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OPTIMAL DESIGN OF BENDING ELEMENTS IN CONDITIONS OF CORROSION AND MATERIAL DAMAGE

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During operation, many of the critical elements of building and engineering structures are in difficult operating conditions (high temperature, aggressive environment, etc.). In this case, they may be subject to a double effect: corrosion and material damage. Corrosion leads to a reduction in the cross-section of a structure, resulting in stress increase therein. In turn, the damage to the material is accompanied by the appearance of microcracks and voids therein due to inelastic deformation (creep), which leads to a deterioration of physical characteristics of the material (for example, elastic modulus) and a sharp decrease in the stress values at which the structure is destroyed. This paper considers the optimization of bending rectangular cross-section elements operated in conditions conducive to the appearance of both corrosion and material damage. As the equation of corrosion, the model of V. M. Dolinsky is taken. This model takes into account the effect of stresses on the corrosion wear of structures. As a kinetic equation describing the change in material damage, the model of Yu. N. Rabotnov is used. The optimality criterion is the minimum mass of the structure. The height of the rectangular cross-section bending element along its length is optimized using the principle of equal damage at the final moment of the lifetime of the structure. The proposed approach can be used to solve similar problems of the optimal design of structures operating in conditions of corrosion and material damage with the use of both analytical solutions and numerical methods.

Keywords: *corrosion, material damage, optimization.*

Introduction

Structures operated in certain conditions (high temperature, aggressive environment, etc.) may be subject to a double effect: corrosion and material damage. The first factor leads to a decrease in the cross-section of a structure and, as a result, to an increase in the stresses of the structure. As for material damage, the appearance of microcracks and voids in the material, as a result of inelastic deformation (creep), leads to a deterioration of physical characteristics (for example, elastic modulus) and a sharp decrease in the stress values at which the structure is destroyed. To take into account damage, L. M. Kachanov [1, 2] proposes a kinetic model of change in material damage, with the model characterized by the parameter of continuity varying from 1 in the initial state to 0 at the moment of destruction. Yu. N. Rabotnov [3] uses a similar equation for the kinetics of material damage, in which the damage value $\overline{\omega}$, varying from 0 to 1, is adopted as a variable parameter. Other modifications of this model are made in the works of J. Lemetri and Ya. L. Cheboshi [4, 5]. Using the principle of "separability" and introducing the normalized time parameter, depending on stress, the above models (in the case of one-dimensional tensile stresses) are modernized by V. P. Golub [6]. A new approach to determining structural damage is illustrated by the example of static and cyclic loads. The original damage model is also proposed by L. A. Sosnovsky and S. S. Shcherbakov [7]. Studies in this area are reviewed in [8, 9].

Problems of optimizing structures used in conditions of material damage are outlined by A. G. Kostyuk [10−14].

Optimization of structural elements in corrosion conditions is considered in [15–19].

This article studies the field of the optimal design of structures, taking into account two factors: corrosion and material damage, with bending elements of a rectangular cross-section used as an example.

Problem Formulation

Consider the optimization of rectangular cross-section bending elements operated in conditions of corrosion and material damage. The optimality criterion is the minimum mass of the structure. The height of the rectangular cross-section bending element is optimized along its length, using the principle of equal damage at the final moment of the lifetime of the structure. The meaning of the principle will be explained below.

As an equation of corrosion, the model of V. M. Dolinsky [20] is taken, with the model taking into account the effect of stresses on the corrosion wear of structures (Fig. 1)

$$\frac{dH}{dt} = -2\left(\alpha + \beta\left|\sigma_{\max}\right|\right), \qquad (1)$$

where α and β are constant coefficients; H_0 and H are the initial and current heights of the rectangular cross-section; $\sigma_{\max}$ are the maximum stresses in the current cross-section.

It is assumed that the upper and lower edges of the cross-section are subject to corrosion, and to the same degree (by taking the maximum stresses as positive in modulus), as indicated by coefficient 2 in (1).

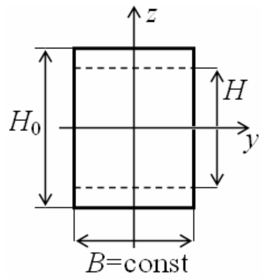

Fig. 1. Current cross-section of a rectangular bending element

As a kinetic equation describing the change (from 0 to 1) of the material damage parameter $\overline{\omega}$, the model of Yu. N. Rabotnov is adopted [3]

$$\frac{d\overline{\omega}}{dt} = a_k \left(\frac{\sigma_{\max}}{1-\overline{\omega}}\right)^{b_k}, \qquad (2)$$

where a_k и b_k are constants.

Since material damage usually occurs at high temperatures, when determining stresses in a beam, we will take into account the creep effect. We assume that the strain rate depends on stress as a power function, i.e. we adopt the creep law in the form [3]

$$\dot{\varepsilon} = A_1 \sigma^n, \qquad (3)$$

where A_1 and n are constant at a given temperature value.

For n=1, we obtain the stress distribution in the elastic element, and for $n \to \infty$, in the element of an ideally plastic material. In practice, the value of n usually does not exceed 12.

Taking into account the hypothesis of flat sections $\dot{\varepsilon} = \dot{x} z$, from (3) we find

$$\sigma = k\dot{\varepsilon}^{1/n} = k\left(\dot{x}z\right)^{1/n}, \qquad (4)$$

where $k = 1/A_1^n$; $\dot{x}$ is the rate of change of the neutral layer curvature.

Assuming the width of the rectangular cross-section is constant along the length of the beam and equal to B, we write the expression for the bending moment in the cross-section x at an arbitrary time t

$$M(x) = \int \sigma z dA = k\left(\dot{x}\right)^{1/n} * 2B \int_0^{H/2} z^{1+1/n} dz = \frac{k\left(\dot{x}\right)^{1/n} BH^{2+1/n}}{2^{1+1/n}\left(2+1/n\right)}.$$

From this $k\left(\dot{x}\right)^{1/n} = \dfrac{M\, 2^{1+1/n}\left(2+1/n\right)}{BH^{2+1/n}}$.

Substituting this expression into (4), with $z=H/2$, we have

$$\sigma_{\max} = \frac{Mm}{BH^2}, \qquad (5)$$

where $m=2(2+1/n)$.

In this case, with (5) taken into account, equations (1) and (2) have the forms

$$\frac{dH}{dt} = -2\left(\alpha + \frac{\beta Mm}{BH^2}\right), \qquad (6)$$

$$\frac{d\overline{\omega}}{dt} = a_k \left(\frac{\frac{Mm}{BH^2}}{1-\overline{\omega}}\right)^{b_k}. \qquad (7)$$

Solving Corrosion and Material Damage Equations

When comparing equations (6) and (7), one can conclude that the corrosion process does not depend on material damage, while $\overline{\varpi}(t)$ does depend on corrosion. Therefore, the corrosion equation can be considered separately. When solving it, the effect of the stress state on the corrosion kinetics is taken into account.

We separate the variables in equation (6) and integrate both sides

$$\int_{H_0}^{H_T} \frac{dH}{1+\frac{\beta Mm}{BH^2\alpha}} = -2\alpha\int_0^T dt,$$

where T is the lifetime of the structure.

After the integration, we have

$$2\alpha T = H_0 - H_T + \sqrt{\frac{\beta Mm}{B\alpha}}\left(\operatorname{arctg} H_T\sqrt{\frac{B\alpha}{\beta mM}} - \operatorname{arctg} H_0\sqrt{\frac{B\alpha}{\beta Mm}}\right). \tag{8}$$

To solve equation (7), we divide the variables and, substituting the expression dt from equation (6), we integrate the two sides

$$\int_0^1 (1-\varpi)^{b_k} d\varpi = -\frac{a_k}{2\alpha}\left(\frac{Mm}{B}\right)^{b_k}\int_{H_0}^{H_T} \frac{dH_1}{H_1^{2b_k-2}\left(H_1^2+\frac{\beta Mm}{B\alpha}\right)}.$$

Taking b_k>1.5 (which is most often encountered in practice), after the integration, we have

$$\frac{1}{b_k+1} = \frac{a_k}{2\beta}\left(\frac{Mm}{B}\right)^{b_k-1}\left[\frac{1}{2b_k-3}\left(\frac{1}{H_T^{2b_k-3}} - \frac{1}{H_0^{2b_k-3}}\right) + \int_{H_0}^{H_T} \frac{dH_1}{H_1^{2b_k-4}\left(H_1^2+\frac{\beta Mm}{B\alpha}\right)}\right]. \tag{9}$$

Optimization Problem

As a result, we obtain two equations, (8) and (9), with two unknowns: H and H_0. Solving them together for each fixed value of x, from the principle of equal damage at the final moment of the lifetime of the structure (when $\overline{\varpi}=1$), we obtain the optimal distribution of bending element heights $H_0(x)$ along its length, with the distribution giving the minimum of the structure mass. The last statement is valid due to the dependence

$$V = \int_o^L A(x)dx = 2B\int_o^L H_0(x)dx.$$

Special Cases of the Solution

The obtained integral expression of material damage (9) can be simplified in two particular cases. Thus, with b_k=2, we have

$$\frac{2\beta B}{3a_k Mm} = \frac{1}{H_T} - \frac{1}{H_0} + \sqrt{\frac{B\alpha}{\beta Mm}}\left(\operatorname{arctg} H_T\sqrt{\frac{B\alpha}{\beta Mm}} - \operatorname{arctg} H_0\sqrt{\frac{B\alpha}{\beta Mm}}\right). \tag{10}$$

When b_k=3, we get

$$\frac{\beta}{2a_k}\left(\frac{B}{mM}\right)^2 = \frac{1}{3H_T^3} - \frac{1}{3H_0^3} + \frac{B\alpha}{\beta Mm}\left(\frac{1}{H_T} - \frac{1}{H_0}\right) + \left(\frac{B\alpha}{\beta Mm}\right)^{3/2}\left(\operatorname{arctg} H_0\sqrt{\frac{B\alpha}{\beta Mm}} - \operatorname{arctg} H_T\sqrt{\frac{B\alpha}{\beta Mm}}\right) \tag{11}$$

As an example, consider the optimization of a cantilever beam with a force F at the end (Fig. 2). In this case, $M=Fx$.

Turning to dimensionless quantities and denoting $T_* = 2\alpha T\sqrt{\frac{B\alpha}{\beta mFL}}$, $\xi = x/L$, $\chi_0 = H_0\sqrt{\frac{B\alpha}{\beta mFL}}$,

$\chi_T = H_T\sqrt{\frac{B\alpha}{\beta mFL}}$, $D = \frac{2\beta}{3a_k}\sqrt{\frac{B\beta}{mFL\alpha}}$, $D_1 = \frac{\beta}{2a_k}\sqrt{\frac{B\beta^3}{mFL\alpha^3}}$, we have the following transformations of equations (8), (10) and (11):

$$T_* = \chi_0 - \chi_T + \sqrt{\xi}\,\mathrm{arctg}\frac{(\chi_T - \chi_0)\sqrt{\xi}}{\xi + \chi_T\chi_0} \quad (12)$$

$$D = \frac{\xi}{\chi_T} - \frac{\xi}{\chi_0} + \sqrt{\xi}\,\mathrm{arctg}\frac{(\chi_T - \chi_0)\sqrt{\xi}}{\xi + \chi_T\chi_0}. \quad (13)$$

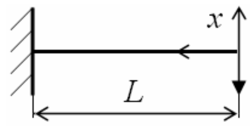

Fig. 2. Design scheme of the beam

$$D_1 = \frac{\xi^2}{3}\left(\frac{1}{\chi_T^3} - \frac{1}{\chi_0^3}\right) + \xi\left(\frac{1}{\chi_0} - \frac{1}{\chi_T}\right) - \sqrt{\xi}\,\mathrm{arctg}\frac{(\chi_T - \chi_0)\sqrt{\xi}}{\xi + \chi_T\chi_0}. \quad (14)$$

We first focus on the solution to the problem in the case of b_k=2. By subtracting equation (13) from (12), we get

$$T_* - D = \chi_0 - \chi_T + \frac{\xi}{\chi_0} - \frac{\xi}{\chi_T}.$$

From this

$$\chi_T = A_2 - \sqrt{A_2^2 - \xi},$$

where

$$A_2 = \left(\chi_0 + \frac{\xi}{\chi_0} + D - T_*\right)/2 \quad (15)$$

By substituting expression (15), say, into (12), we have a transcendental equation with one unknown χ_o. By setting 0≤ξ≤1, its solution can be found, for example, using one of the efficient algorithms of the random search method [21].

In the case of b_k=3, by adding (12) and (14), we get

$$T_* + D = \chi_0 - \chi_T + \frac{\xi^2}{3}\left(\frac{1}{\chi_T^3} - \frac{1}{\chi_0^3}\right) + \xi\left(\frac{1}{\chi_0} - \frac{1}{\chi_T}\right).$$

By writing this expression relative to χ_T, we have the following equation:

$$\chi_T^4 + \left(T_* + D_1 - \chi_0 + \frac{\xi^2}{3\chi_0^3} - \frac{\xi}{\chi_0}\right)\chi_T^3 + \xi\chi_T^2 - \frac{\xi^2}{3} = 0.$$

To solve it, we apply the Ferrari method. Then $\chi_T = y - b$, where $b = \left(T_* + D_1 - \xi/\chi_0 + \xi^2/3\chi_0^3\right)/4$; $y = \sqrt{t/2} - \sqrt{-p - t/2 - q/\sqrt{2t}}$; $p = 3c - 3b^2$; $q = 4b^3 - 6cb$; $c = \xi/6$; $t = z_1 - r_1/3$; $z_1 = \sqrt[3]{-q_1/2 + \sqrt{D_*}} + \sqrt[3]{-q_1/2 - \sqrt{D_*}}$; $D_* = (p_1/3)^3 + (q_1/2)^2$; $p_1 = \left(3s_1 - r_1^2\right)/3$; $r_1 = 6c - 6b^2$; $q_1 = 2r_1^3/27 - r_1 s_1/3 + t_1$; $t_1 = -q^2$; $s_1 = p^2 - r$; $r = 6cb^2 + e - 3b^4$; $e = -\xi^2/3$.

After the dependence between χ_0 and χ_T, is found, by setting 0≤ξ≤1, equation (12) can be solved, as in the previous case, using the random search method. As a result, in both cases (with b_k=2 and b_k=3), we have the optimal height shape of a minimum-mass cantilever beam.

Numerical Results

As a numerical illustration, consider the following calculation options: with b_k=2: T^*_a=1, D_a=0.5 and T^*_b=0.8, D_b=0.4; with b_k=3: T^*_c =1, D_{1c}=0.375 and T^*_d=0.8, D_{1d}=0.3. Conventionally, we assume that variants *b* and *d* correspond to the elastic element (*n*=1, *m*=6), and variants *a* and *c* correspond to the element of the ideally plastic material ($n \to \infty$; *m*=4). Therefore, $T^*_a/T^*_b = D_a/D_b = T^*_c/T^*_d = D_{1c}/D_{1d}$. The choice of the coefficients D_1 in relation to the coefficients D in the respective variants can be carried out as follows. Since D_1/D=3/4·β/α, then, taking β/α=1, we have D_1=0.5D.

The optimal outlines of the cantilever beam height $\chi_o(\xi)$ and its view at the moment of destruction (the final moment of the lifetime of the structure) $\chi_T(\xi)$ for all the variants are shown in Fig. 3.

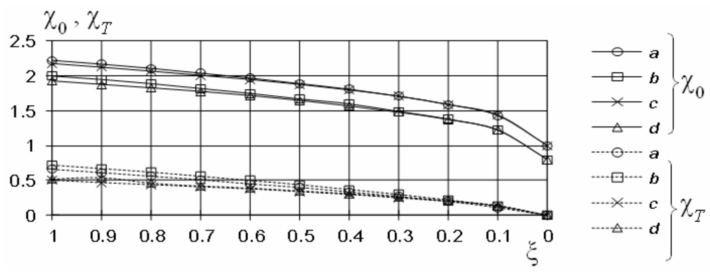

Fig. 3. Optimal shapes of the beam height and the height view at the moment of destruction

Conclusions

The problem of the optimal design of rectangular cross-section bending elements operated in conditions of corrosion and material damage is set and solved.

As for the obtained results, then, as can be seen from Fig. 3, in all the optimal projects $\chi_T(0)=0$, a $\chi_o(0)=T^*$. The first equality indicates that there is no waste of material in the cross-section with zero stress. In turn, the inequality to zero $\chi_o(0)$ can be explained by the fact that corrosion also acts in a non-stressed section, and its value follows directly from equation (12) at $\chi_T=0$ and $\xi\to 0$.

By comparing optimal projects *a* and *c*, respectively, with projects *b* and *d*, one can come to the conclusion that taking creep into account gives an increase in the initial mass of cantilever beams by an average of 15%. At the same time, the magnitude of the stress, at which failure occurs, also increases as shown by the parameter $\chi_T(\xi)$.

A change in the value of b_k (with equal values of T^*) slightly affects the optimal shape $\chi_o(\xi)$. However, it significantly affects the magnitude of the stress at the moment of destruction, and the higher b_k, the higher the stress.

The proposed approach can be used to solve similar problems of the optimal design of structures operating in conditions of corrosion and material damage with the use of both analytical solutions and numerical methods.

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Оптимальне проектування елементів, що згинаються, в умовах корозії й пошкоджуваності матеріалу

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Багато відповідальних елементів будівельних і машинобудівних конструкцій під час своєї експлуатації перебувають в складних умовах роботи (висока температура, агресивне середовище тощо). У цьому випадку вони можуть бути схильні до подвійного ефекту: корозії і пошкодження матеріалу. Корозія призводить до зменшення перерізу конструкції, через що в ній збільшуються напруження. У свою чергу, пошкодженість матеріалу супроводжується появою в ньому мікротріщин і порожнеч, в результаті непружної деформації (повзучості), що призводить до погіршення його фізичних характеристик (наприклад, модуля пружності) і різкого зниження величин напружень, за яких відбувається руйнування конструкції. У цій роботі розглядається оптимізація елементів прямокутного перерізу. що згинаються та експлуатуються в умовах, які сприяють появі як корозії, так і пошкодженню матеріалу. Як рівняння корозії приймається модель В. М. Долинського, що враховує вплив напружень на корозійний знос конструкцій. Як кінетичне рівняння, що описує зміну пошкодження матеріалу, використовується модель Ю. М. Работнова. Критерієм оптимальності служить мінімум маси конструкції. Оптимізується висота згинального прямокутного елемента за його довжиною з використанням принципу рівнопошкоджуваності в кінцевий момент життя конструкції. Запропонований в роботі підхід може бути використаний під час розв'язання аналогічних задач оптимального проектування конструкцій, що працюють в умовах корозії і пошкодження матеріалу, з використанням як аналітичних розв'язків, так і числових методів.

***Ключові слова**: корозія, антикорозійні покриття, оптимізація.*

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