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STUDY OF THE STRESSED STATE NEAR THE CRACK THAT INITIATES AT THE INCLUSION UNDER LONGITUDINAL SHIFT WAVE INFLUENCE

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Modern elements of building structures and machine parts often contain structural elements or technological defects that can be considered as thin inclusions of high rigidity. Reinforcing elements of composite materials can also be thin rigid inclusions. But studies show that thin rigid inclusions cause a significant stress concentration in the environment, which can lead to the formation of cracks at the inclusion. The problems of determining the stress state in the vicinity of complex defects were solved, as a rule, in a static formulation and for the case of rectilinear defects. This is due to the difficulties that arise in the case of their solution by the common method of boundary integral equations, which consists in reducing such problems to singular integral or integro-differential equations with fixed singularities. Such equations require that special methods be created for their numerical solution. Recently, there has been a continuous growth in the number of papers where special quadrature formulas are used for singular integrals with fixed singularities, for example, for cracks or inclusions in the form of broken or branched defects. These works propose a collocation method that takes into account the real feature of the solution, and in order to calculate integrals with fixed singularities special quadrature formulas are used. The problems of determining the stress state around the defects, which are thin inclusions from whose edge a crack propagates at a certain angle, have been barely solved. The purpose of this paper is to study the stress state near the crack that initiates at the inclusion when subjected to a longitudinal shear wave. The formulated problem is reduced to a system of singular integro-differential equations with fixed singularities with respect to the unknown voltage surges and displacements on the surface of a defect. To solve this system, a similar collocation method is used. There have been shown dependences of the change in the dimensionless values of the stress intensity factors (SIF) on the dimensionless value of the wave number in the case of wave propagation at different angles. For numerical experiments, different values of the angle between the inclusion and crack were taken. In all cases, there was found the value of the dimensionless wave number at which SIFs for the crack reach their peaks. With an increase in the angle between the inclusion and crack, SIF values for the inclusion, up to certain oscillation frequency values, decrease. For the case when the defects are on the same straight line, SIF values for the inclusion are smallest. Conversely, when the angle between the defects increases, SIF values for the crack increase too. In general, as a result of the complexity of the wave field created by the reflection of waves from a defect, SIF dependence on frequency has significant maxima, whose magnitude and position are influenced by the configuration of the defect.

Keywords: *stress intensity factors, singular integro-differential equations, harmonic oscillations, fixed singularity, inclusion, crack.*

Introduction

In the field of building technology and engineering, structures and machine parts often contain elements or technological defects that can be considered as thin inclusions of high rigidity. However, as studies [1] show, thin rigid inclusions cause a significant stress concentration in the environment, which can lead to cracks at the inclusion. The problems of determining the stress state in the vicinity of complex defects were solved, as a rule, in a static formulation and for the case of rectilinear defects in [2–5]. This is due to the difficulties that arise in the case of their solution by the common method of boundary integral equations, which consists in reducing such problems to singular integral or integro-differential equations with fixed singularities. Similar problems were solved in a static formulation, but the real feature of the solutions was either ignored, or the Gauss-Jacobi formulas were applied to integrals with fixed singularities, resulting in a rather slow convergence of numerical solutions.

Recently, there has been a continuous growth in the number of papers where special quadrature formulas are used for singular integrals with fixed singularities, for example, for cracks or inclusions in the form of broken or branched defects. These works propose a collocation method that takes into account the real feature of the solution, and in order to calculate integrals with fixed singularities special quadrature formulas are used. The problems of determining the stress state around the defects, which are thin inclusions from whose edge a crack propagates at a certain angle, have been barely solved. The purpose of this paper is to study the stress state near the crack that initiates at the inclusion when subjected to a longitudinal shear wave. The formulated problem is reduced to a system of singular integro-differential equations with fixed singularities with respect to the unknown voltage surges and displacements on the surface of a defect. To solve this system, a collocation method is used. This method is similar to the one applied in [6– 8].

Problem Formulation

An elastic isotropic medium in a state of antiplane deformation is considered. In the environment, there is a penetration defect in the form of absolutely rigid inclusion. From its end a crack propagates at an arbitrary angle. The inclusion and crack in the Oxy plane occupy the segments $2d_l$, forming the angles α_l, l=1, 2 with the Ox axis (Fig. 1).

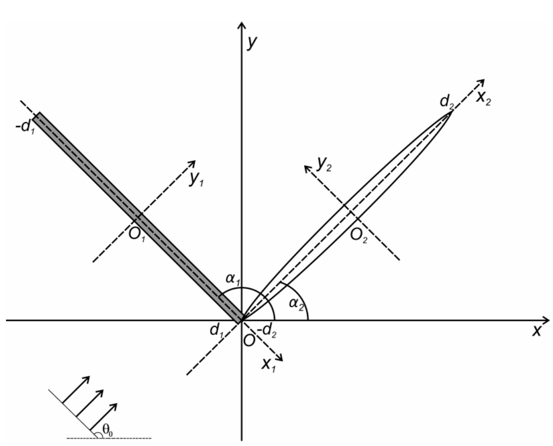

Fig. 1. Inclusion with a crack propagating from its edge

The defects interact with a flat longitudinal displacement wave that causes in the environment the following displacements along the Oz axis:

$$w_0(x, y) = Ae^{i\kappa_2(x\cos\theta_0 + y\sin\theta_0)}, \quad \kappa_2^2 = \frac{\rho\omega^2}{G},$$

where ω is the oscillation frequency, ρ and G – are the density and environment shear modulus. Dependence on time is determined by the factor $e^{-i\omega t}$ ignored here and to be ignored later. Under these conditions, the only non-zero z – component of the displacement vector satisfies the Helmholtz equation

$$\Delta w + \kappa_2^2 w = 0\,, \tag{1}$$

where Δ is the Laplace operator in the Oxy coordinate system.

To formulate the boundary conditions for a defect, we associate both the crack and inclusion with the local $O_l x_l y_l$, $l = 1,2$ coordinate system whose center coincides with the middle of the corresponding defect (Fig. 1). The relationship between the coordinate systems is given by the formulas

$$\begin{cases} x = (d_l + (-1)^l x_l)\cos\alpha_l - (-1)^l y_l \sin\alpha_l, \\ y = (d_l + (-1)^l x_l)\sin\alpha_l + (-1)^l y_l \cos\alpha_l, \end{cases}$$
$$\begin{cases} x_l = (-1)^l x\cos\alpha_l + (-1)^l y\sin\alpha_l - (-1)^l d_l, \\ y_1 = (-1)^{l-1} x\sin\alpha_l + (-1)^l y\cos\alpha_l,\ l = 1,2. \end{cases} \tag{2}$$

Let $w_l(x_l, y_l)$, $l = 1,2$, be obtained from $w(x, y)$ as a result of the transition to local coordinates by formulas (2). We formulate the crack boundary conditions on the basis of the fact that there are no stresses on the crack faces. This results in the equality

$$\tau_{y_2 z}(x_2, 0) = 0\,,\ x_2 \in [-d_2, d_2] \tag{3}$$

On the surface of the crack, the displacements $w_2(x_2, y_2)$ have a gap with an unknown jump for which the following designation is introduced.

$$w_2(x_2, +0) - w_2(x_2, -0) = \chi_2(x_2), \chi_2(d_2) = 0,\ x_2 \in [-d_2, d_2]. \tag{4}$$

On the inclusion, under the condition of ideal adhesion there must be fulfilled the equality

$$w_1(x_1, 0) = a, \quad x_1 \in [-d_1, d_1], \tag{5}$$

where a is the unknown amplitude of the longitudinal (along the axis Oz) oscillations of the inclusion.

On the surface of the inclusion, the stress $\tau_{y_1 z}(x_1, y_1)$ has a gap with an unknown jump for which the following designation is introduced

$$\tau_{y_1z}(x_1,+0)-\tau_{y_1z}(x_1,-0)=\chi_1(x_1),\ x_1\in[-d_1,d_1]. \tag{6}$$

The unknown amplitude of the inclusion oscillations is determined from the equation

$$-m\omega^2 a=\int_{-d_1}^{d_1}\chi_1(\eta)d\eta,\ m=2d_1\rho_v h.$$

Under these conditions, it is necessary to determine the stress state in the vicinity of the defect.

Method of Solution

To solve this problem for a crack and inclusion in a coordinate system connected with defects, discontinuous solutions of equation (1) with jumps (4), (6) are constructed. These solutions are based on the formulas

$$w_1^d(x_1,y_1)=\int_{-d_1}^{d_1}\frac{\chi_1(\eta)}{G}r_2(\eta-x_1,y_1)d\eta\,,\quad w_2^d(x_2,y_2)=\int_{-d_2}^{d_2}\chi_2(\eta)\frac{\partial}{\partial y_2}r_2(\eta-x_2,y_2)d\eta\,, \tag{7}$$

$r_2(\eta-x_l,y_l)=-\dfrac{i}{4}H_0^{(1)}\left(\kappa_2\sqrt{(\eta-x_l)^2+y_l^2}\right),\ l=1{,}2$, where $H_0^{(1)}(x)$ is Hankel's function.

After this, the displacement of the diffraction field in the Oxy system is given in the form

$$w(x,y)=w_1^g(x,y)+w_2^g(x,y)\,,$$

where $w_l^g(x,y)$, $l=1{,}2$ are obtained by formulas (7) after the transformation of coordinates (2). In order to finally determine the displacement and strain in a semi-space, it is necessary to find the unknown jumps of displacements and strains. To do this, conditions (3), (5) should be used.

After realizing the boundary conditions on the defects, we obtain a system of singular integro-differential equations with respect to the unknown jumps. This system, after separating the kernel functions of singularities and transitioning to the gap $[-1,1]$, has the form

$$\begin{cases}\dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\varphi_1(\tau)\left[\frac{1}{\tau-\zeta}+R_1(\tau,\zeta)\right]d\tau-\frac{1}{2\pi}\int_{-1}^{1}\varphi_2'(\tau)\,g_2(1+\tau,1-\zeta)\,d\tau-\\ -\dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\varphi_2(\tau)\,R_2(\tau,\zeta)\,d\tau=f_1(\zeta),\\ \dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\varphi_2'(\tau)\left[\frac{1}{\tau-\zeta}+R_3(\tau,\zeta)\right]d\tau+\frac{\kappa_0^2\gamma_2^2}{2\pi}\int_{-1}^{1}\varphi_2(\tau)\,\left[\ln|\tau-\zeta|+R_4(\tau,\zeta)\right]d\tau-\\ -\dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\varphi_1(\tau)\,[g_1(1-\tau,1+\zeta)+R_5(\tau,\zeta)]d\tau=f_2(\zeta),\\ \dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\varphi_1(\tau)\,[\ln|1+\tau|+R_6(\tau)]d\tau+\frac{\gamma_2^2}{\pi}\int_{-1}^{1}\varphi_2(\tau)\,R_7(\tau)\,d\tau=f_3,\end{cases} \tag{8}$$

where

$$\varphi_1(\tau)=\chi_1(\eta)/G,\ \varphi_2(\tau)=\chi_2(\eta)/d_2\,,\ \varphi_2'(\tau)=\chi_2'(\eta),$$
$$g_l(x,y)=x/p_l(x,y),\ p_l(x,y)=\gamma_l^2y^2-2\gamma_l\,yx\cos(\alpha_1-\alpha_2)+x^2,\ l=1{,}2,$$
$$f_1(\zeta)=-A_0 i\kappa_0\cos(\alpha_1-\theta_0)e^{i\kappa_0\gamma_1(1-\zeta)\cos(\alpha_1-\theta_0)},$$
$$f_2(\zeta)=A_0 i\kappa_0\sin(\alpha_2-\theta_0)e^{i\kappa_0\gamma_2(1+\zeta)\cos(\alpha_1-\theta_0)},$$
$$f_3=-A_0e^{i\kappa_0\gamma_1\cos(\alpha_1-\theta_0)},\ A_0=A/d,\ d=\max(d_1,d_2).$$

As can be seen from the above, the functions $g_l(x,y)$ have features at $\tau=\pm1,\ \zeta=\mp1$.

Approximate solution of the system of integro-differential equations

The presence of fixed singularities (at $\tau=1, \zeta=-1$ and $\tau=-1, \zeta=1$) in a singular part of system (8) influences the behavior of its solutions in the vicinity of the points $\zeta=\pm1$. The asymptotics of the solutions in the vicinity of these points is determined by the method described in [9]. The result is that unknown functions need to be searched for in the form

$$\varphi_1(\tau)=(1+\tau)^{-1/2}(1-\tau)^{-\delta}\psi_1(\tau), \varphi_2'(\tau)=(1+\tau)^{-\delta}(1-\tau)^{-1/2}\psi_2(\tau), \tag{9}$$

where the singularity indicator is determined by the equality

$$\delta=\frac{2\beta-3\pi}{2(\beta-2\pi)}, \beta=|\alpha_1-\alpha_2|, 0\le\beta\le\pi .$$

For the functions with such features to be a solution to system (8), there must be fulfilled the equality

$$\psi_1(1)=(\gamma_1/\gamma_2)^{-\delta}\psi_2(-1),$$

and the functions $\psi_l(\tau)$, $l=1,2$ are to be considered as satisfying the Hölder condition at the gap $[-1,1]$. The further solution is based on the approximation of these functions by interpolation polynomials

$$\psi_l(\tau)=\sum_{m=1}^{n}\psi_{lm}\frac{P_{ln}(\tau)}{(\tau-\tau_{lm})[P_{lm}(\tau_{lm})]'}, \tag{10}$$

where $\psi_{lm}=\psi_l(\tau_{lm})$, $l=1,2$, $P_{1n}(\tau)=P_n^{-\delta,-1/2}(\tau)$, $P_{2n}(\tau)=P_n^{-1/2,-\delta}(\tau)$ – are the Jacobi polynomials, and τ_{lm} are their roots.

For integrals with the Cauchy kernel, we use the following quadrature formulas [10]:

$$\int_{-1}^{1}\frac{\varphi_l^{(l-1)}(\tau)}{\tau-\zeta_{lk}}d\tau=\sum_{m=1}^{n}\psi_{lm}\frac{A_{lm}}{\tau_{lm}-\zeta_{lk}}, \tag{11}$$

where $l=1,2, k=1,2,...,n-1$, ζ_{lk} – are the zeros of the Jacobi functions of the second kind $J_n^{-\delta,-1/2}(\tau)$ and $J_n^{-1/2,-\delta}(\tau)$, and A_{lm} are the coefficients of the corresponding Gauss-Jacobi quadrature formulas [11].

Next, analogous formulas need to be obtained for integrals with fixed singularities

$$E_l=\int_{-1}^{1}\varphi_l^{(l-1)}(\tau)g_l(1+(-1)^l\tau,1-(-1)^l\zeta)d\tau, l=1,2 . \tag{12}$$

If $1-\zeta>\varepsilon, 1+\zeta>\varepsilon$, where $0<\varepsilon<1$ is some approximate number, then the functions $g_l(\tau,\zeta)$ are infinitely smooth and to integrals (12) the Gauss-Jacobi quadrature formulas can be applied. The main difficulty is the computation of these integrals at $1\pm\zeta\to0$. For this purpose, representations (9), (10) are used and the following transformations with sub-integral functions are performed:

$$\begin{aligned}\frac{g_l(\tau,\zeta)}{\tau-\tau_{lm}}&=\frac{g_l(1+(-1)^l\tau_{lm},1-(-1)^l\zeta)}{\tau-\tau_{lm}}-\\&-(-1)^l\frac{g_l(1+(-1)^l\tau,1-(-1)^l\zeta)(1+(-1)^l\tau_{lm})}{p_l(1+(-1)^l\tau_{lm},1-(-1)^l\zeta)}+\\&+\frac{(-1)^l\gamma_l^{\,2}\left(1-(-1)^l\zeta\right)^2}{p_l(1+(-1)^l\tau_{lm},1-(-1)^l\zeta)\,p_l(1+(-1)^l\tau,1-(-1)^l\zeta)}, l=1,2.\end{aligned} \tag{13}$$

Integrals of the functions included in representation (13) can be found using a method based on the application of the convolution theorem for the Mellin integral transform. Finally, the formulas for calculating integrals with fixed singularities $\zeta=\zeta_{lk}$ are of the form

$$E_l=(-1)^l\sum\nolimits_{m=1}^{n}\psi_{lm}S_{mk}^l, l=1,2 \tag{14}$$

where

$$S_{mk}^{l}=\sum\nolimits_{p=1}^{3}B_{p}^{l}(\tau_{lm},\zeta_{jk})h_{p}^{l}(r_{l}^{k}),\varepsilon>r_{l}>0,\; r_{l}=\gamma_{l}(1+(-1)^{l}\zeta_{lk})/(2\gamma_{3-l}),$$

$$B_{1}^{l}(x,y)=\frac{A_{l}^{m}(\gamma_{l}/\gamma_{3-l})^{2}(1+(-1)^{l}x)\cos\beta}{p_{l}(x,y)},\; h_{1}^{l}(y)\equiv 1,$$

$$B_{p}^{l}(x,y)=\frac{[(1+(-1)^{l}x)^{3-p}(\gamma_{l}/\gamma_{3-l})^{2}y^{2}+((-1)^{l}p_{l}(x,y))^{3-p}]\cos\beta}{p_{l}(x,y)[P_{ln}(x)]'},$$

$$h_{p}^{l}(y)=-\frac{2^{-\delta-0.5}\gamma_{3-l}^{2}\Gamma(n+0.5)}{\gamma_{l}^{2}\sin\beta}\left(\frac{\pi}{\sin(\pi\delta)}\sum_{s=0}^{\infty}c_{sp}^{l}y^{s-\delta}-\sum_{s=0}^{\infty}d_{sp}^{l}y^{s}\right),$$

$$c_{sp}^{l}=\frac{\Gamma(-\delta+s+n+1)\sin(\beta(-\delta+s+p-2))}{s!\,\Gamma(s-\delta+1)\Gamma(n-s+0.5)},$$

$$d_{sp}^{l}=\frac{(-1)^{s+1}\Gamma(-\delta+s+p-2)\sin(\beta(s+1))}{(s+p-2)!\,\Gamma(-\delta-s+n+0.5)},\; l=1,2,\, p=2,3.$$

Thus, at $1+\zeta_{1k}\to 0$ and $1-\zeta_{2k}\to 0$ integrals (12) can be calculated using fast convergent power series.

For an integral with a logarithmic function, as a result of integration by parts and using the representation of derivative (9), we obtain the following quadrature formula:

$$\int_{-1}^{1}\varphi_{2}(\tau)\ln\left|\tau-\zeta_{2k}\right|dx=\sum\nolimits_{m=1}^{n}A_{2m}\psi_{2m}B_{km} \tag{15}$$

$$B_{km}=-(1+\zeta_{2k})(\ln\left|1+\zeta_{2k}\right|-1)-(\tau_{2m}-\zeta_{2k})(\ln\left|\tau_{2m}+\zeta_{2k}\right|-1).$$

To calculate an integral immediately containing an unknown function, it is necessary to find its approximate value by means of the equality

$$\varphi_{2}(\tau)=-\int_{\tau}^{1}\varphi_{2}'(x)dx.$$

Next, we use the representation for derivative (9), as well as the Christophel-Darboux identity. As a result, after the integration, there is obtained the expression

$$\varphi_{2}(\tau)=-(1-\tau)^{1/2}\sum\nolimits_{m=1}^{n}A_{2m}S_{km}(\tau), \tag{16}$$

$$S_{km}=2^{1-\delta}F\left(\delta,\frac{1}{2},\frac{3}{2},\frac{1-\tau}{2}\right)\Big/\sigma_{0}^{2}+(1+\tau)^{1-\delta}\sum\nolimits_{j=1}^{n-1}P_{j}^{-1/2,-\delta}(\tau_{2m})P_{j-1}^{1/2,1-\delta}(\tau)\Big/2j\sigma_{j}^{2}.$$

Representation (16) is the basis for such quadrature formulas with an unknown function $\varphi_{2}(\tau)$:

$$\int_{-1}^{1}\varphi_{2}(x)R(\tau,\zeta_{lk})dx=\sum\nolimits_{m=1}^{n}A_{2m}\psi_{2m}U_{mi}^{2}(\tau_{0i})R(\tau_{2m},\zeta_{lk}), \tag{17}$$

$$U_{mi}^{2}(\tau_{0i})=-\sum_{i=1}^{n}A_{0i}S_{km}(\tau_{0i}),\; A_{0i}=\frac{2\sqrt{2}}{(1-\tau_{0i}^{2})((P_{n}^{1/2,0}(\tau_{0i}))')^{2}},l=1,2,$$

where $P_{n}^{1/2,0}(\tau)$ are the Jacobi polynomials, and τ_{0i} are their roots.

The application of quadrature formulas (11), (14), (15), (17), as well as the Gauss-Jacobi formulas leads to the replacement of the system of integro-differential equations (8) by a system of linear algebraic equations with respect to the values of the functions ψ_{l}, $l=1,2$ in the interpolation nodes.

For fracture mechanics, of greatest interest is the stress intensity factors

$$K_{1}=\lim_{\eta\to -d_{1}+0}\sqrt{d_{1}+\eta}\cdot\chi_{1}(\eta),K_{2}=\lim_{x_{2}\to d_{1}+0}\sqrt{x_{2}-d_{2}}\cdot\tau_{y_{2}z}^{d}(x_{2},0).$$

Due to the solution of the system, the approximate values of the SIF are determined by the formulas

$$K_{1}=-G\sqrt{d_{1}}2^{-\delta}\psi_{1}(-1),K_{2}=-G\sqrt{d_{2}}2^{-1-\delta}\psi_{2}(1).$$

Analysis of Numerical Results and Conclusions

There have been considered the defects of the same length d_l=d, l=1, 2, starting at the coordinate origin and located symmetrically with respect to the Oy axis. An in-going wave propagates along a positive Oy axis direction. Figure 2 shows the dependences of the change in the dimensionless values of SIF ($k_l = K_l / G\sqrt{d_l}$, $l = 1,2$) from the dimensionless value of the wave number in the case of wave propagation at an angle θ_0=90° and in Fig. 3 – at an angle θ_0=270°. The angle between the defects was taken consistently (β=30°, 60°, 90°, 120°, 175°) and corresponds to the curves in the figures under numbers 1, 2, 3, 4, 5.

In all cases, there has been found the value of the dimensionless wave number at which SIF values for the crack reach their maxima. In the case of an increase in the angle β between the inclusion and crack, the value of SIF for the inclusion, up to certain values of the oscillation frequency, is reduced. For the case where the defects lie on the same line, SIF values for the inclusion are smallest. It can be seen that for small frequencies ($\kappa_0 \le 2$), with increasing angle β, the value of SIF for the crack increases, and the greatest values can be observed when the angle approaches 180°. In general, due to the complexity of the wave field generated by the reflection of the waves from the defects, the dependence of SIF on frequency reaches significant peaks, whose value and position are affected by the configuration of the defect.

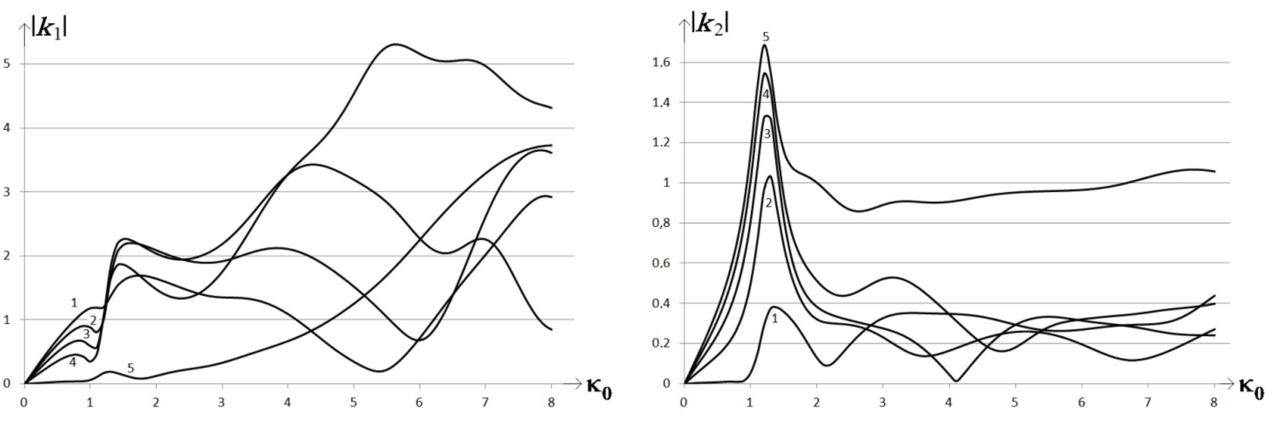

Fig.2. Dependence of SIF for the inclusion and crack on the wave number under the influence of longitudinal displacement (angle θ_0=90°)

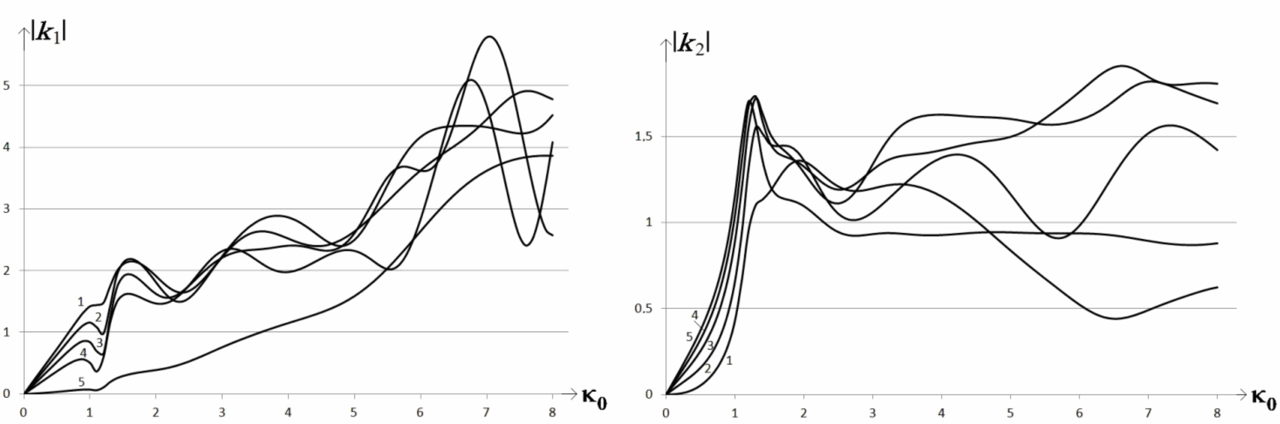

Fig. 3. Dependence of SIF for the inclusion and crack on the wave number under the influence of longitudinal wave displacement (angle θ_0=270°)

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Дослідження напруженого стану біля тріщини, що відходить від включення під впливом хвилі поздовжнього зсуву

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Сучасні елементи будівельних конструкцій і деталі машин досить часто містять конструктивні елементи або технологічні дефекти, які можна розглядати як тонкі включення великої жорсткості. Армуючі елементи композитних матеріалів теж можуть являти собою тонкі жорсткі включення. Але як показують дослідження, тонкі жорсткі включення спричиняють значну концентрацію напружень у навколишньому середовищі, яка може призвести до утворення тріщин на його продовженні. Задачі з визначення напруженого стану в околі складних дефектів розв'язувались, як правило, у статичній постановці і для випадку прямолінійних дефектів. Це пов'язано з труднощами, які виникають під час їх розв'язання поширеним методом граничних інтегральних рівнянь, що полягає у зве-

денні подібних задач до сингулярних інтегральних або інтегро-диференціальних рівнянь з нерухомими особливостями. Такі рівняння вимагають створення спеціальних методів їхнього числового розв'язання. Останнім часом все більше з'являється робіт, де для сингулярних інтегралів з нерухомими особливостями використовуються спеціальні квадратурні формули, наприклад, для тріщин або включень у вигляді ламаних або розгалужених дефектів. В цих роботах запропоновано колокаційний метод, який враховує справжню особливість розв'язку, а для обчислення інтегралів з нерухомими особливостями використано спеціальні квадратурні формули. Задачі з визначення напруженого стану навколо дефектів, що являють собою тонке включення, від краю якого під деяким кутом відходить тріщина, майже не розв'язувались. Метою цієї роботи є дослідження напруженого стану біля тріщини, що відходить від включення під впливом хвилі поздовжнього зсуву. Сформульована задача приведена до системи сингулярних інтегро-диференціальних рівнянь з нерухомими особливостями відносно невідомих стрибків напружень і переміщень на поверхні дефекту. Для розв'язання цієї системи використовується аналогічний колокаційний метод. Показано залежності зміни безрозмірних значень коефіцієнтів інтенсивності напружень (КІН) від безрозмірного значення хвильового числа у випадку поширення хвилі під різними кутами. Для числових експериментів бралися різні значення кута між включенням і тріщиною. У всіх випадках знайдено значення безрозмірного хвильового числа, за якого значення КІН для тріщини досягають максимуму. У разі зростання кута між включенням і тріщиною значення КІН для включення, до певних значень частоти коливань, зменшуються. Для випадку, коли дефекти лежать на одній прямій, значення КІН для включення найменші. І навпаки, коли кут між дефектами зростає, значення КІН для тріщини також зростають. В цілому, внаслідок складності хвильового поля, створеного відбиттям хвиль від дефекту, залежність КІН від частоти має істотні максимуми, на величину і положення яких впливає конфігурація дефекту.

***Ключові слова**: коефіцієнти інтенсивності напружень, сингулярні інтегро-диференціальні рівняння, гармонічні коливання, нерухома особливість, включення, тріщина.*

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