UDC 539.3

MAJOR STRESS-STRAIN STATE OF DOUBLE SUPPORT MULTILAYER BEAMS UNDER CONCENTRATED LOAD

PART 1. MODEL CONSTRUCTION

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The development of composite technologies contributes to their being widely introduced into the practice of designing modern different-purpose structures. Reliable prediction of the stress-strain state of composite elements is one of the conditions for creating reliable structures with optimal parameters. Analytical theories for determining the stress-strain state of multilayer rods (bars, beams) are significantly inferior in development to those for composite plates and shells, although their core structural elements are most common. The purpose of this paper is to design an analytical model for bending double support multilayer beams under concentrated load based on the previously obtained solution of the elasticity theory for a multi-layer cantilever. The first part of the article includes a statement of the problem, accepted prerequisites and main stages of constructing a model for bending a double-support multi-layer beam with a concentrated load (normal, tangential force and moment) and general-view supports in the extreme cross-sections. When building the model, the double support beam was divided across the loaded cross-section and presented in the form of two separate sections with equivalent loads on the ends. Using the general solution of the elasticity theory for a multilayer cantilever with a load on the ends, the main stress-strain state of the design sections was described without taking into account the local effects of changing the stress state near the concentrated load application points and supports. The obtained relations contain 12 unknown initial parameters. To determine them on the basis of the conditions of joint deformation (static and kinematic) of design sectors, a system of algebraic equations has been constructed. The constructed model allows one to determine the components of the main stress-strain state of double support beams each consisting of an arbitrary number of orthotropic layers, taking into account the amenability of their materials to lateral shear deformations and compression.

Keywords: *multilayer beam, orthotropic layer, concentrated load, stresses, displacements.*

Introduction

With the development of technology, composite materials are increasingly being used in various-purpose constructions. Reliable determination of the stress-strain state (SSS) of composite elements of various types is one of the conditions for creating durable and reliable structures. Modern numerical methods and software systems built on their basis have a large arsenal of tools for solving this problem when performing checking calculations. However, at the stage of designing and solving optimization problems it is more convenient to use analytical methods for determining the SSS of composite elements.

A significant number of scientific papers [1–8] are devoted to separate types of composite, in particular, multilayer elements such as plates and shells, in which various analytical, numerical, and analytical methods for determining SSS are constructed.

The deformation of composite rods (bars, beams) has not been studied well enough, although such structural elements are most common. As it is difficult to take into account the inhomogeneous structure of multilayer composite rods when constructing analytical theories of their deformation, approximate methods for solving problems in the theory of elasticity, in particular, the iterative method [9–11], are very common. Despite the introduction of simplifications, the deformation models constructed using this method remains cumbersome and complex for practical application.

Exact solutions to elasticity theory problems have been obtained only for those of bending narrow cantilevers with separate types of loads [12, 13]. Such solutions are quite limited in terms of accounting for various types of supports and loads. However, on their basis it is possible to build relatively simple, but fairly accurate applied solutions to typical problems of bending beams.

The purpose of this paper is to build an analytical bending model for double-support composite multilayer beams under the action of a concentrated load, based on the general solution of the theory of elasticity for a multilayer cantilever with a load on the free end [12].

Main part

Consider the general case of the plane transverse bending of a straight multi-layer beam under the action of a concentrated load, taking into account that, in the general case, the beam has rigid supports excluding all the displacements of the extreme cross-sections (Fig. 1).

The beam consists of m parallel layers P_k ($k = \overline{1,m}$), made of various materials and rigidly connected on the contact surfaces. The cross-sections (Fig. 1, b) along the beam axis have a uniform structure and dimensions that meet the condition $b = h = l$.

The beam is related to the Cartesian coordinate system xyz, whose origin O coincides with the leftmost section rigidity center. The Ox axis coincides with the longitudinal axis of beam stiffness, and the plane xOz coincides with the beam symmetry plane and the external load plane.

Beam layers are made of homogeneous orthotropic materials with elastic symmetry planes parallel to the coordinate planes. The elastic material properties of all the beam layers are known. For an arbitrary k -st layer they are represented by a set of constants

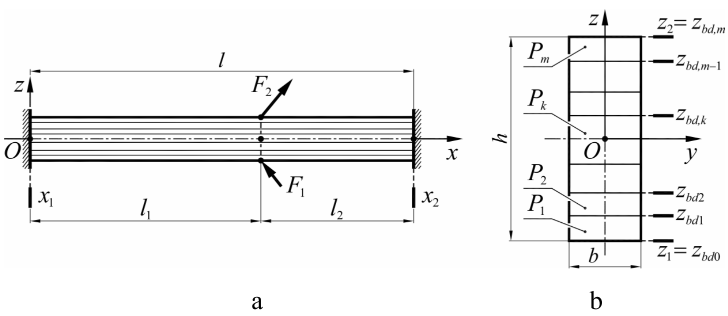

Fig. 1. Double support multi-layer beam:
a – load and support diagram; b – cross-section diagram

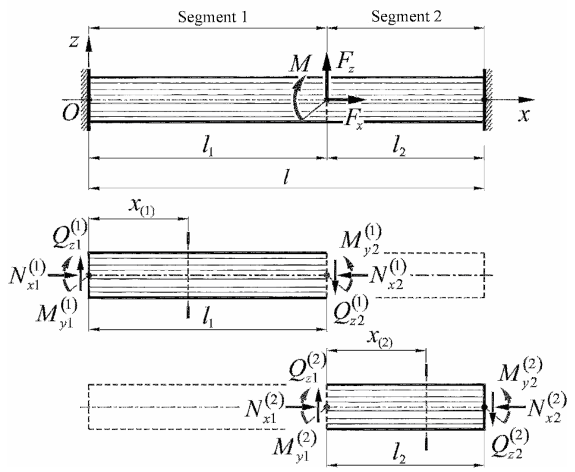

Fig. 2. Beam design diagram

$$\left\|S_a^{[k]}\right\| = \left\|E_x^{[k]}, E_y^{[k]}, E_z^{[k]}, G_{xy}^{[k]}, G_{yz}^{[k]}, G_{xz}^{[k]}, \nu_{xy}^{[k]}, \nu_{yx}^{[k]}, \nu_{zy}^{[k]}, \nu_{yz}^{[k]}, \nu_{xz}^{[k]}, \nu_{zx}^{[k]}\right\|,$$

where $E_x^{[k]}, E_y^{[k]}, E_z^{[k]}$ are the elastic moduli along the coordinate axes of the system xyz ; $G_{xy}^{[k]}, G_{yz}^{[k]}, G_{xz}^{[k]}$ are the shear moduli in the planes parallel to the coordinate planes; $\nu_{xy}^{[k]}, \ldots, \nu_{zx}^{[k]}$ are Poisson's ratios.

For an entire multi-layer beam, the elastic material properties will be piecewise constant functions $\mu_a^S(z)$, which, by analogy with [12] and [13], will be represented using the Heaviside functions $H(z)$

$$\mu_a^S = \sum_{k=1}^{m} \left(S_a^{[k]} \left(H\left(z - z_{bd,k-1}\right) - H\left(z - z_{bd,k}\right) \right) \right). \tag{1}$$

Consider SSS of such a beam during the elastic work of the materials of its layers, neglecting the local distortions of the stress and displacement distribution near the concentrated load application points and near the fixing supports. Such SSS, by analogy with the theory of shells [14], will be called the main one.

We present the concentrated load to the beam stiffness axis and represent it in the form of components F_x , F_z and M , applied to the section stiffness center (Fig. 2). Next, we divide the beam across the loaded section ($x = l_1$) into two design segments. In so doing, we replace the support and the discarded part of the beam with the corresponding internal force factors for each section and introduce our own reference system of the section coordinates ($i = 1, 2$ – section number).

If we consider the beam segments separately, then at some distance from the extreme sections their stress state (SS) will be similar to that of the cantilever beam with the load on the end, for which an exact solution was obtained in [12]. Using this solution, we write (for the i -th design segment) the relations for the SSS components

$$\sigma_x^{(i)} = \mu_x^E \left(\frac{N_{x1}^{(i)}}{bB_0} + \frac{Q_{z1}^{(i)}}{bB_2} z x_{(i)} + \frac{M_{y1}^{(i)}}{bB_2} z \right), \qquad \sigma_z^{(i)} = 0, \qquad \tau_{xz}^{(i)} = -\frac{Q_{z1}^{(i)}}{bB_2} \int_{z_1}^{z} \left(\mu_x^E z \right) dz, \tag{2}$$

$$u^{(i)} = \frac{N_{x1}^{(i)}}{bB_0} x_{(i)} - \frac{Q_{z1}^{(i)}}{bB_2} \int_{z_1}^{z} \left(\frac{1}{\mu_{xz}^G} \int_{z_1}^{z} \left(\mu_x^E z \right) dz - \int_{z_1}^{z} \left(\mu_{xz}^{\nu} z \right) dz \right) dz +$$

$$+ \frac{Q_{z1}^{(i)}}{bB_2} \left(\frac{1}{2} x_{(i)}^2 z + \frac{D_2}{h} (z - z_1) \right) + \frac{M_{y1}^{(i)}}{bB_2} x_{(i)} z + \frac{u_{11}^{(i)}}{h} (z_2 - z) + \frac{u_{12}^{(i)}}{h} (z - z_1), \tag{3}$$

$$w^{(i)} = -\frac{N_{x1}^{(i)}}{bB_0} \int_{z_1}^{z} \mu_{xz}^{\nu} \, dz - \frac{Q_{z1}^{(i)}}{bB_2} \left(\frac{1}{6} x_{(i)}^3 + x_{(i)} \int_{z_1}^{z} \left(\mu_{xz}^{\nu} z \right) dz + \frac{D_2}{h} x_{(i)} \right) -$$

$$- \frac{M_{y1}^{(i)}}{bB_2} \left(\frac{1}{2} x_{(i)}^2 + \int_{z_1}^{z} \left(\mu_{xz}^{\nu} z \right) dz \right) + \frac{u_{11}^{(i)}}{h} x_{(i)} - \frac{u_{12}^{(i)}}{h} x_{(i)} + w_{11}^{(i)}, \tag{4}$$

where $N_{x1}^{(i)}$, $Q_{z1}^{(i)}$, $M_{y1}^{(i)}$ – internal force factors in the initial section of the i-th segment (Fig. 2); $u_{11}^{(i)} = u^{(i)} |_{x_{(i)}=0,\, z=z_1}$, $u_{12}^{(i)} = u^{(i)} |_{x_{(i)}=0,\, z=z_2}$, $w_{11}^{(i)} = w^{(i)} |_{x_{(i)}=0,\, z=z_1}$ – displacements of the i-th segment extreme points (Fig. 3, a); $i = 1, 2$ – number of the beam segment.

In relations (2) – (4), the integral cross-section stiffness characteristics are determined by the formulas:

$$B_0 = \int_{z_1}^{z_2} \mu_x^E dz, \quad B_1 = \int_{z_1}^{z_2} \left(\mu_x^E z \right) dz, \quad B_2 = \int_{z_1}^{z_2} \int_{z_1}^{z} \left(\mu_x^E z \right) dz dz, \quad D_2 = \int_{z_1}^{z_2} \left(\frac{1}{\mu_{xz}^G} \int_{z_1}^{z} \left(\mu_x^E z \right) dz - \int_{z_1}^{z} \left(\mu_{xz}^{\nu} z \right) dz \right) dz.$$

Relations (2) – (4) are obtained for the main coordinate system xyz in which the condition $B_1 = 0$ is satisfied. The original position of the beginning of such a system relative to an arbitrary parallel auxiliary coordinate system xzy' is determined by the relation

$$z_{B_1} = B_1' / B_0',$$

where B_1', B_0' are the characteristics determined in an arbitrary auxiliary coordinate system xzy'.

The internal strength factors within the boundaries of the i-th segment are related to their initial values by the relations

$$N_x^{(i)} = N_{x1}^{(i)}, \qquad Q_z^{(i)} = Q_{z1}^{(i)}, \qquad M_y^{(i)} = Q_{z1}^{(i)} x_{(i)} + M_{y1}^{(i)}. \tag{5}$$

For the entire beam, the expressions for the SSS components and internal force factors can be combined using the Heaviside function

$$f = f^{(1)} |_{x_{(1)}=x} \left(1 - H(x - l_1) \right) + f^{(2)} |_{x_{(2)}=x-l_1} H(x - l_1), \tag{6}$$

where $f^{(1)}, f^{(2)}$ is the SSS component distribution in the first and second design segments, respectively.

Relations (2) – (4) contain 3 unknown internal force factors (static initial parameters) and 3 unknown displacements of the points in the initial section (kinematic initial parameters). With the help of these unknowns, one can specify the displacements of 4 points of the beam design segments, making it possible to simulate various restrictions on the displacements of its extreme sections.

For the two design segments of the beam under consideration, in the general case, we will have 12 unknown constants. Such a number of the unknowns is not sufficient both for the exact fulfillment of the compatibility conditions for the displacement of the adjacent design segments of said beam ($u^{(1)} |_{x_{(1)}=l_1} = u^{(2)} |_{x_{(2)}=0}$, $w^{(1)} |_{x_{(1)}=l_1} = w^{(2)} |_{x_{(2)}=0}$), and absolutely rigid fixation of its ends ($\left(u^{(1)}, w^{(1)} \right) |_{x_{(1)}=0} = 0$, $\left(u^{(2)}, w^{(2)} \right) |_{x_{(2)}=l_2} = 0$).

Therefore, the kinematic conditions on the boundary between the beam design segments will be described in a simplified way, combining the displacement of only the extreme cross-section points according to the diagram in Fig. 3, b.

In this case, we will have the following system of kinematic conditions for the beam design segment joint deformation:

$$u^{(1)}\Big|_{\substack{x_{(1)}=l_1,\\ z=z_1}} = u^{(2)}\Big|_{\substack{x_{(2)}=0,\\ z=z_1}},$$

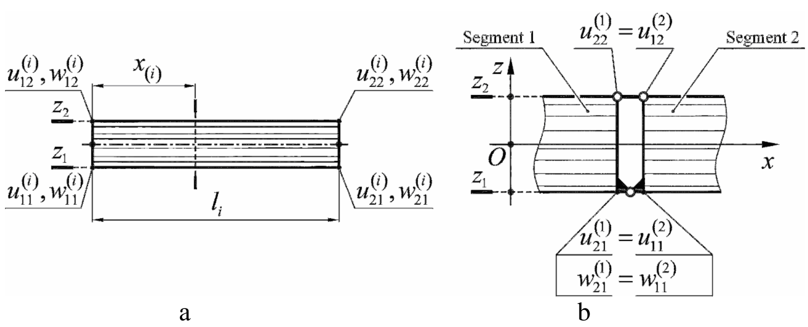

Fig. 3. Restrictions on beam design segment displacements:
a – segment kinematic parameters;
b – diagram of the boundary section kinematic relations

$$w^{(1)}\Big|_{\substack{x_{(1)}=l_1,\\ z=z_1}} = w^{(2)}\Big|_{\substack{x_{(2)}=0,\\ z=z_1}}, \quad u^{(1)}\Big|_{\substack{x_{(1)}=l_1,\\ z=z_2}} = u^{(2)}\Big|_{\substack{x_{(2)}=0,\\ z=z_2}},$$

or, taking into account the accepted conditions

$$u_{21}^{(1)} = u_{11}^{(2)}, \qquad w_{21}^{(1)} = w_{11}^{(2)}, \qquad u_{22}^{(1)} = u_{12}^{(2)}. \tag{7}$$

In addition to the compatibility of displacements of beam segments, we will integrally ensure the compatibility of the SSS components. To do this, on the boundary between the segments, we will require that the internal force factors be equal, taking into account the acting concentrated load

$$N_{x1}^{(2)} = N_{x1}^{(1)} - F_x, \qquad Q_{z1}^{(2)} = Q_{z1}^{(1)} + F_z, \qquad M_{y1}^{(2)} = l_1 Q_{z1}^{(1)} + M_{y1}^{(1)} + M\ . \tag{8}$$

Using conditions (7), (8) and relations (3) – (5), we construct a system of equations to determine all the unknown constants in solution (2) – (4).

For this, we replace the variables in relations (3) – (5) with the values: $x_{(i)} = l_i$, $z = z_1, z_2$

$$\begin{aligned} &N_{x2}^{(i)} = N_{x1}^{(i)},\ \ Q_{z2}^{(i)} = Q_{z1}^{(i)},\ \ M_{y2}^{(i)} = l_i Q_{z1}^{(i)} + M_{y1}^{(i)}, \\ &u_{21}^{(i)} = \frac{l_i}{bB_0} N_{x1}^{(i)} + \frac{z_1 l_i^2}{2bB_2} Q_{z1}^{(i)} + \frac{z_1 l_i}{bB_2} M_{y1}^{(i)} + u_{11}^{(i)}, \\ &u_{22}^{(i)} = \frac{l_i}{bB_0} N_{x1}^{(i)} + \frac{z_2 l_i^2}{2bB_2} Q_{z1}^{(i)} + \frac{z_2 l_i}{bB_2} M_{y1}^{(i)} + u_{12}^{(i)}, \\ &w_{21}^{(i)} = -\frac{h l_i^3 + 6D_2 l_i}{6hbB_2} Q_{z1}^{(i)} - \frac{l_i^2}{2bB_2} M_{y1}^{(i)} + \frac{l_i}{h} u_{11}^{(i)} - \frac{l_i}{h} u_{12}^{(i)} + w_{11}^{(i)}, \end{aligned} \tag{9}$$

where $u_{21}^{(i)} = u^{(i)}\big|_{x_{(i)}=l_i, z=z_1}$, $u_{22}^{(i)} = u^{(i)}\big|_{x_{(i)}=l_i, z=z_2}$, $w_{21}^{(i)} = w^{(i)}\big|_{x_{(i)}=l_i, z=z_1}$, $i = 1, 2$.

Substituting (7) and (8) into (9), upon transformations, we obtain such a system of relations between the initial ($N_{x1}^{(1)}, Q_{z1}^{(1)}, M_{y1}^{(1)}, u_{11}^{(1)}, w_{11}^{(1)}, u_{12}^{(1)}$) and final ($N_{x2}^{(2)}, Q_{z2}^{(2)}, M_{y2}^{(2)}, u_{21}^{(2)}, w_{21}^{(2)}, u_{22}^{(2)}$) parameters of the beam design segments:

$$\begin{aligned} &N_{x2}^{(2)} = N_{x1}^{(1)} - F_x;\ \ Q_{z2}^{(2)} = Q_{z1}^{(1)} + F_z;\ \ M_{y2}^{(2)} = l Q_{z1}^{(1)} + M_{y1}^{(1)} + l_2 F_z + M; \\ &u_{21}^{(2)} = \frac{l}{bB_0} N_{x1}^{(1)} + \frac{z_1 l^2}{2bB_2} Q_{z1}^{(1)} + \frac{z_1 l}{bB_2} M_{y1}^{(1)} + u_{11}^{(1)} - \frac{l_2 F_x}{bB_0} + \frac{z_1 l_2^2 F_z}{2bB_2} + \frac{z_1 l_2 M}{bB_2}; \\ &u_{22}^{(2)} = \frac{l}{bB_0} N_{x1}^{(1)} + \frac{z_2 l^2}{2bB_2} Q_{z1}^{(1)} + \frac{z_2 l}{bB_2} M_{y1}^{(1)} + u_{12}^{(1)} - \frac{l_2 F_x}{bB_0} + \frac{z_2 l_2^2 F_z}{2bB_2} + \frac{z_2 l_2 M}{bB_2}; \\ &w_{21}^{(2)} = -\frac{h l^3 + 6D_2 l}{6hbB_2} Q_{z1}^{(1)} - \frac{l^2}{2bB_2} M_{y1}^{(1)} + \frac{l}{h} u_{11}^{(1)} - \frac{l}{h} u_{12}^{(1)} + w_{11}^{(1)} - \frac{h l_2^3 + 6D_2 l_2}{6hbB_2} F_z - \frac{l_2^2 M}{2bB_2} \end{aligned} \tag{10}$$

Discussion of the results

System (10) consists of 6 equations, which together contain 12 static and kinematic parameters. For each particular type of supports, the values of 6 parameters will be known or can be expressed in terms of other parameters and known values. This allows one to bring the original system (10) to the correct form and determine the remaining static and kinematic parameters.

The first segment initial parameters obtained by the solution of system (10) are the input data for determining the initial parameters of the second segment, using relations (7) – (9)

$$N_{x1}^{(2)} = N_{x1}^{(1)} - F_x, \;\; Q_{z1}^{(2)} = Q_{z1}^{(1)} + F_z, \;\; M_{y1}^{(2)} = l_1 Q_{z1}^{(1)} + M_{y1}^{(1)} + M,$$

$$u_{11}^{(2)} = \frac{l_1}{bB_0} N_{x1}^{(1)} + \frac{z_1 l_1^2}{2bB_2} Q_{z1}^{(1)} + \frac{z_1 l_1}{bB_2} M_{y1}^{(1)} + u_{11}^{(1)},$$

$$u_{12}^{(2)} = \frac{l_1}{bB_0} N_{x1}^{(1)} + \frac{z_2 l_1^2}{2bB_2} Q_{z1}^{(1)} + \frac{z_2 l_1}{bB_2} M_{y1}^{(1)} + u_{12}^{(1)}, \tag{11}$$

$$w_{11}^{(2)} = -\frac{h l_1^3 + 6 D_2 l_1}{6hbB_2} Q_{z1}^{(1)} - \frac{l_1^2}{2bB_2} M_{y1}^{(1)} + \frac{l_1}{h} u_{11}^{(1)} - \frac{l_1}{h} u_{12}^{(1)} + w_{11}^{(1)}.$$

Substituting the initial parameters known and defined using (10) and (11) into the initial relations (2) – (4) allows one to obtain expressions for the components of the main SSS of all the design segments of the beam. In the future, the solutions for design segments using (6) can be combined into general expressions for an entire multi-layer beam.

Conclusions

Thus, there has been constructed an analytical model of flat bending of double-support multi-layer beams under the action of a concentrated load, which are represented by relations (2) – (4), (10) and (11). The model allows determining the components of the main SSS of multilayer beams each consisting of an arbitrary number of orthotropic layers, taking into account the amenability of their materials to transverse shear deformations and compression.

The obtained relations can be used to solve the problems of deforming multilayer beams with different types of supports on the extreme cross-sections.

The approach used to construct the model can be generalized and extended to the case of multi-span beams with an arbitrary number of concentrated forces and intermediate supports, as well as beams with different rigidity.

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Received 26 September 2018

Основний напружено-деформований стан двохопорних багатошарових балок під дією зосередженого навантаження. Частина 1. Побудова моделі

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Розвиток технологій композитів сприяє їх широкому впровадженню в практику проектування сучасних конструкцій різного призначення. Достовірне прогнозування напружено-деформованого стану композитних елементів є однією із умов створення надійних конструкцій з оптимальними параметрами. Аналітичні теорії визначення напружено-деформованого стану багатошарових стержнів (брусів, балок) значно поступаються у розвитку теоріям для композитних плит і оболонок, хоча стрижневі елементи конструкцій є найпоширенішими. Метою цієї роботи є побудова аналітичної моделі вигину двохопорних багатошарових балок під дією зосередженого навантаження на основі отриманого раніше розв'язку теорії пружності для багатошарової консолі. У першій частині статті наведено постановку задачі, прийнято передумови і основні етапи побудови моделі згину багатошарової двохопорної балки із зосередженим навантаженням (нормальна, дотична сила і момент) і закріпленнями загального вигляду в крайніх перетинах. Під час побудови моделі двохопорна балка була розділена по навантаженому перерізу і подана у вигляді двох окремих ділянок з еквівалентними навантаженнями на торцях. З використанням загального розв'язку теорії пружності для багатошарової консолі з навантаженням на торцях був описаний основний напружено-деформований стан розрахункових ділянок без урахування локальних ефектів зміни напруженого стану поблизу точок прикладання зосередженого навантаження і закріплень. Отримані співвідношення містять 12 невідомих початкових параметрів, для визначення яких з умов спільного деформування (статичних і кінематичних) розрахункових ділянок побудована система алгебраїчних рівнянь. Побудована модель дозволяє визначати компоненти основного напружено-деформованого стану двохопорних балок, що складаються з довільної кількості ортотропних шарів, з урахуванням податливості їх матеріалів деформаціям поперечного зсуву і обтиснення.

Ключові слова: *багатошарова балка, ортотропний шар, зосереджене навантаження, напруження, переміщення.*

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