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SOLUTION TO NON-STATIONARY INVERSE HEAT CONDUCTION PROBLEMS FOR MULTI-LAYER BODIES, BASED ON EFFECTIVE SEARCH FOR THE REGULARIZATION PARAMETER

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To obtain a stable solution to the inverse heat conduction problem (IHCP), the article uses A. N. Tikhonov's method with an effective algorithm for finding the regularization parameter. The required heat flux at the boundary and the thermal contact resistance in the time coordinate are approximated by Schoenberg splines of the third degree, with the sum of the squares of the desired value, its first and second derivatives, being used as a stabilizing functional. The object of this study is multi-layer plates or shells, such as solid-fuel rocket engine bodies. To a first approximation, the problem is considered in a one-dimensional non-stationary linear formulation. The shell thickness-to-radius ratio will be considered such that in the heat equation, the curvature of the shell can be neglected and considered as a flat plate. This assumption was chosen to simplify the presentation of the material, and it does not limit the applicability of the methodology under consideration for the case of axially symmetrical shells, as well as for the case when a mathematical model is converted from the rectangular coordinate system to the cylindrical one. Three inverse problems are considered. In the first two, heat fluxes in a composite body with the ideal and real thermal contacts are determined. In the third IHCP, with the real thermal contact, thermal contact resistance is determined. Heat fluxes in multi-layer bodies are represented as linear combinations of Schoenberg splines of the third degree with unknown coefficients, which are calculated by solving a system of linear algebraic equations. This system is a consequence of the necessary condition for the minimum functional based on the principle of the least squares of the deviation of the temperature being simulated from the one obtained as a result of a thermophysical experiment. To regularize the solutions to the IHCP, in this functional, the stabilizing functional with the regularization parameter, as a multiplicative factor, is used as the summand to the sum of squares. This functional is the sum of the squares of heat fluxes, their first and second derivatives with the corresponding multipliers. These multipliers are selected according to the previously known properties of the desired solution. The search for the regularization parameter is carried out using the algorithm similar to the one for searching for the root of a nonlinear equation.

Keywords: *inverse heat conduction problem, heat flux, thermal contact resistance, A. N. Tikhonov regularization method, functional, stabilizer, regularization parameter, identification, approximation, Schoenberg spline of the third degree.*

Introduction

Solving inverse heat conduction problems (IHCP) for identifying the parameters of mathematical models is of particular importance to ensure the adequacy of these models in the presence of experimental information on the thermal process being studied. The effectiveness of the decisions made in designing various power equipment depends both on the depth and reliability of the knowledge of heat transfer phenomena and on the adequacy of modeling various thermophysical processes. In order to create effective methods of diagnosing and identifying such processes, experimental studies are carried out and study results are processed. These methods can be based on solutions to IHCPs for both homogeneous and composite media. Virtually, in some cases, methods for solving IHCPs are the only means of obtaining the necessary information

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on the object under study. In this work, the boundary IHCP is reduced to determining heat fluxes on the surface of a body from the temperature measurements at one or more internal points.

An IHCP can be formalized as follows:

$$A[f(T,M,\tau)] = T^{\exp},$$

where $f(T,M,\tau)$ are the desired dependencies, which, in the general case, may depend on the temperature T, spatial coordinates of the point M and time coordinate τ; $T^{\exp}$ is the temperature specified, which has the form $T^{\exp} = T(M,\tau)$ and in most cases is known from the experiment (initial data); A is the operator that associates the desired dependencies with the source data $T^{\exp}$. Such a problem, like any other IHCP, due to the cause-effect relationship violation, is ill-conditioned according to Hadamard, which can cause instability of the solution to be obtained.

To solve such an ill-conditioned problem, it is either reduced to a conditionally correct one, and no regularization is carried out; or it remains ill-conditioned, but one of the regularization methods is used [1–6]. If there is no thermophysical experiment, then $T^{\exp}$ are obtained from the corresponding direct problem solution with the addition of some random variable.

Problem Formulation

The thermal process in a two-layer plate with the real contact between the layers was described by the following system of equations:

$$C_1 \frac{\partial T}{\partial \tau} = \lambda_1 \frac{\partial^2 T}{\partial x^2},\ 0 < x < L_1,\ C_2 \frac{\partial T}{\partial \tau} = \lambda_2 \frac{\partial^2 T}{\partial x^2},\ L_1 < x < L,\ \ \tau > 0, \tag{1}$$

$$\lambda_1 \frac{\partial T}{\partial x} = 0, \quad x = 0, \quad \lambda_2 \frac{\partial T}{\partial x} = Q(\tau), \quad x = L,\ \tau > 0, \tag{2}$$

$$\lambda_1 \left.\frac{\partial T}{\partial x}\right|_{x=L_1-0} = \lambda_2 \left.\frac{\partial T}{\partial x}\right|_{x=L_1+0}, \quad \left.\left(\lambda_1 \frac{\partial T}{\partial x} + \frac{1}{R(\tau)} T\right)\right|_{x=L_1-0} = \left.\frac{1}{R(\tau)} T\right|_{x=L_1+0},\ \tau > 0, \tag{3}$$

$$T(x,0) = T_0, \tag{4}$$

$$T(x_j,\tau_k) = T_{j,k}^{\exp}, \quad \tau_k = k\Delta\tau,\ j = \overline{1,n},\ k = \overline{1,m}, \tag{5}$$

where $Q(\tau)$ is the required heat flux; x is the spatial coordinate; τ is the time; $\lambda_i, C_i,\ i = 1,2$ are the thermal conductivity and mass heat capacity for each layer, respectively; L is the thickness of the two-layer plate; L_1 is the thickness of the first layer; T is the temperature; T_0 is the initial temperature; $x_j, j = \overline{1,n}$ are the spatial coordinates of thermometer points; $R(\tau) > 0$ is the thermal contact resistance (with the ideal thermal contact $R = 0$); $\Delta\tau$ is the time interval between measurements; m is the number of measurements; n is the number of measurement points; $T_{j,k}^{\exp},\ j = \overline{1,n},\ k = \overline{1,m}$, are the temperatures obtained as a result of the thermophysical experiment with the error that is characterized by a random variable distributed according to the normal law with zero mathematical expectation and the dispersion σ^2.

Because of the absence of any thermophysical experiment, data (5) were obtained from the solution to the model direct problem (1)–(4) with the known heat flux $Q(\tau)$.

Regularization Algorithm for Solving IHCPs and the Influence Function Method

A. N. Tikhonov's regularization algorithm [5] for solving the linear IHCP (1)–(5) reduces to minimizing the functional

$$J = \sum_{i=1}^{n} \sum_{k=1}^{m} [T(x_i,\tau_k) - T_{i,k}^{\exp}]^2 + \alpha\Omega[Q], \tag{6}$$

where α is the regularization parameter; $\Omega[Q]$ is the stabilizing functional; $T(x_i,\tau_k)$ and $T_{i,k}^{\exp}$ are the temperature being simulated and the temperature from the hermophysical experiment at the thermometry points x_j at the time instances τ_k .

If the required function $Q(\tau)$ is represented as

$$Q(\tau)=\sum_{j=1}^{n_Q} q_{Q,j}\varphi_{Q,j}(\tau), \tag{7}$$

where $\varphi_{Q.j}(\tau)$, $j=\overline{1,n_Q}$ is some finite basis over the entire temperature measurement interval, and $q_{Q,j}$ are the desired coefficients, then, using the principle of superposition, the solution $T(x,\tau)$ can be written as

$$T(x,\tau)=\overline{T}(x,\tau)+\sum_{j=1}^{n_Q} q_{Q,j}T_j(x,\tau)\,, \tag{8}$$

where $\overline{T}(x,\tau)$ is the solution to the boundary value problem (1)–(4) with an inhomogeneous initial condition (4) and homogeneous boundary conditions (2), and $T_j(x,\tau)$ is the solution to the boundary value problem (1)–(4) with a homogeneous initial condition and the boundary conditions of the form

$$\lambda_2\frac{\partial T_j}{\partial x}=0,\quad x=0,\quad \lambda_2\frac{\partial T_j}{\partial x}=\varphi_{Q,j}(\tau),\quad x=L,\quad \tau>0.$$

Substituting expression (8) into functional (6) and taking as the stabilizing functional

$$\Omega[Q]=\omega_{Q,0}\int_0^{\tau_0}\left[\sum_{j=1}^{n_Q} q_{Q,j}\varphi_{Q,j}(\tau)\right]^2 d\tau+\omega_{Q,1}\int_0^{\tau_0}\left[\sum_{j=1}^{n_Q} q_{Q,j}\varphi'_{Q,j}(\tau)\right]^2 d\tau+\omega_{Q,2}\int_0^{\tau_0}\left[\sum_{j=1}^{n_Q} q_{Q,j}\varphi''_{Q,j}(\tau)\right]^2 d\tau\,, \tag{9}$$

by differentiating (9) with the required coefficients $q_{Q,j}$, we can obtain a system of linear equations

$$(\mathbf{A}+\alpha\mathbf{B})\mathbf{q}=\mathbf{C}, \tag{10}$$

where **A** is the symmetric matrix with the elements

$$a_{ij}=\sum_{l=1}^{n}\sum_{k=1}^{m} T_i(x_l,\tau_k)T_j(x_l,\tau_k); \tag{11}$$

B is the symmetric matrix with the elements

$$b_{ij}=\omega_{Q,0}\int_0^{\tau_0}\varphi_{Q,i}(\tau)\varphi_{Q,j}(\tau)\,d\tau+\omega_{Q,1}\int_0^{\tau_0}\varphi'_{Q,i}(\tau)\varphi'_{Q,j}(\tau)\,d\tau+\omega_{Q,2}\int_0^{\tau_0}\varphi''_{Q,i}(\tau)\varphi''_{Q,j}(\tau)\,d\tau\,;$$

C is the vector of the right-hand side of the system of linear equations (10)

$$c_i=\sum_{l=1}^{n}\sum_{k=1}^{m} T_i(x_l,\tau_k)\left(T_{l,k}^{\text{экс}}-\overline{T}(x_l,\tau_k)\right). \tag{12}$$

$\varphi_{Q,j}(\tau)$, $j=\overline{1,n_Q}$ are Schoenberg splines of the third degree $B_3(\tau)$.

The system of linear equations (10) includes the regularization parameter α , which is determined in the same way as in [1, 7÷10]. It is believed that the regularization parameter is chosen correctly, providing, for the solution obtained according to the iterative scheme proposed above, fulfilled is the following two-sided inequality:

$$\left(1-\sqrt{2/N}\right)\sigma^2\le\delta^2\le\left(1+\sqrt{2/N}\right)\sigma^2\,, \tag{13}$$

where N is the total number of thermometric measurements; δ^2 is the standard deviation of the model solution from the exact solution. The search algorithm for the regularization parameter α is based on some iterative process of finding the root of a nonlinear equation.

Figs. 1 and 2 show the temperature graphs at the point L_1 (contact of the plate layers) and at the boundary L with the ideal thermal contact between the layers, and Fig. 3, the identified heat flux at the boundary L.

The results are obtained for the following values of the dimensionless parameters of the problem: $m = 100$, $\Delta\tau = 0.01$, $n = 2$, $x_1 = L_1$, $x_2 = L$, $C_1 = 1$, $C_2 = 1$, $\lambda_1 = 1$, $\lambda_2 = 50$, $L = 1$, $L_1 = 0{,}5$, $T_0 = 1$, $n_Q = 23$, $\sigma = 0.1$, $\omega_{Q,0} = 1$, $\omega_{Q,1} = 0$, $\omega_{Q,2} = 10$.

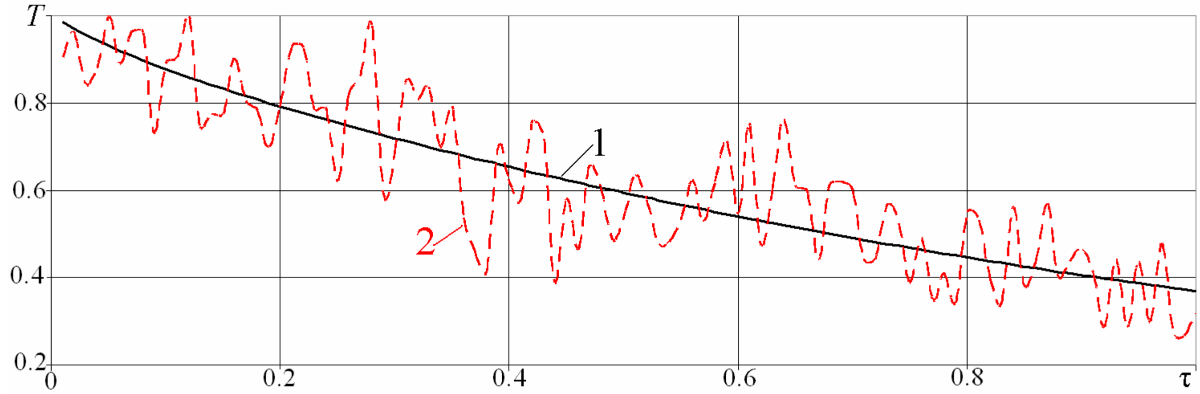

Fig. 1. Temperature at the point of the ideal thermal contact:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

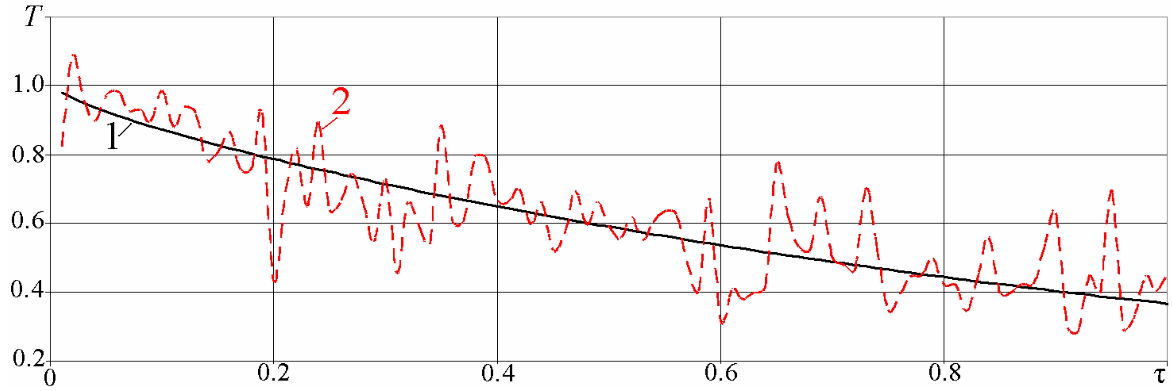

Fig. 2. Temperature at the boundary of the plate with the required heat flux:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

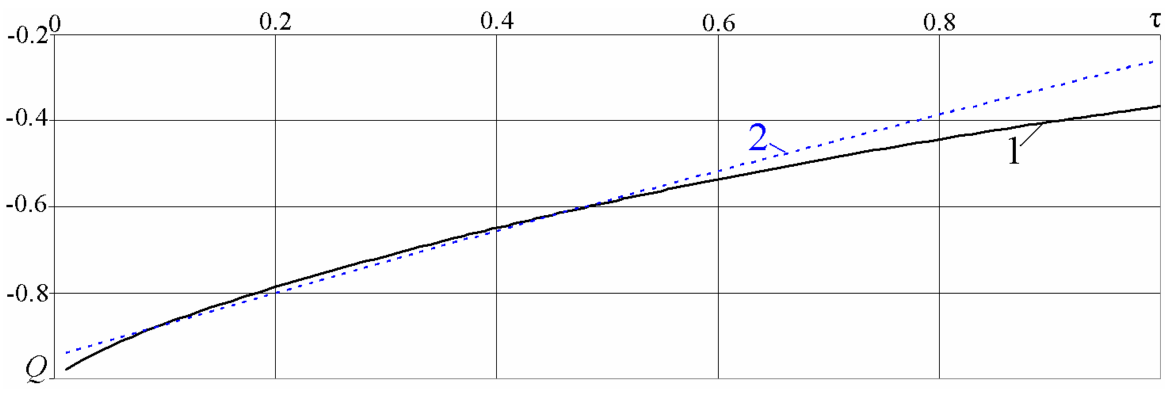

Fig. 3. Heat flux graphs in the problem with the ideal thermal contact:
1 – refers to the specified heat flux in solving the model problem; 2 – refers to the identified heat flux

For the thermal process with the real thermal contact, the temperature graphs both at the point of the real contact and at the plate boundary are shown in Figs. 4 and 5, and the identified heat flux obtained according to the above approach, at the boundary L, in Fig. 6.

The graphs presented in Fig. 6 are obtained with the same dimensionless parameters as for the ideal contact. In this case, thermometry was performed at space points $x_1 = L_1$ (the point in the first layer) and $x_2 = L$.

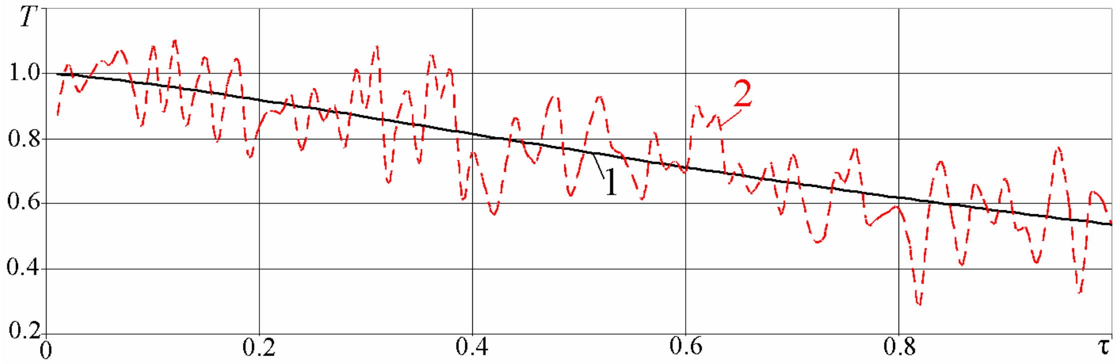

Fig. 4. Temperature at the point of the real thermal contact:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

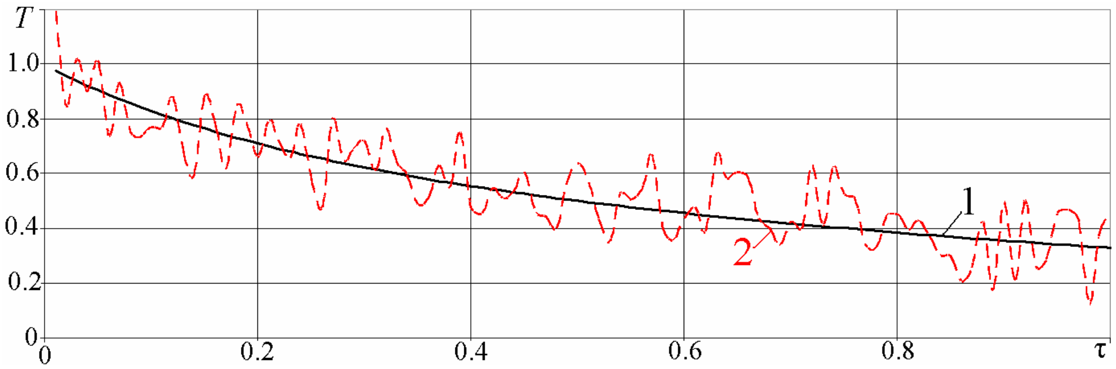

Fig. 5. Temperature at the boundary of the plate with the required heat flux in the problem with the real heat contact:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

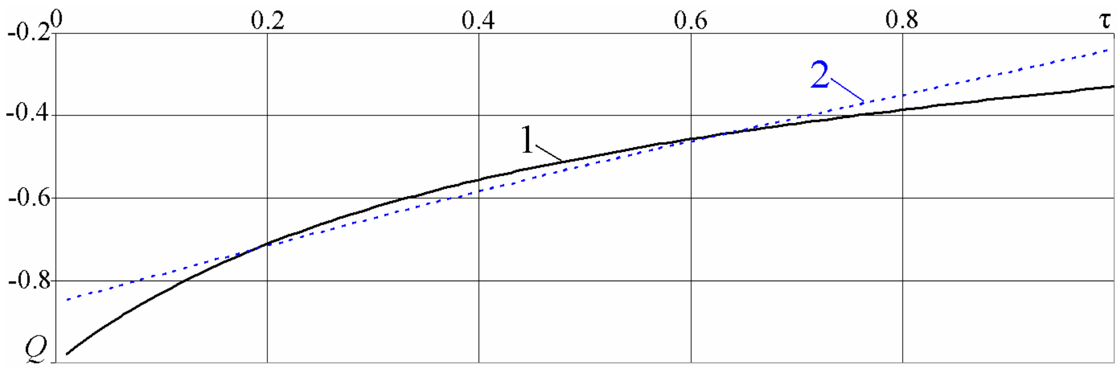

Fig. 6. Heat flux graphs at the plate boundary in the problem with the real heat contact:
1 – refers to the heat flux specified in solving the model problem; 2 – refers to the identified heat flux

The third IHCP was the inverse non-stationary boundary value heat conduction problem (1–5), in which, according to the data of the thermophysical experiment, it is necessary to determine both the heat flux $Q(\tau)$ and the thermal contact resistance $R(\tau)$. Since thermometry points are present in both layers, this problem was divided into two connected external inverse boundary value problems for the first and second layers.

The heat flux $Q(\tau)$ at the outer boundary (L) was presented in form (7), and the heat flux at the contact boundary $Q_R(\tau)$, in the form

$$\lambda_1 \frac{\partial T}{\partial x} = Q_R(\tau) = \sum_{j=1}^{n_R} q_{R,j} \varphi_{R,j}(\tau), \quad x = L_1,$$

where $\varphi_{R.j}(\tau)$, $j = \overline{1, n_R}$ is some finite basis over the entire temperature measurement interval, $q_{R,j}$ $j = \overline{1, n_R}$ are the required coefficients.

If the vector $\mathbf{q}$ of the dimension $n_Q + n_R$ is written in the form $\mathbf{q} = (q_{Q,1}, \ldots, q_{Q,n_Q}, q_{R,1}, \ldots, q_{R,n_R})$, then, due to the linearity of the problem, using the principle of superposition, the temperature field for the first layer can be represented as

$$T(x,\tau) = \overline{T}(x,\tau) + \sum_{j=n_Q+1}^{n_Q+n_R} q_j T_j(x,\tau),$$

where $\overline{T}(x,\tau)$ is the solution to the boundary value problem (1–4) with an inhomogeneous initial condition (4) and homogeneous boundary conditions, and $\{T_j(x,\tau)\}_{j=n_Q+1}^{j=n_Q+n_R}$ is the solution to the boundary value problem (1–4) with a homogeneous initial condition and an inhomogeneous boundary condition of the second kind

$$\lambda_1 \left.\frac{\partial T_j}{\partial x}\right|_{x=L_1-0} = \varphi_{R,j-n_Q}(\tau), \;\; \lambda_1 \left.\frac{\partial T_j}{\partial x}\right|_{x=0} = 0, \;\; j = \overline{n_Q+1, n_Q+n_R}, \quad \tau > 0.$$

The temperature field for the second layer can be written as follows:

$$T(x,\tau) = \overline{T}(x,\tau) + \sum_{j=1}^{n_Q+n_R} q_j T_j(x,\tau),$$

where $\overline{T}(x,\tau)$ is the same solution to the boundary value problem (1–4) as for the first layer, and $T_j(x,\tau)$, $j = \overline{1, n_Q+n_R}$ is the solution to the boundary value problem (1–4) with a homogeneous initial condition and inhomogeneous boundary conditions of the second kind

$$\lambda_2 \left.\frac{\partial T_j}{\partial x}\right|_{x=L} = \varphi_{Q,j}(\tau), \;\; \lambda_2 \left.\frac{\partial T_j}{\partial x}\right|_{x=L_1+0} = 0, \;\; j = \overline{1, n_Q}, \quad \tau > 0;$$

$$\lambda_2 \left.\frac{\partial T_j}{\partial x}\right|_{x=L_1+0} = \varphi_{R,j-n_Q}(\tau), \;\; \lambda_2 \left.\frac{\partial T_j}{\partial x}\right|_{x=L} = 0, \; j = \overline{n_Q+1, n_Q+n_R}, \quad \tau > 0.$$

Then, following the reasoning given above, it is possible to obtain a system of linear equations (10) with the $\mathbf{A}$ matrix elements in form (11), with the $\mathbf{C}$ vector elements in form (12) and with the $\mathbf{B}$ matrix elements in the form

$$b_{ij} = \omega_{Q,0} \int_0^{\tau_0} \varphi_{Q,i}(\tau) \varphi_{Q,j}(\tau) d\tau + \omega_{Q,1} \int_0^{\tau_0} \varphi'_{Q,i}(\tau) \varphi'_{Q,j}(\tau) d\tau + \omega_{Q,2} \int_0^{\tau_0} \varphi''_{Q,i}(\tau) \varphi''_{Q,j}(\tau) d\tau, \quad i, j = \overline{1, n_Q},$$

$$b_{ij} = \omega_{R,0} \int_0^{\tau_0} \varphi_{R,i}(\tau) \varphi_{R,j}(\tau) d\tau + \omega_{R,1} \int_0^{\tau_0} \varphi'_{R,i}(\tau) \varphi'_{R,j}(\tau) d\tau + \omega_{R,2} \int_0^{\tau_0} \varphi''_{R,i}(\tau) \varphi''_{R,j}(\tau) d\tau, \; i, j = \overline{n_Q+1, n_Q+n_R}.$$

By selecting the regularization parameter α so that as a result of solving the system of linear equations (10), condition (13) is satisfied, it is possible to determine the thermal contact resistance at this boundary from the restored heat flux and temperatures at the contact boundary (Fig. 7).

Figs. 7–9 show temperature graphs on the left and right of the contact boundary as well as on the outer plate boundary.

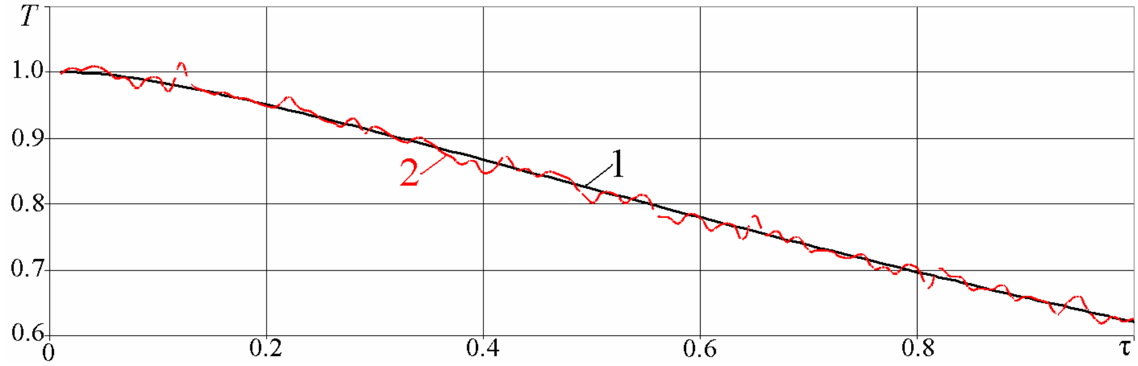

Fig. 7. Temperature at the contact boundary in the first medium:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

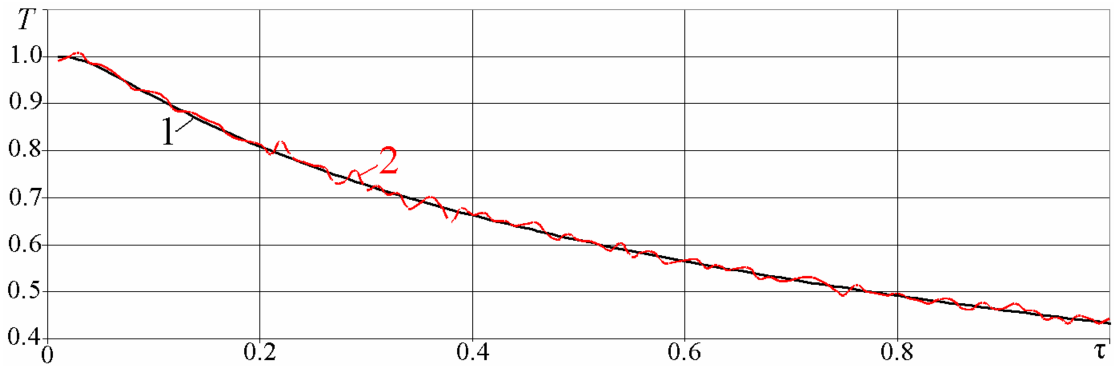

Fig. 8. Temperature at the contact boundary in the second medium:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

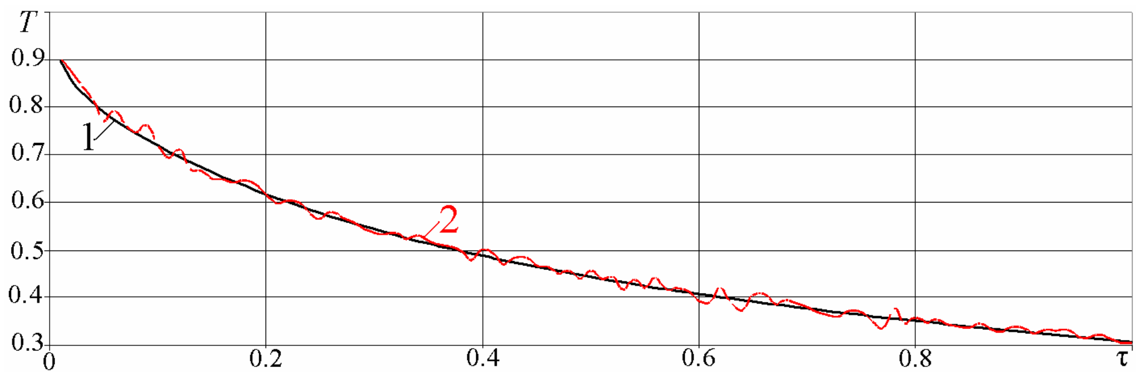

Fig. 9. Temperature at the boundary of the plate with the required heat flux:
1 – is obtained from the solution to the model problem; 2 – is the result of the thermophysical experiment with noise

The identified heat fluxes are given at the outer plate boundary (Fig. 10) and at the boundary of the thermal contact (Fig. 11); and the reciprocal of the thermal contact resistance is shown in Fig. 12.

The results are obtained for the following values of dimensionless parameters $n=3,\ x_1=L_1-0,\ x_2=L_1+0,\ x_3=L,\ C_1=1,\ C_2=1,\ \lambda_1=1,\ \lambda_2=2,\ L=1,\ L_1=0.5,\ T_0=1, n_Q=23,$ $n_R=23,\ m=100,\ \sigma=0.01, \omega_{Q,0}=1,\ \omega_{Q,1}=0,\ \omega_{Q,2}=10,\ \omega_{R,0}=1,\ \omega_{R,1}=0,\ \omega_{R,2}=10,\ \Delta\tau=0.01,\ R=1$.

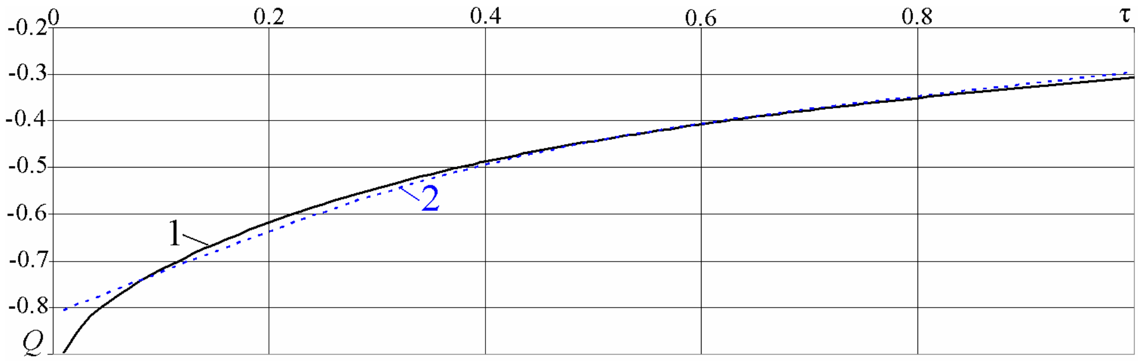

Fig. 10. Heat flux graphs at the plate boundary:
1 – refers to the heat flux specified in solving the model problem; 2 – is identified

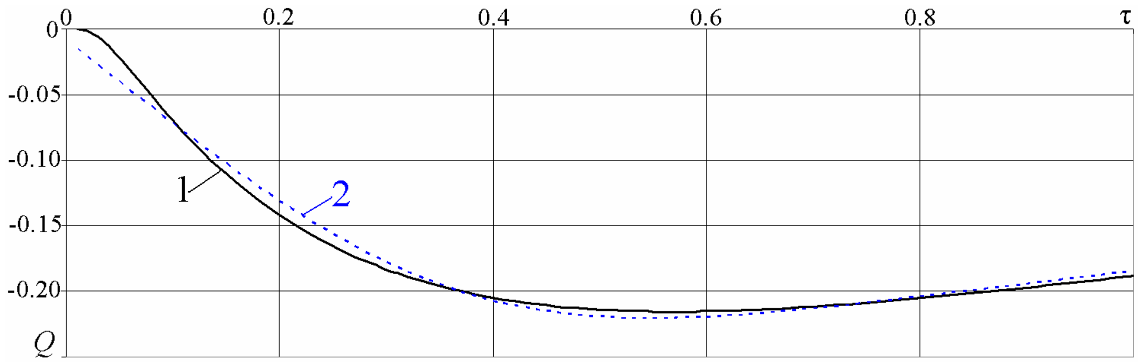

Fig. 11. Heat flow graphs at the contact boundary:
1 – refers to the heat flux specified in solving the model problem; 2 – is identified

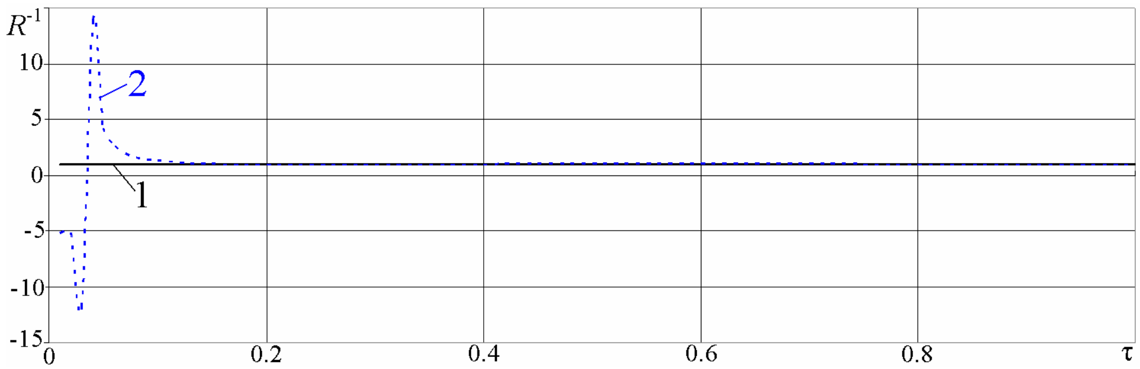

Fig. 12. Graphs of the thermal contact resistance reciprocal:
1 – refers to the resistance specified in solving the model problem; 2 – refers to the resistance identified

Conclusions

The described approach for the joint use of Tikhonov's regularization algorithm with the influence function method allows us to identify complex dependencies of heat fluxes with a certain error in the results of a thermophysical experiment. Its advantages include: weak sensitivity to measurement errors; possibility of using experimental information from one or several sensors; applicability for heterogeneous environments; possibility of simultaneous restoration of the heat flux on different parts of the surface of a structural element; simplicity of programming and the ability to parallelize the computing process, which meets modern requirements for methods and algorithms for solving direct and inverse problems.

The graphs presented in the article demonstrate stable solutions to the inverse problems of thermal conductivity for heterogeneous layers with both the ideal and real thermal contacts. The solid lines in Fig. 1, 2, 4, 5, 7, 8, and 9 represent both the temperature obtained by solving the model direct problem and the temperature identified by solving the inverse problem, since these two curves practically coincide in the graphs.

Analyzing the deviation of the identified temperature from the temperature obtained by solving the direct problem, we can conclude that they are in good agreement. As for the heat fluxes, the error of their identification is noticeable, and more significant at the ends of the time interval.

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Розв'язання нестаціонарних обернених задач теплопровідності для багатошарових тіл на основі ефективного пошуку регуляризуючого параметра

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У статті для отримання стійкого розв'язання оберненої задачі теплопровідності (ОЗТ) застосовується метод регуляризації А. М. Тихонова з ефективним алгоритмом пошуку регуляризуючого параметра. Шукані теп-

ловий потік на границі та термічний контактний опір за часовою координатою апроксимуються сплайнами Шьонберга третього ступеня. Як стабілізуючий функціонал використовується сума квадратів шуканої величини, її першої та другої похідних. Як об'єкт дослідження розглядаються багатошарові пластини або оболонки, до яких можна віднести і корпус твердопаливних ракетних двигунів. У першому наближенні задача розглядається в одновимірній нестаціонарній лінійній постановці. Співвідношення товщини оболонки до її радіуса будемо вважати таким, що в рівнянні теплопровідності кривизною оболонки можна знехтувати і розглядати її як плоску пластину. Таке припущення вибрано для спрощення викладення матеріалу і не обмежує застосовності викладеної методики в разі осьової симетрії оболонки, а також під час перекладу математичної моделі з прямокутної в циліндричну систему координат. Розглядаються три обернені задачі. У перших двох визначаються теплові потоки в складеному тілі з ідеальним і реальним тепловим контактом. У третій ОЗТ за реального теплового контакту визначається термічний контактний опір. Теплові потоки в багатошарових тілах розглядаються у вигляді лінійних комбінацій сплайнів Шьонберга третього ступеня з невідомими коефіцієнтами, які обчислюються шляхом розв'язання системи лінійних алгебраїчних рівнянь. Ця система є наслідком необхідної умови мінімуму функціонала, в основу якого покладено принцип найменших квадратів відхилення модельованої температури від температури, отриманої в результаті теплофізичного експерименту. Для регуляризації розв'язків ОЗТ використовується стабілізуючий функціонал з параметром регуляризації як мультиплікативним множником. Він являє собою суму квадратів теплових потоків, їх перших і других похідних з відповідними множниками. Ці множники вибираються згідно із заздалегідь відомими властивостями шуканого розв'язку. Пошук регуляризуючого параметра здійснюється за допомогою алгоритму, аналогічного алгоритму пошуку кореня нелінійного рівняння.

Ключові слова: *обернена задача теплопровідності, тепловий потік, термічний контактний опір, метод регуляризації А. М. Тихонова, функціонал, стабілізатор, параметр регуляризації, ідентифікація, апроксимація, сплайн Шьонберга третього ступеня.*

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