UDC 539.375

SLIT SYSTEM PARTIAL CLOSURE SIMULATION IN A STRINGER REINFORCED PERFORATED ISOTROPIC MEDIUM

Minavar Mir-Salim-zade
minavar.mirsalimzade@imm.az

Institute of Mathematics and Mechanics of Azerbaijan National Academy of Sciences,
9 F. Agaev St., Baku, AZ1141, Azerbaijan

On the basis of the methods of the theory of elasticity, a mathematical description of the model of partial closure of a system of slits in a perforated isotropic medium with foreign transverse inclusions is given. Such a medium can be considered as a perforated unrestricted plate, reinforced by a system of stringers of a very narrow cross section. It is believed that the medium is weakened by a periodic system of circular holes and rectilinear variable width slits. The variable width of the slits is comparable to elastic deformations. A method of solving the periodic elastic problem and an explicit method of constructing complex potentials corresponding to the unknown normal displacements along rectilinear slits are applied. General representations of solutions are constructed, that describe a class of problems with a periodic distribution of stresses outside circular holes and slits with contact zones. To determine the unknown contact stresses and sizes of contact zones, a singular integral equation is obtained, that reduces to a system of nonlinear algebraic equations. The system of algebraic equations can be solved by the method of successive approximations. As a result, the contact stresses and sizes of contact zones have been found.

Keywords: *perforated plate, stringers, rectilinear variable width slits, contact stresses, contact zones.*

Introduction

The problem of closing the existing crack in a medium is of great interest in fracture theory. It is known [1−3] that stiffening ribs help to slow down the growth of a crack and even achieve its closure. Reducing deformation in the direction perpendicular to the crack, stiffening ribs reduce the stress intensity factor in the vicinity of the crack end. As a result, a zone of compressive stresses may appear, sufficient for crack faces to come into contact. Problems of deformation of infinite plates reinforced by a regular system of ribs, whose cross-sections are very narrow rectangles, is dealt with in extensive literature [4–9]. Considerable attention is paid to the investigation of plate destruction, strengthened by a regular stringer system [10–15]. In the papers mentioned, the Griffiths fracture (model) is considered, i.e. a crack with non-interacting edges. At that, it is found that the stress intensity factors under the combined action of the tensile stress and reinforcement elements can have a negative value. This means the emergence of compressive stress zones in the vicinity of crack vertices, in which (in some areas) the crack faces come into contact, which leads to the appearance of contact stresses.

The problems of the partial contacting of slit faces in a reinforced plate have by now been little studied. The contacting of crack faces, taking account their width variability, was considered in [16–25]. The main task of this paper is to construct a mathematical model of a partial closure of variable width slits in a perforated isotropic plate reinforced by stiffening ribs.

Formulation of the problem

We consider an elastic isotropic medium with a system of foreign transverse rectilinear inclusions and circular holes. Such a medium can be considered as an infinite perforated unrestricted plate, reinforced by a system of stringers of very narrow cross-sections.

It is considered that the stringers are riveted to the plate at discrete points at a fixed pitch along the entire length of the stringer, symmetrically relative to the plate surface (Fig. 1). The material of the stiffening ribs is assumed to be elastic. At infinity, the reinforced plate is subject to uniform stretching along the stringers by the stress $\sigma_y^\infty = \sigma_0$. The contours of the circular holes are free from external forces. From the contours of the holes, variable width symmetrical rectilinear slits emanate. It is assumed that slit widths are comparable with elastic deformations.

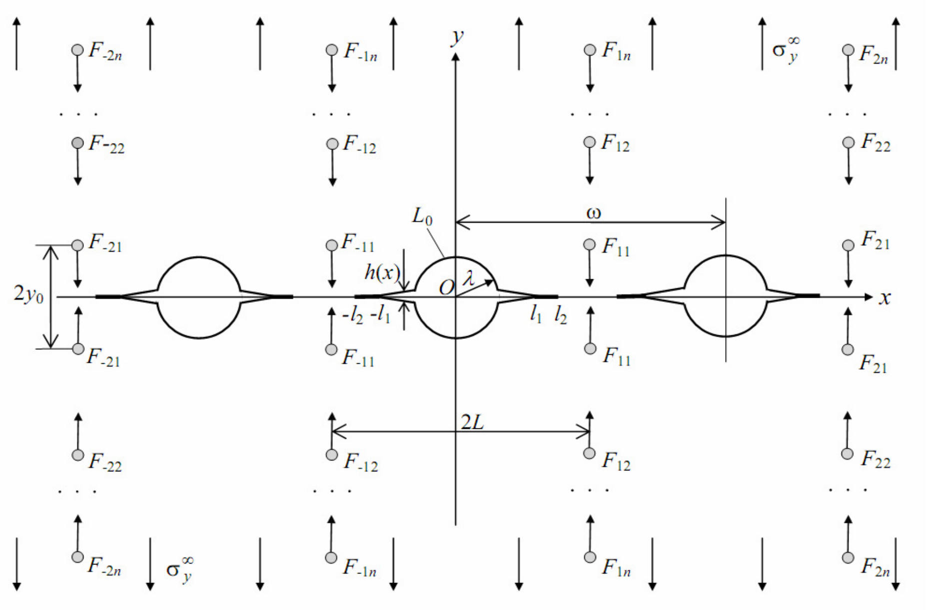

Fig. 1. Calculation scheme of the problem

With regard to stringers, the one-dimensional continuum hypothesis is accepted, i.e. the thickness of a stringer under deformation is considered unchanged, and its stress state is considered uniaxial. Stringers are not subject to bending and only work in tension. The following assumptions are accepted: a) in a medium (plate), a plane stress state is realized; b) neither the stringer truss reinforcing system nor the weakening of stringers due to the mounting of attachment points is taken into account; c) the plate and stringers interact with each other in the same plane and only at the attachment points; d) all attachment points are identical and have the radius a_0 (clutch platform) which is small in comparison with their pitch and other characteristic dimensions. The action of the attachment points is replaced by the action of the equivalent unknown concentrated forces applied at the points corresponding to the centers of the attachment points.

Let there be a reinforced isotropic medium with a periodic system of circular holes with a radius λ (λ<1) and centers:

$$P_m = m\omega \quad (m = \pm1, \pm2, \ldots), \quad \omega = 2.$$

Under the action of the external tensile load σ_0 and unknown concentrated forces F_{mn} ($m, n = \pm1, \pm2, \ldots$), the slit faces will come into contact in some areas of the compressive stress region, where contact stresses will arise. Outside these areas, the slit faces will be free of loads. The contact area between the slit faces is unknown in advance, but it is obvious that it will always start from the end points of the slits in the compressive stress region. It is believed that the unknown size of the contact areas is comparable with the slit lengths. Thus, the problem set is an elasticity theory problem with an unknown boundary, which must be determined in the course of the solution.

The problem under consideration consists in developing a mathematical model that makes it possible to determine the contact areas, contact stresses in the contact areas, magnitude of concentrated forces, and the medium stress-strain state outside circular holes and slits.

The boundary conditions on slit faces have the form

$$\sigma_y = 0, \ \tau_{xy} = 0 \ \text{ in } L', \ \sigma_y = p(x), \ \tau_{xy} = 0, \ \upsilon^+(x,0) - \upsilon^-(x,0) = -h(x) \ \text{ in } L'', \tag{1}$$

on the contours of circular holes

$$\sigma_r - i\tau_{r\theta} = 0 .$$

Here, L' is the set of load free slit zones; L'' is the set of contact edge zones; $\upsilon^+(x,0) - \upsilon^-(x,0)$ is the opening of slit faces; $h(x)$ is the width of slits; σ_x, σ_y, τ_{xy} are the stress tensor components; u, υ are the vector displacement components along the x, y axes, respectively; $i^2 = -1$.

Because of the symmetry of the boundary conditions and geometry of the domain D occupied by the medium, the stresses are periodic functions with the fundamental period ω. Based on the Kolosov-Muskhelishvili formulas [26] and the boundary conditions on the contours of circular holes and slit faces, the problem reduces to the determination (in the D domain) of two analytic functions $\Phi(z)$ and $\Psi(z)$ from the conditions

$$\Phi(\tau)+\overline{\Phi(\tau)}-\left[\bar{\tau}\Phi'(\tau)+\Psi(\tau)\right]e^{2i\theta}=0\,,\qquad \Phi(x)+\overline{\Phi(x)}+x\overline{\Phi'(x)}+\overline{\Psi(x)}=f\,, \tag{2}$$

where $\tau=\lambda e^{i\theta}+m\omega$ ($m=\pm1,\pm2,\ldots$); x is the affix of the points of slit faces; f=0 in L' and f=$p(x)$ in L''.

Solution to the boundary value problem

We seek the solution to the boundary value problem (1) –2) in the form

$$\Phi(z) = \Phi_0(z) + \Phi_1(z) + \Phi_2(z),\ \Psi(z) = \Psi_0(z) + \Psi_1(z) + \Psi_2(z). \tag{3}$$

Here, the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ determine the stress and strain field in a solid reinforced plate under the action of the tensile stress σ_0 and the concentrated forces F_{mn}, and they can be determined by the following formulas:

$$\Phi_0(z)=\frac{1}{4}\sigma_0-\frac{i}{2\pi h_*(1+\kappa)}\sum_{m,n}{}'F_{mn}\left(\frac{1}{C_1}-\frac{1}{C_2}\right), \tag{4}$$

$$\Psi_0(z)=\frac{1}{2}\sigma_0-\frac{i\kappa}{2\pi h_*(1+\kappa)}\sum_{m,n}{}'F_{mn}\left(\frac{1}{C_1}-\frac{1}{C_2}\right)+\frac{i}{2\pi h_*(1+\kappa)}\sum_{m,n}{}'F_{mn}\left(\frac{\overline{C}_3}{C_2^2}-\frac{C_3}{C_1^2}\right).$$

where h_* is the plate thickness; $\kappa=(3-\nu)/(1+\nu)$ is the plate material Poisson's ratio; $C_1=z-mL+iny_0$; $C_2=z-mL-iny_0$; $C_3=mL+iny_0$. The summation symbol with a prime indicates that the summation excludes the index m=n=0.

The functions $\Phi_1(z)$ and $\Psi_1(z)$, corresponding to the unknown normal displacements along the slit, are sought in the explicit form

$$\Phi_1(z)=\frac{1}{2\omega}\int_{L_1}g(t)\operatorname{ctg}\frac{\pi}{\omega}(t-z)dt\,, \tag{5}$$

$$\Psi_1(z)=-\frac{\pi z}{2\omega^2}\int_{L_1}g(t)\sin^{-2}\frac{\pi}{\omega}(t-z)dt\,,\qquad L_1=[-l_2,-\lambda]+[\lambda,l_2].$$

The required function $g(x)$ describes the derivative of the opening of slit faces

$$g(x)=\frac{2\mu}{1+\kappa}\frac{\partial}{\partial x}\left[\upsilon^+(x,0)-\upsilon^-(x,0)\right],$$

where μ is the shear modulus of the reinforced plate material.

To find the complex potentials $\Phi_2(z)$ and $\Psi_2(z)$, we represent the first of the boundary conditions (2)

$$\Phi_2(\tau)+\overline{\Phi_2(\tau)}-\left[\bar{\tau}\Phi_2'(\tau)+\Psi_2(\tau)\right]e^{2i\theta}=f_1(\theta)+if_2(\theta)+\varphi_1(\theta)+i\varphi_2(\theta)\,, \tag{6}$$

where $f_1(\theta)+if_2(\theta)=-\Phi_0(\tau)-\overline{\Phi_0(\tau)}+\left[\bar{\tau}\Phi_0'(\tau)+\Psi_0(\tau)\right]e^{2i\theta}$,

$\varphi_1(\theta)+i\varphi_2(\theta)=-\Phi_1(\tau)-\overline{\Phi_1(\tau)}+\left[\bar{\tau}\Phi_1'(\tau)+\Psi_1(\tau)\right]e^{2i\theta}$.

To solve the boundary value problem (6), we seek the complex potentials $\Phi_2(z)$ and $\Psi_2(z)$ in the form

$$\Phi_2(z)=\alpha_0+\sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}\rho^{(2k)}(z)}{(2k+1)!}\,, \tag{7}$$

$$\Psi_2(z)=\sum_{k=0}^{\infty}\beta_{2k+2}\frac{\lambda^{2k+2}\rho^{(2k)}(z)}{(2k+1)!}-\sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}S^{(2k+1)}(z)}{(2k+1)!}\,.$$

Here $\rho(z)=\left(\frac{\pi}{\omega}\right)^2\sin^{-2}\left(\frac{\pi}{\omega}z\right)-\frac{1}{3}\left(\frac{\pi}{\omega}\right)^2$, $\quad S(z)=\sum_{m,n}{}'\left[\frac{P_m}{(z-P_m)^2}-\frac{2z}{P_m^2}-\frac{1}{P_m}\right]$.

From the symmetry conditions with respect to the coordinate axes, we find that

$$\operatorname{Im} \alpha_{2k+2} = 0, \qquad \operatorname{Im} \beta_{2k+2} = 0 \qquad k = 0, 1, 2, \ldots .$$

The relations (3) – (5), (7) define a class of symmetric problems with periodic stress distribution. From the condition for the constancy of the principal vector of all the forces acting on the arc joining two congruent points in the domain D, it follows that

$$\alpha_0 = \frac{\pi^2}{24}\beta_2\lambda^2 .$$

The unknown coefficients α_{2k}, β_{2k} must be determined from the boundary condition (6). With respect to the functions $f_1(\theta) + if_2(\theta)$ и $\varphi_1(\theta) + i\varphi_2(\theta)$, we assume that on the contour $|\tau|=\lambda$ they expand into Fourier series, which in view of the symmetry of the problem have the form

$$f_1(\theta) + if_2(\theta) = \sum_{k=-\infty}^{\infty} A_{2k} e^{2ik\theta}, \qquad \operatorname{Im} A_{2k} = 0, \tag{8}$$

$$A_{2k} = \frac{1}{2\pi}\int_0^{2\pi}\left[f_1(\theta) + if_2(\theta)\right]e^{-2ik\theta}d\theta \qquad (k = 0, \pm 1, \pm 2, \ldots),$$

$$\varphi_1(\theta) + i\varphi_2(\theta) = \sum_{k=-\infty}^{\infty} B_{2k} e^{2ik\theta}, \qquad \operatorname{Im} B_{2k} = 0, \tag{9}$$

$$B_{2k} = \frac{1}{2\pi}\int_0^{2\pi}\left[\varphi_1(\theta) + i\varphi_2(\theta)\right]e^{-2ik\theta}d\theta \qquad (k = 0, \pm 1, \pm 2, \ldots).$$

The unknown function $g\,(x)$ and the coefficients α_{2k}, β_{2k} of the functions $\Phi_2(z)$, $\Psi_2(z)$ must be determined from the boundary conditions (2) and (6). Since the problem is periodic, the boundary conditions (6) degenerate into one functional equation, for example, on the contour $\tau=\lambda e^{i\theta}$, and the system of conditions (2) – to the boundary condition in the basic period. To form the equations for the coefficients α_{2k}, β_{2k}, we expand the functions $\Phi_2(z)$, $\Psi_2(z)$ into Laurent series in the neighborhood of the point z=0. W substitute the functions $\Phi_2(z)$, $\Psi_2(z)$ in the left-hand part of the boundary condition (6) on the contour $z=\lambda\exp(i\theta)$ by their expansions into the Laurent series in the neighborhood of the zero point, and the functions $f_1(\theta)+if_2(\theta)$ and $\varphi_1(\theta)+i\varphi_2(\theta)$ in the right-hand part of said condition – by the Fourier series (8) and (9), respectively. Comparing the coefficients of the same powers of $\exp(i\theta)$, we obtain two infinite systems of algebraic equations with respect to the coefficients α_{2k}, β_{2k}. After some transformations, we arrive at an infinite system of linear algebraic equations with respect to α_{2k+2}

$$\alpha_{2j+2} = \sum_{k=0}^{\infty} A_{j,k}\alpha_{2k+2} + b_j \qquad (j = 0, 1, 2, \ldots), \tag{10}$$

$$b_0 = M_2 - \sum_{k=0}^{\infty}\frac{g_{k+2}\lambda^{2k+4}}{2^{2k+4}}M_{-2k-2},$$

$$b_j = M_{2j+2} - \frac{(2j+1)M_0 g_{j+1}\lambda^{2j+2}}{K_1 2^{2j+2}} - \sum_{k=0}^{\infty}\frac{(2j+2k+3)\,g_{j+k+2}\lambda^{2j+2k+4}}{(2j)!(2k+3)!2^{2j+2k+4}}M_{-2k-2},$$

$$A_{j,k} = (2j+1)\gamma_{j,k}\lambda^{2j+2k+2}, \qquad g_j = 2\sum_{m=1}^{\infty}\frac{1}{m^{2j}},$$

$$\gamma_{0,0} = \frac{3}{8}g_2\lambda^2 + \sum_{i=1}^{\infty}\frac{(2i+1)g_{i+1}^2\lambda^{4i+2}}{2^{4i+4}},$$

$$\gamma_{j,k} = -\frac{(2j+2k+2)!\,g_{k+j+1}}{(2j+1)!(2k+1)!2^{2j+2k+2}} + \frac{(2j+2k+4)!\,g_{j+k+2}\lambda^2}{(2j+2)!(2k+2)!2^{2j+2k+4}} +$$

$$+\sum_{i=0}^{\infty}\frac{(2j+2i+1)!(2k+2i+1)!\,g_{j+i+1}g_{k+i+1}\lambda^{4i+2}}{(2j+1)!(2k+1)!(2i+1)!(2i)!2^{2j+2k+4i+4}} + b_{j,k}\,,$$

$$b_{0,k}=0\,,\qquad b_{j,0}=0\,,\qquad b_{j,k}=\frac{g_{j+1}g_{k+1}\lambda^2}{2^{2j+2k+4}}\left(1-\frac{\pi^2\lambda^2}{12}\right)^{-1}\qquad (j,\,k=1,\,2,\ldots),$$

$$M_{2k} = A_{2k} + B_{2k}\,.$$

The constants β_{2k+2} are determined from the following relations:

$$\beta_2 = \left[1-\frac{\pi^2\lambda^2}{12}\right]^{-1}\left[-M_0 + 2\sum_{k=0}^{\infty}\frac{g_{k+1}\lambda^{2k+2}}{2^{2k+2}}\alpha_{2k+2}\right], \tag{11}$$

$$\beta_{2j+4} = (2j+3)\alpha_{2j+2} + \sum_{k=0}^{\infty}\frac{(2j+2k+3)!\,g_{j+k+2}\lambda^{2j+2k+4}}{(2j+2)!(2k+1)!2^{2j+2k+4}}\alpha_{2k+2} - M_{-2j-2}\,.$$

Requiring that the functions (3) satisfy the boundary condition (1), after some transformations, we obtain a singular integral equation with respect to the function $g(x)$

$$\frac{1}{\omega}\int_{L_1} g(t)\,\mathrm{ctg}\frac{\pi}{\omega}(t-x)\,dt + H(x) = f(x)\,. \tag{12}$$

Here $H(x) = \Phi_0(x) + \Phi_2(x) + \overline{\Phi_0(x)} + \overline{\Phi_2(x)} + x\Phi_0'(x) + x\Phi_2'(x) + \Psi_0(x) + \Psi_2(x)$.

The singular integral equation (12), as well as the algebraic systems (10), (11) contain the unknown values of the concentrated forces F_{mn}(m=1,2,…; n=1,2,…). To determine them, we use Hooke's law and the method of pasting together two asymptotic forms of the desired solution. According to Hooke's law, the magnitude of the concentrated force F_{mn}, acting on each attachment point from the side of the stringer

$$F_{mn} = \frac{E_S A_S}{2y_0 n}\Delta\upsilon_{m,n}\qquad (m=1,2,\ldots;\ n=1,2,\ldots),$$

where E_S is the Young's modulus of the stringer material; A_S is the cross-sectional area of the stringer; $2y_0n$ is the distance between the attachment points; $\Delta\upsilon_{m,n}$ is the mutual displacement of the considered attachment points, equal to the elongation of the corresponding stringer section.

We assume that the mutual elastic displacement of the points $z=mL+i(ny_0-a_0)$ and $z=mL-i(ny_0-a_0)$ is equal to the mutual displacement of the attachment points $\Delta\upsilon_{m,n}$. This additional displacement compatibility condition makes it possible to find an effective solution to the problem set. Using the complex potentials (3) – (5), (7) and the Kolosov-Muskhelishvili formula for displacements [26], after performing the elementary calculations, we find the mutual displacement $\Delta\upsilon_{m,n}$ in the following form:

$$\Delta\upsilon_{p,r} = \Delta\upsilon_{p,r}^{(0)} + \Delta\upsilon_{p,r}^{(1)} + \Delta\upsilon_{p,r}^{(2)}\,. \tag{13}$$

In view of some cumbersomeness, the values $\Delta\upsilon_{p,r}^{(0)}$, $\Delta\upsilon_{p,r}^{(1)}$, and $\Delta\upsilon_{p,r}^{(2)}$ are not given.

The required value of the force F_{mn} is determined using the formulas (13) from the infinite system

$$F_{pr} = \frac{E_S A_S}{2y_0 n}\Delta\upsilon_{p,r}\qquad (p = 1,2,\ldots;\ r = 1,2,\ldots), \tag{14}$$

degenerating into one infinite algebraic system because of the periodicity of the problem.

The resulting equation (14), the algebraic systems (10), (11), and the singular integral equation (12) are connected and they must be solved jointly. Solving them together on condition that there is no opening of the slit faces in the contact edge zone and taking into account the condition of the limitedness of contact stresses, we find the required function $p(x)$, the values F_{mn}, and the contact zone of the slit faces.

Numerical solution and its analysis

Using the expansion $\frac{\pi}{\omega}\mathrm{ctg}\frac{\pi}{\omega}z=\frac{1}{z}-\sum_{j=0}^{\infty}g_{j+1}\frac{z^{2j+1}}{\omega^{2j+2}}$, we bring equation (12) to the usual form

$$\frac{1}{\pi}\int_{L_1}\frac{g(t)}{t-x}dt+\frac{1}{\pi}\int_{L_1}g(t)K(t-x)dt+H(x)=f(x),\quad K(t)=-\sum_{j=0}^{\infty}g_{j+1}\frac{t^{2j+1}}{\omega^{2j+2}}. \tag{15}$$

Taking into account that $g(x)=-g(-x)$ and changing the variables, we bring the equation (15) to the standard form

$$\frac{1}{\pi}\int_{-1}^{1}\frac{g_*(\tau)}{\tau-\eta}d\tau+\frac{1}{\pi}\int_{-1}^{1}g_*(\tau)B(\eta,\tau)\,d\tau+H_*(\eta)=f_*(\eta), \tag{16}$$

$$g_*(\tau)=g(\xi),\qquad H_*(\eta)=H(\xi_0),\qquad f_*(\eta)=f(\xi_0),$$

$$B(\eta,\tau)=-\frac{1-\lambda_1^2}{2}\sum_{j=0}^{\infty}g_{j+1}\left(\frac{l_2}{2}\right)^{2j+2}\cdot u_0^jA_j^*,$$

$$A_j^*=(2j+1)+\frac{(2j+1)(2j)(2j-1)}{1\cdot2\cdot3}\left(\frac{u}{u_0}\right)+\ldots+\frac{(2j+1)(2j)(2j-1)\ldots[(2j+1)-(2j+1-1)]}{1\cdot2\cdot3\ldots(2j+1)}\left(\frac{u}{u_0}\right)^j.$$

To construct the solution to the singular integral equation (16), the method of direct solution of such equations is used [27, 28]. The singular integral equation (16), in addition to the singularity in the Cauchy kernel, has a fixed singularity at the exit point of the slit to the surface of a circular hole. At such points the function $g\,(x)$ has the singularity $x=\pm\lambda$ that differs from the root one. The character of this singularity can be established from the analysis of the integral equation (16) [29].

The integral $\int_{\lambda}^{l}g(t)dt$, unlike the case of an internal slit, is equal to a non-zero constant, which is expressed through the opening of the slip on the surface of a circular hole and which must be determined after solving the singular integral equation (16).

Because of the awkwardness of the expressions for the functions included into the singular integral equation, it is difficult to establish the true singularity of the function $g_*(\eta)$ at the end (16). Therefore, for its numerical solution, a simplified numerical method is used [27, 28, 30]. We represent the solution in the form

$$g_*(\eta)=g_0(\eta)\sqrt{1-\eta^2},$$

where $g_0(\eta)$ is an unknown regular function.

Using quadrature formulas, the equation (16) can be reduced to the system of M+1 algebraic equations

$$\sum_{m=1}^{M}\frac{g_0(\tau_m)}{M+1}\sin^2\frac{\pi m}{M+1}\left[\frac{1}{\tau_m-\eta_r}+B(\tau_m,\eta_r)\right]=\pi[f_*(\eta_r)-H_*(\eta_r)]. \tag{17}$$

Here, $\tau_m=\cos\frac{\pi m}{M+1}$ ($m=1,2,\ldots,M$), $\eta_r=\cos\frac{2r-1}{2(M+1)}\pi$ ($r=1,2,\ldots,M+1$).

The obtained algebraic system (17) satisfies the additional condition under which there exists a solution in the class of everywhere bounded functions [29]. The right-hand sides of the system (17) include the unknown values of the contact stresses $f_*(\eta_m)$ at the node points belonging to the contact edge zone.

The condition that determines the unknown contact stresses arising on the slit faces in the contact edge zones is the absence of slit opening in these zones (the second condition on L''). In the problem under consideration, this additional condition can be more conveniently written for the derivative of the slit face opening displacements

$$g(x)=\frac{2\mu}{1+\kappa}\frac{\partial}{\partial x}\left[\upsilon^{+}(x,0)-\upsilon^{-}(x,0)\right]=-\frac{2\mu}{1+\kappa}h'(x)\,, \tag{18}$$

where x is the affix of the slit contact edge zone face points (l_1,l_2).

Requiring that the conditions (18) be fulfilled at the node points contained in the contact edge zone (l_1, l_2), we obtain the missing equations for determining the approximate values of the contact stresses $p(t_{m_1})$ at the nodal points.

$$g(t_{m_1})=-\frac{2\mu}{1+\kappa}h'(t_{m_1}) \qquad m_1 = 1, 2, \ldots, M_1. \tag{19}$$

Due to the unknown size of the contact edge zone, the combined algebraic system consisting of (10), (11), (14), (17), (19) is nonlinear. The obtained systems of equations with respect to α_{2k}, β_{2k}, g_k^0, F_{mn}, $p(t_{m_1})$,and l_2 make it possible, at a given external tensile load, to find the stress-strain state of the perforated stringer plate in the presence of slits with partially contacting faces, contact stresses, as well as the size of the contact edge zone. The algebraic systems (10), (11), (14), (17), (19) were solved by the method of successive approximations in the following way. The system was solved from the equations (10), (11), (14), (19) and the M equations of the system (17) with respect to the unknowns α_{2k}, β_{2k}, g_1^0, $g_2^0,\ldots,$ g_M^0, $p_1, p_2,\ldots,$ p_{M_1} and $N_1{\times}N_2$ and unknown concentrated forces $N_1{\times}N_2$ at some value l_{2*}. Further, the found values l_{2*} were substituted into the unused equation of the combined system, i.e. into the M+1 equation of the system (17). Since the chosen parameter value l_{2*}, the corresponding values α_{2k}, β_{2k}, g_1^0, $g_2^0,\ldots,$ g_M^0, $p_1, p_2,\ldots,$ p_{M_1} и $N_1{\times}N_2$ the values of concentrated forces will not generally satisfy this equation, then, selecting new parameter l_{2*} values, the calculations are repeated until this equation is not satisfied with a given accuracy.

Calculations were performed depending on the geometric parameters of the problem at ν=0,3; $\varepsilon_1=a_0/L$=0,01; $\varepsilon=y_0/L$=0,25; E=7.1·10^4 MPa (V95/B95 alloy); E_S =11,5·10^4 MPa (Al-steel composite), A_S/y_0h=1. The number of stringers and attachment points was assumed to be finite: 6, 10, 14. A parametric analysis of the contact stress p(x) dependence on the size of the slits and other geometric parameters of the problem was performed. The results of calculating the contact stresses p/σ_0 for different values of the slip lengths along the edge zone are shown in figure 2. Curve 1 corresponds to the value of the hole radius λ=0.3; curve 2–λ=0.5. In the calculations, the dimensionless coordinates of x' were used:

$$x=\frac{l_2+l_1}{2}+\frac{l_2-l_1}{2}x'\,.$$

The greatest values of contact stresses are in the middle part of the contact zone, where the slit faces close.

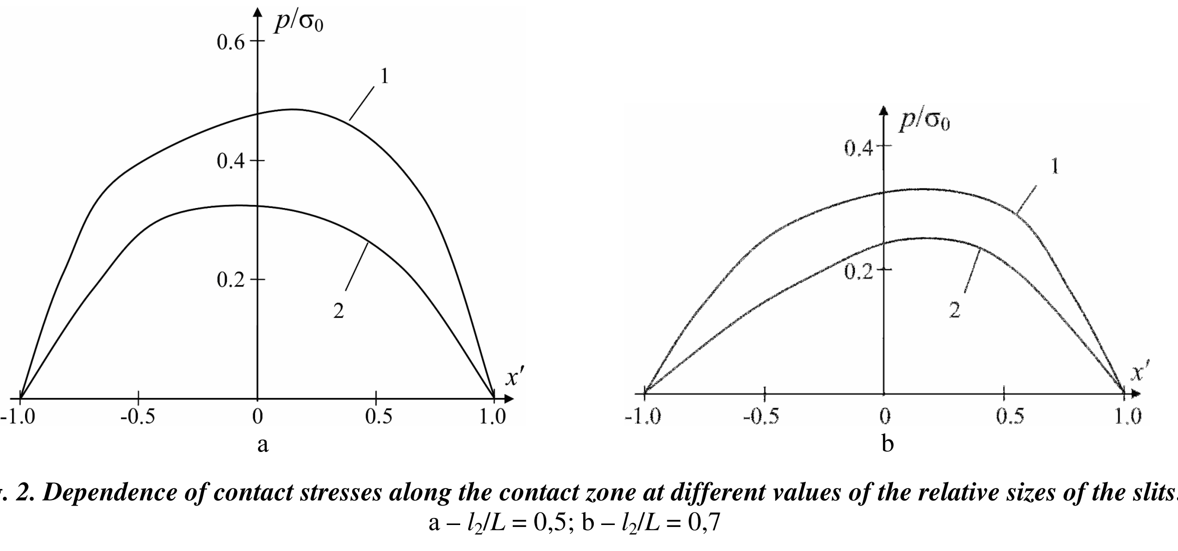

Fig. 2. Dependence of contact stresses along the contact zone at different values of the relative sizes of the slits: a – l_2/L = 0,5; b – l_2/L = 0,7

Conclusions

The analysis of the partial closure model of the varied width slit in a perforated isotropic plate reinforced by stringers is reduced to a parametric joint study of the algebraic systems (10), (11), (14), (17), (19) at various geometric and physical parameters of the plate. The relations obtained make it possible to solve the inverse problem, i.e. determine the characteristics of the perforated plate reinforcement and its stress state, at which a given contact region of the slit faces is reached.

References

1. Finkel, V. M. (1977). *Fizicheskiye osnovy tormozheniya razrusheniya* [Physical principles of inhibition of fracture]. Moscow: Metallurgiya, 360 p. [in Russian].
2. Parton, V. Z., & Morozov, Ye. M. (1985*). Mekhanika uprugo-plasticheskogo razrusheniya* [Mechanics of elastic-plastic fracture]. Moscow: Nauka, 504 p. [in Russian].
3. Mirsalimov, V. M. (1986*). Nekotoryye zadachi konstruktsionnogo tormozheniya treshchin* [Some problems of structural inhibition of cracks]. *Fiziko-khim. Mekhanika Mterialov – Materials Science,* vol. 22, no. 1, pp. 94–98.[in Russian].
4. Tolkachev, V. M. (1964). *Peredacha nagruzki ot stringera konechnoy dliny k beskonechnoy i polubeskonechnoy plastine* [Transfer of a load from a stringer of finite length to an infinite and semi-infinite plate]. *Dokl. AN SSSR,* vol. 154, no. 4, pp. 86–88 [in Russian].
5. Dolgikh, V. N., & Filshtinskiy, L. A. (1976). *Ob odnoy modeli regulyarnoy kusochno-odnorodnoy sredy* [On a model of a regular piecewise homogeneous medium]. *Izv. AN SSSR. Mekhanika Tverdogo Tela,* no. 2, pp. 158–164 [in Russian].
6. Vanin, G. A. (1985). *Mikromekhanika kompozitsionnykh materialov* [Micromechanics of Composite Materials]. Kyiv: Nauk. dumka, 302 p. [in Russian].
7. Broyek, D. (1980). *Osnovy mekhaniki razrusheniya* [Fundamentals of fracture mechanics]. Moscow: Vyssha shkola, 368 pp. [in Russian].
8. Cherepanov, G. P. (1983). *Mekhanika razrusheniya kompozitsionnykh materialov* [Mechanics of destruction of composite materials]. Moscow: Nauka, 296 p. [in Russian].
9. Maksimenko, V. N. (1988). *Vliyaniye priklepannykh reber zhestkosti na razvitiye treshchin vozle otverstiya* [The effect of riveted stiffeners on the development of cracks near the hole]. *Prikl. Mekhanika i Tekhn. Fizika – Journal of Applied Mechanics and Technical Physics,* vol. 29, no. 2, pp. 133–140 [in Russian].
10. Savruk, M. P., & Kravets, V. S. (1994). Reinforcement of a thin cracked plate by a system of parallel stringers. *Materials Sci.,* vol. 30, iss. 1, pp. 95–104.
11. Savruk, M. P., & Kravets, V. S. (1995). Two-dimensional problems of the theory of elasticity for reinforced cracked plates. *Materials Sci*., vol. 31, iss. 3, pp. 350–362.
12. Savruk, M. P, & Kravets, V. S. (1999). Effect of breaks in riveted stringers on the elastic and limiting equilibrium of a cracked plate. *Materials Sci*., vol. 35, iss. 3, pp. 339–348.
13. Mir-Salim-zade, M. V. (2005). *Treshchina so svyazyami mezhdu beregami v izotropnoy srede, usilennoy regulyarnoy sistemoy stringerov* [Fissure with connections between the banks in an isotropic medium, reinforced by a regular stringers system]. *Mekhanika Kompozit. Materialov – Mechanics of Composite Materials,* vol. 41, no. 6, pp. 773–782 [in Russian].
14. Mir-Salim-zade, M. V. (2007). *Razrusheniye izotropnoy sredy, usilennoy regulyarnoy sistemoy stringerov* [Destruction of an isotropic medium reinforced by a regular stringer system]. *Mekhanika Kompozit. Materialov – Mechanics of Composite Materials*, vol. 51, no.1, pp. 59–72 [in Russian].
15. Mirsalimov, V. M., & Mustafayev, A. B. (2014). *Tochnoye resheniye kontaktnoy zadachi o chastichnom vzaimodeystvii beregov shcheli peremennoy shiriny pri deystvii temperaturnogo polya* [Exact solution of the contact problem on the partial interaction of variable width slit faces under the action of a temperature field]. *Problemy Mashinostroyeniya –Journal of Mechanical Engineering*, vol. 17, no. 3, pp. 33–37 [in Russian].
16. Mustafayev, A. B. (2014). *Vzaimodeystviye beregov shcheli peremennoy shiriny pri izgibe polosy (balki) pod vozdeystviyem temperaturnogo polya* [Interaction of variable width slit faces with the bending of a strip (beam) under the influence of a temperature field]. *Mekhanika Mashin, Mekhanizmov i Materialov – Mechanics of Machines, mechanisms and Materials*, no. 3, pp. 30–36 [in Russian].
17. Mirsalimov, V. M., & Mustafayev, A. B. (2015). *Resheniye zadachi o chastichnom kontaktirovanii beregov shcheli peremennoy shiriny pod deystviyem temperaturnogo polya* [Solution of the problem of partial contact of variable width slit faces under the action of a temperature field]. *Fiziko-khim. Mekhanika Materialov – Physico-Chemical Mechanics of Materials*, vol. 51, no.1, pp. 86–92 [in Russian].
18. Mirsalimov, V. M., & Mustafayev, A. B. (2015). A contact problem on partial interaction of faces of a variable thickness slot under the influence of temperature field. *Mechanika*, vol. 21, iss. 1, pp. 19–22.

19. Mirsalimov, V. M. (2016). Simulation of partial closure of a variable width slot with interfacial bonds in end zones in an isotropic medium. *Int. J. Damage Mech.*, vol. 25, iss. 2, pp. 266–279.
20. Mir-Salim-zade, M. V. (2016). *Zakrytiye shcheli, iskhodyashchey iz kontura krugovogo otverstiya v stringernoy plastine* [Closure of a slit originating from the contour of a circular hole in a stringer plate]. *Vestn. Chuvash. ped. un-ta im. I .Ya. Yakovleva. Ser. Mekhanika predelnogo sostoyaniya – I. Yakovlev Chuvash State Pedugogical University Bulletin. Series: Mechanics of a Limit State*, no. 1(27). , pp. 78–89 [in Russian].
21. Mir-Salim-zade, M. V. (2016). *Chastichnyy kontakt beregov shcheli peremennoy shiriny v podkreplennoy stringerami plastine* [Partial contact of variable width slit faces in a plate reinforced by stringer]. *Fiziko-khim. Mekhanika Materialov – Physico-Chemical Mechanics of Materials*, vol. 52, no. 3, pp. 29–34 [in Russian].
22. Gasanov, Sh. G. (2017). *Resheniye kontaktnoy zadachi dlya ploskosti, oslablennoy shchelyu peremennoy shiriny, v neodnorodnom napryazhennom pole* [Solution of the contact problem for a plane weakened by variable width slip in an inhomogeneous stressed field]. *Problemy Mashinostroyeniya – Journal of Mechanical Engineering*, vol. 20, no. 2, pp. 29–36 [in Russian].
23. Mir-Salim-zade, M. V. (2017). Contact problem for a stringer plate weakened by a periodic system of variable width slots. *Structural Eng. and Mech*, vol. 62, no 6. pp. 719–724.
24. Mustafayev, A. B. (2017). *Zamedleniye rosta shcheli peremennoy shiriny pod deystviyem temperaturnogo polya* [Slowing growth of a variable width slit under the action of a temperature field]. *Prikl. Mekhanika i Tekhn. Fizika – Journal of Applied Mechanics and Technical Physics*, vol. 58, no. 1, pp. 168–176 [in Russian].
25. Muskhelishvili, N. I. (1966). *Nekotoryye osnovnyye zadachi matematicheskoy teorii uprugosti* [Some basic problems of the mathematical theory of elasticity]. Moscow: Nauka, 707 p. [in Russian].
26. Panasyuk, V. V., Savruk, M. P., & Datsyshin, A. P. (1976). *Raspredeleniye napryazheniy okolo treshchin v plastinakh i obolochkakh* [Distribution of stresses near cracks in plates and shells]. Kyiv: Nauk. dumka, 444 p. [in Russian].
27. Ladopoulos, E. G. (2000). Singular Integral Equations, Linear and Non-Linear Theory and its Applications in Science and Engineering. Berlin: Springer Verlag, 552 p.
28. Muskhelishvili, N. I. (1968). *Singulyarnyye integralnyye uravneniya* [Singular integral equations]. Moscow: Nauka, 512 p. [in Russian].
29. Mirsalimov, V. M. (1987). *Neodnomernyye uprugoplasticheskiye zadachi* [Non-dimensional elastic-plastic problems]. Moscow: Nauka, 256 p. [in Russian].
30. Savruk, M. P., & Kazberuk, A. (2017). Stress concentration at notches. Springer International Publishing, 498 p.

Received 24 May 2018

Моделювання часткового закриття системи щілин у перфорованому ізотропному середовищі, що підкріплене стрингерами

Мір-Салім-заде М. В.

Інститут математики та механіки НАН Азербайджану,
Азербайджан, AZ1141, м. Баку, вул. Ф. Агаєва, 9

На основі методів теорії пружності проведено математичний опис моделі часткового закриття системи щілин у перфорованому ізотропному середовищі зі сторонніми поперечними включеннями. Таке середовище можна розглядати як перфоровану необмежену пластину, підсилену системою стрингерів надто вузького поперечного перерізу. Вважається, що середовище послаблене періодичною системою кругових отворів і прямолінійних щілин змінної ширини. Змінну ширину щілин можна порівняти з пружними деформаціями. В роботі застосовані метод розв'язання періодичної пружної задачі та метод побудови в явній формі комплексних потенціалів, що відповідають невідомим нормальним зміщенням вздовж прямолінійних щілин. Будуються загальні подання розв'язків, що описують клас задач з періодичним розподілом напружень поза круговими отворами та щілин з контактними зонами. Для визначення невідомих контактних напружень та розмірів зон контакту отримано сингулярне інтегральне рівняння, що зводиться до системи нелінійних алгебраїчних рівнянь. Система алгебраїчних рівнянь розв'язується методом послідовних наближень. В результаті знайдено контактні напруження та розміри зон контакту.

***Ключові слова**: перфорована пластина, стрингери, прямолінійні щілини змінної ширини, контактні напруження, контактні зони.*

Література

1. Финкель В. М. Физические основы торможения разрушения. М.: Металлургия, 1977. 360 с.
2. Партон В. З., Морозов Е. М. Механика упруго-пластического разрушения. М.: Наука, 1985. 504 с.
3. Мирсалимов В. М. Некоторые задачи конструкционного торможения трещин. *Физико-хим. механика материалов.* 1986. Т. 22. № 1. С. 94–98.
4. Толкачев В. М. Передача нагрузки от стрингера конечной длины к бесконечной и полубесконечной пластине. *Докл. АН СССР*. 1964. Т. 154. №4. С.86–88.
5. Долгих В. Н., Фильштинский Л. А. Об одной модели регулярной кусочно-однородной среды. *Изв. АН СССР. Механика твердого тела.* 1976. №2. С. 158–164.
6. Ванин Г. А. Микромеханика композиционных материалов. Киев: Наук. думка. 1985. 302 с.
7. Броек Д. Основы механики разрушения. М.: Высш. шк., 1980. 368 с.
8. Черепанов Г. П. Механика разрушения композиционных материалов. М.: Наука, 1983. 296 с.
9. Максименко В. Н. Влияние приклепанных ребер жесткости на развитие трещин возле отверстия. *Прикл. механика и техн. физика.* 1988. Т. 29. №2. С. 133–140.
10. Savruk M. P., Kravets V. S. Reinforcement of a thin cracked plate by a system of parallel stringers. *Materials Sci.* 1994. Vol. 30. Iss. 1. P. 95–104.
11. Savruk M. P., Kravets V. S. Two-dimensional problems of the theory of elasticity for reinforced cracked plates. Materials Sci. 1995. Vol. 31. Iss. 3. P. 350–362.
12. Savruk M. P, Kravets V. S. Effect of breaks in riveted stringers on the elastic and limiting equilibrium of a cracked plate. *Materials Sci*. 1999. Vol. 35. Iss. 3. P. 339–348.
13. Мир-Салим-заде М. В. Трещина со связями между берегами в изотропной среде, усиленной регулярной системой стрингеров. *Механика композит. материалов.* 2005. Т. 41. №6. С. 773–782.
14. Мир-Салим-заде М. В. Разрушение изотропной среды, усиленной регулярной системой стрингеров. *Механика композит. материалов.* 2007. Т. 51. №1. С. 59–72.
15. Мирсалимов В. М., Мустафаев А. Б. Точное решение контактной задачи о частичном взаимодействии берегов щели переменной ширины при действии температурного поля. *Проблемы машиностроения*. 2014. Т. 17. № 3. С. 33–37.
16. Мустафаев А. Б. Взаимодействие берегов щели переменной ширины при изгибе полосы (балки) под воздействием температурного поля. *Механика машин, механизмов и материалов*. 2014. №3. С. 30–36.
17. Мирсалимов В. М., Мустафаев А. Б. Решение задачи о частичном контактировании берегов щели переменной ширины под действием температурного поля. *Физико-хим. механика материалов*. 2015. Т. 51. №1. С. 86–92.
18. Mirsalimov V. M., Mustafayev A. B. A contact problem on partial interaction of faces of a variable thickness slot under the influence of temperature field. *Mechanika*. 2015. Vol. 21. Iss. 1. P. 19–22.
19. Mirsalimov V. M. Simulation of partial closure of a variable width slot with interfacial bonds in end zones in an isotropic medium. *Int. J. Damage Mech.*. – 2016. Vol. 25. Iss. 2. P. 266–279.
20. Мир-Салим-заде М. В. Закрытие щели, исходящей из контура кругового отверстия в стрингерной пластине. *Вестн. Чуваш. пед. ун-та им. И .Я. Яковлева.* Сер. Механика предельного состояния. 2016. № 1(27). С.78–89.
21. Мир-Салим-заде М. В. Частичный контакт берегов щели переменной ширины в подкрепленной стрингерами пластине. *Физико-хим. механика материалов*. 2016. Т. 52. № 3. С. 29–34.
22. Гасанов Ш. Г. Решение контактной задачи для плоскости, ослабленной щелью переменной ширины, в неоднородном напряженном поле. *Проблемы машиностроения*. 2017. Т. 20. № 2. С. 29–36.
23. Mir-Salim-zada M. V. Contact problem for a stringer plate weakened by a periodic system of variable width slots. *Structural Eng. and Mech.* 2017. Vol. 62. № 6. P. 719–724.
24. Мустафаев А. Б. Замедление роста щели переменной ширины под действием температурного поля. *Прикл. механика и техн. физика*. 2017. Т. 58. № 1. С. 168–176.
25. Мусхелишвили Н. И. Некоторые основные задачи математической теории упругости. М.: Наука, 1966. 707 с.
26. Панасюк В. В., Саврук М. П., Дацышин А. П.. Распределение напряжений около трещин в пластинах и оболочках. Киев: Наук. думка, 1976. 444 с.
27. Ladopoulos E. G. Singular Integral Equations, Linear and Non-Linear Theory and its Applications in Science and Engineering. Berlin: Springer Verlag, 2000. 552 p.
28. Мусхелишвили Н. И. Сингулярные интегральные уравнения. М.: Наука, 1968. 512 с.
29. Мирсалимов В. М. Неодномерные упругопластические задачи. М.: Наука, 1987. 256 с.
30. Savruk M. P., Kazberuk A. Stress concentration at notches. Springer International Publishing, 2017. 498 p.