2(LCC) F'2(7) entropy + risk + profit – projection on the plane (profit, risk) $(R,\ D)$

The results of experiment 5 were atypical, since the solution by the method of concessions according to model 2 led to an invalid solution $X = (0.095; 0.277; 0.069; 0.325; 0.233)$, at which the values of the criteria: $\mathrm{Min}\,R = 0.000030$, $\mathrm{Max}\,E = 1$ are achieved. The results of computer modeling for this case are shown in Fig. 3.

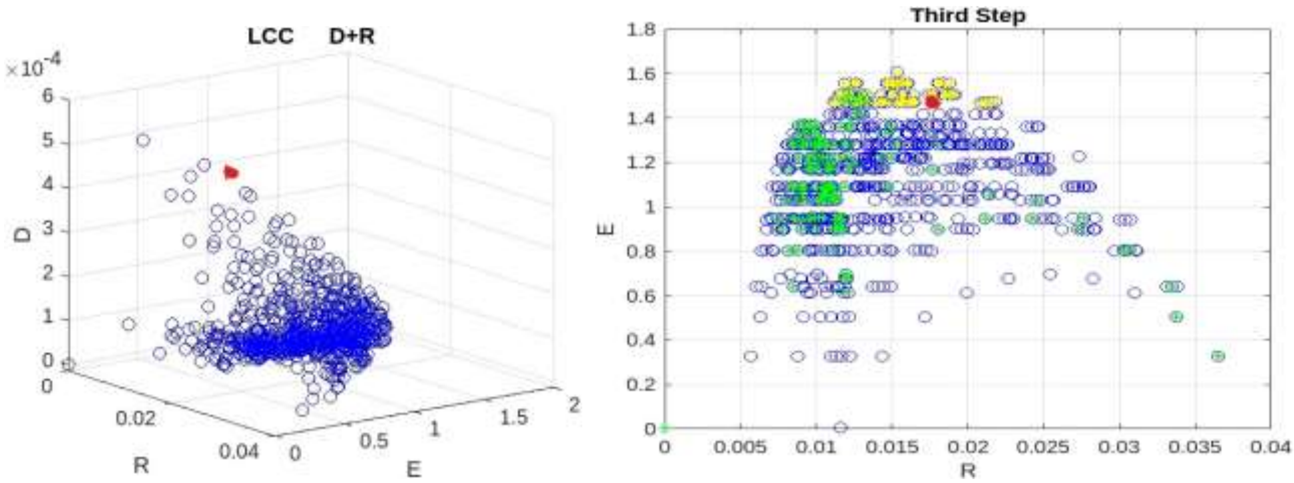

Fig. 3. Experiment 5: Model 2 (concession method): $E > D > R$.
Yellow color – the area of concession in terms of entropy E, green color – the area of concession in terms of risk D

The results of experiment 7 are related to model 3 and are shown in Fig. 4. The optimal solution has the form $X = (0.038; 0.206; 0.063; 0.679; 0.013)$, at which the optimal values of the criteria: $\mathrm{Min}\,D = 0.000028$, $\mathrm{Max}\,R = 0.015$, $\mathrm{Max}\,E = 0.9425$ are achieved.

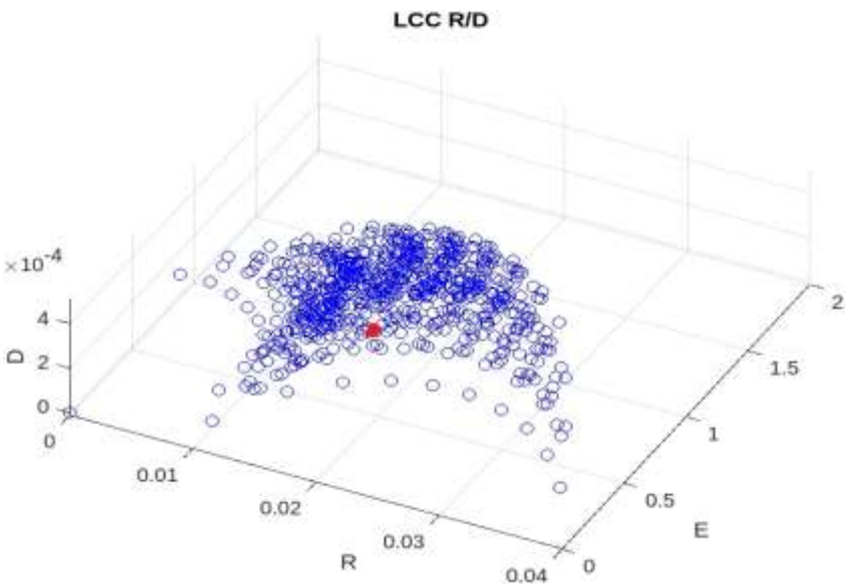

Fig. 4. Experiment 7, model 3 with R/D convolution

The results of the experiments conducted on the same indicators using different software are shown in Table 2.

The analysis of Table 2 shows that the results obtained belong to the Pareto set in all experiments and are non-dominant and non-comparable, except for the non-Pareto result of Experiment 7. A comparison of results 7 and 8 shows that the risk is reduced when the entropy criterion is introduced. The fact that experiment 8 dominates the result of 7 proves the importance of adding entropy. Comparison of the results of experiments 1 and 2 proves the necessity of introducing the profitability criterion. The analysis of the results when applying the method of linear convolution of criteria proves that

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only the application of all three criteria allows to obtain an adequate result (experiments 1–3, 7). Experiments 4 and 8 demonstrated tolerance to the results of the entropy and risk criteria when using the concession method, and in the process of implementing the LCC, the result improves in terms of entropy and deteriorates in terms of risk (experiments 1 and 4, 8). The result of experiment 5 is that in the concession method we set 10% of the possible deviation. The numerical method did not allow us to find this result. The deviation in the second step is 40%. That is, by analogy with the concept of intractability by the convolution method, we can consider this example as intractable by the concession method.

Table 2. *Results of solving the problem of enterprise diversification*

MS Excel				MATLAB			
№exp	**Max E**	**Max R**	**Min D**	**№exp**	**Max E**	**Max R**	**Min D**
1	0,69088	0,015	4,09E–05	1	1,3729	0,015	2,05 E–05
4	0,62907	0,015	2,39E–05	4	1,456153	0,015122	2,50 E–05
8	0,62907	0,015	2,39E–05	8	0,973765	0,015	1,40 E–05
5	0,62907	0,01575	2,63E–05	**5**	**1,469858**	**0,017697**	**3,40 E–05**
6	0,62907	0,02067	4,45E–05	6	1,452399	0,017933	3,30 E–05
2	0,61663	0,02301	5,98E–05	2	0,27729	0,037512	1,30 E–05
3	4,05E–08	0,0393	1,10E–05	3	0,000011	0,0393	1,45 E–05
7	**3,80E–08**	**0,0393**	**1,10E–05**	**7**	**0,942531**	**0,015**	**2,80 E–05**

Different results in *MS Excel* and *Matlab* indicate the peculiarities of the numerical solution and the problem of software selection. Built-in *MS Excel* services and built-in *Matlab* functions apply numerical methods, which produces standard features of numerical solutions, such as the accumulation of numerical method errors and calculation errors.

Discussions and Conclusions

To formalize the problem, five models are proposed, which differ in vector objective functions both in terms of the number and quality of the selected criteria.

Two methods of solving multicriteria problems are considered. The first is the construction of a generalized criterion based on the components of the objective function vector of a multicriteria problem. The second method is a step-by-step solution based on the lexicographic order of the criteria in terms of importance to the decision maker. The results of the experiments allow us to draw general conclusions: the use of the entropy criterion reduces risk; model 2 helps to obtain the highest profitability when solving both criteria and concessions by the linear convolution method; the multiplicative convolution produces a non-Pareto solution.

Thus, using the conclusions of the classical portfolio theory, which have been tested by experience and time, the authors of the article have developed a methodology for the efficient allocation of resources between branches of a retail network, which takes into account the expected profitability and diversification of the distribution and minimizes risks.

The scientific novelty of this work lies in the formalization of diversification models for a wholesale trading network based on portfolio theory and multicriteria optimization methods.

The practical value is that the results were obtained on real indicators for the network and they demonstrated the possibility of using the developed tool for automatic resource allocation in the form of Pareto optimal portfolios to minimize risks. The areas for further research include conducting a number of experiments with different ways of formalizing risk in portfolio models and finding appropriate analytical dependencies.

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References

1. "The Digital Economy: A Look Into the 2022 Digital Frontier, U.S. Chamber of Commerce’s 2nd Annual Global Forum". URL: https://www.uschamber.com/on-demand/technology/digital-economy-the-global-competition-to-write-the-rules (last accessed 22.03.2023).

2. Zanjirdar, M., (2020), Overview of Portfolio Optimization Models. *Advances in mathematical finance and applications*. 5(4). P. 419–435. DOI: 10.22034/amfa.2020.1897346.1407

3. Ghandehari, M., Azar, A., Yazdanian, A., Golarzi, Gh. (2019), "A Hybrid Model of Stochastic Dynamic Programming and Genetic Algorithm for Multistage Portfolio Optimization with Glue VaR Risk Measurement". *Industrial Management Journal*. No. 11 (3). P. 517–542. DOI: 10.22059/IMJ.2019.278912.1007579

4. Kwon, R., Butler, A. (2021), "Covariance Estimation for Risk-Based Portfolio Optimization". *An Integrated Approach. Journal of Risk*. No. 24 (2). P. 11–41. DOI: 10.21314/JOR.2021.020

5. Chaweewanchon, A., Chaysiri, R. (2011), "Markowitz Mean-Variance Portfolio Optimization with Predictive Stock Selection Using Machine Learning", *International Journal of Financial Studies*. No. 10 (3), P. 64–73. DOI: https://doi.org/10.3390/ijfs10030064

6. Lim, Q.Y.E., Cao, Q. Quek, C. (2022), "Dynamic portfolio rebalancing through rein for cement learning". *Neural Computing and Applications*. Vol. 34, P. 7125–7139. DOI:10.1007/s00521-021-06853-3

7. Sharma, M., Shekhawat, H.S. (2022), "Portfolio optimization and return prediction by integrating modified deep belief network and recurrent neural network". *Knowledge-Based Systems*. Vol. 250, P. 1–19. DOI:10.1016/j.knosys.2022.109024

8. Escobar-Anel, M., Kschonnek, M., Zagst, R. (2022), "Portfolio optimization: not necessarily concave utility and constraints on wealth and allocation". *Mathematical Method sof Operations Research*. Vol. 95. P. 101–140. https://doi.org/10.1007/s00186-022-00772-2

9. Grechuk, B., Hao, D. (2022), "Individual and cooperative portfolio optimization as line ar program". *Optimization Letters*. Vol.16. P. 2569–2589. DOI:10.1007/s11590-022-01901-w

10. Mazin, A. M. Al Janabi (2021), "M.A.M.: Multivariate portfolio optimization under illiquid market prospects: a review of theoretical algorithms and practical techniques for liquidity risk management". *Journal of Modellingin Management*. No.16(1). P. 288–309. DOI:10.1108/JM2-07-2019-0178

11. Ahmadi-Javid, A., Fallah-Tafti, M. (2019), "Portfolio optimization with entropic value-at-risk". *European Journal of Operational Research*. No. 279(1). P. 225–241. https://doi.org/10.1016/j.ejor.2019.02.007

12. Markowitz, H. M., Blay, K. "Risk–Return Analysis. The Theory and Practice of Rational Investing (a four-volume series), McGraw-Hill". 2014. 208 p. URL: https://books.google.com.ua/books/about/Risk_Return_Analysis_The_Theory_and_Prac.html?id=_GknVPOReYoC&redir_esc=y

13. Xidonas, P., Steuer, R. Hassapis, C. (2020), "Robust portfolio optimization: a categorized bibliographic review". *Annalsof Operations Research*. Vol. 292. P. 533–552. DOI: 10.1007/s10479-020-03630-8

14. Perepelitsa, V. A., Kozin I. V., Tereshchenko, E. V. (2012), Classification tasks: approaches, methods, algorithms [Zadachi classifikatsii i formirovanie znaniy. – Saarbrucken, Germany] LAP LAMBERT Academic Publishing Gmbh&Co. KG. 196 p.

15. Ehrgott, M. (2005), "Multicriteria Optimization". *Springer, Heidelberg*. Vol. XIII. 323 p. DOI: https://doi.org/10.1007/3-540-27659-9

16. Engau A., Sigler D. (2020), "Pareto solution sin multi criteria optimization underrun certainty". *European Journal of Operational Research*. No.281 (2). P. 357–368. DOI: 10.1016/j.ejor.2019.08.040

17. Zhou W., Zhu W., Chen Y., Chen J. (2022), "Dynamic changes and multi-dimensional evolution of portfolio optimization". *Economic Research-Ekonomska Istraživanja*. Vol. 35(11):1-26. P. 1431–1456. DOI:10.1080/1331677X.2021.1968308

18. Bakurova, A. V., Ropalo, H. M., Tereschenko, E. V. (2021), "Analysis of the Effectiveness of the Successive Concessions Method to Solve the Problem of Diversification". MoMLeT+DS 2021: 3rd International Work shop on Modern Machine Learning Technologies and Data Science. P. 231–242. URL: https://ceur-ws.org/Vol-2917/paper21.pdf

19. Mathworks, "MATLAB for Artificial Intelligence". URL: https://www.mathworks.com/campaigns/products.

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АНАЛІЗ ЗАДАЧІ ВИБОРУ КРИТЕРІЇВ У МОДЕЛЯХ ДИВЕРСИФІКАЦІЇ

Цифровізація економіки знижує вартість ведення бізнесу завдяки автоматизації відповідних процесів, але будь-яка трансформація генерує нові ризики, нестійкість економіки. Економічна нестабільність призводить до падіння рівня життя та, як наслідок, негативно впливає на діяльність торговельних підприємств. Особливо чутливими до будь-яких змін є середній та малий бізнес. Зниження попиту на більшість товарів повсякденного вживання болісно позначається на діяльності торговельних підприємств малого та середнього бізнесу, призводить до появи нових ризиків. Ці ризики істотно впливають на зниження прибутковості підприємств. Тому важливим для кожного підприємства є завдання диверсифікації діяльності, що передбачає розширення товарного асортименту, переорієнтацію ринків збуту та оптимальний розподіл товарів між підрозділами одного підприємства. **Предметом дослідження** статті є багатокритеріальні моделі диверсифікованого портфеля, що мінімізують ризики, які виникають в епоху цифрової економіки в управлінні мережами торговельних підприємств. Для формалізації задачі запропоновано п’ять моделей, що різняться векторними цільовими функціями як за кількістю, так і за якістю обраних критеріїв. **Метою роботи** є аналіз проблеми вибору критеріїв у відповідних багатокритеріальних, або векторних, задачах диверсифікації. У статті досліджуються переваги введення до складу критеріїв класичної двокритеріальної моделі портфельної теорії критерію максимізації ентропії, що визначає ступінь різноманітності складу портфеля. Застосовується комплексне поєднання методів класичної теорії портфеля та багатокритеріальної оптимізації. **Результатами дослідження** є порівняння трьох методів розв’язування таких задач: згортка критеріїв, послідовні поступки та комп’ютерне моделювання множини Парето. **Висновки:** здобуті результати будуть корисними для автоматизації управління ризиками торговельних мереж. Практична цінність роботи полягає в тому, що досягнуті результати на реальних даних для мережі продемонстрували можливість застосування розробленого інструменту для автоматичного розподілу ресурсів у вигляді паретооптимальних портфелів із метою мінімізації ризиків.

Ключові слова: багатокритеріальна задача; задача оптимального портфеля; згортка критеріїв; метод послідовних поступок; множина Парето; ентропія.

Бібліографічні описи / Bibliographic descriptions

Бакурова А. В., Савранська А. В., Терещенко Е. В., Широкорад Д. В., Шевчук М. В. Аналіз задачі вибору критеріїв у моделях диверсифікації. *Сучасний стан наукових досліджень та технологій в промисловості*. 2023. № 4 (26). С. 5–15. DOI: https://doi.org/10.30837/ITSSI.2023.26.005

Bakurova, A., Savranska, A., Tereschenko, E., Shyrokorad, D., Shevchuk, M. (2023), "Analysis of the criteria selection problem in diversification models", *Innovative Technologies and Scientific Solutions for Industries*, No. 4 (26), P. 5–15. DOI: https://doi.org/10.30837/ITSSI.2023.26.005