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R-FUNCTIONS IN THE ANALYTICAL DESCRIPTION OF THE SURFACE OF A FLYING WING UNMANNED AERIAL VEHICLE

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Unmanned aerial vehicles (UAVs) are becoming increasingly demanded worldwide. The scope of their use is very extensive. They are used for military purposes, delivery of goods, environmental monitoring, border patrolling, aerial reconnaissance and mapping, traffic control, etc. UAVs have a number of important advantages over manned aircraft. These advantages include relatively low costs of UAVs at their long flight durations and ranges, their low operating costs, and the ability to perform maneuvers with overloads that exceed the physical capabilities of a human being, making their development more active. One cannot imagine the designing of UAVs and control systems without mathematical modeling. To build mathematical models, high-speed computers and modern software tools have been created, Solid Works, Ansys CFX, POLYE software systems being among them. There arises a problem of specifying and quickly changing geometric information to create a mathematical and computer model of the UAV being designed. At the design stage, there can be solved a lot of tasks that are put before researchers as regards the use of UAVs. At the same time, insufficient attention is paid to the parametric representation of aircraft surfaces. Expanding the scope of using the apparatus of the theory of R-functions for modeling UAV surfaces is an urgent scientific and technical task. In this paper, for the first time, using the theory of R-functions, we build up the equation of the surface of a flying wing UAV in the form of a single analytical expression with alphabetic parameters. This equation can be used in solving various practical problems as well as developing and manufacturing the product itself, for example, on a 3D printer. The proposed method for specifying the shapes of products by using a limited number of parameters can significantly reduce the complexity of work in CAD systems in cases where it is required to view a large number of design options in search of an optimal solution. In this paper, we build a 14-parameter family of flying wing UAV surfaces. By changing the values of alphabetic parameters, we can quickly explore its various forms.

Keywords: *unmanned aerial vehicle, R-functions, alphabetic parameters, standard primitives.*

Introduction

Among the advantages of unmanned aerial vehicles (UAVs), commonly known as drones, one should note their relatively low costs at long flight durations and ranges, low operating costs, and the ability to perform maneuvers with overloads that exceed the physical capabilities of a human being. UAVs have a lot of positive qualities. UAV design characteristics are different, which affects the scope of their application. Unlike light ones, heavy UAVs are mainly involved in military actions, carrying out reconnaissance or destroying specific targets. According to estimates of most Western experts, in future wars and conflicts, the United States and other NATO countries will rely on the use of UAVs, which are capable of delivering goods, monitoring environment and forest fires, patrolling borders, preventing drugs from being trafficked, carrying out aerial reconnaissance and mapping, controlling traffic, etc. [1–4]. In the Chinese province of Heilongjiang, UAVs are used for training Amur tigers who, while hunting for UAVs, maintain their physical form. A significant advantage of drones is their mobility and transport accessibility – they will reach those land areas that are very difficult to reach by land or air. The speed of cargo delivery is another good argument for using drones. A drone can reach a remote land area in 30 minutes, and a helicopter, in 2 hours. For manned aircraft, it is important to have a long runway, while drones can land on 500 to 600 meter long landing strips, and miniature models can easily land even on a step near the threshold of a house. UAVs economically consume fuel due to their compact dimensions, which is also an advantage.

Thus, expanding the scope of applying the apparatus of the theory of R-functions for modeling UAV surfaces is an urgent scientific and technical task. Based on the theory of R-functions, new approaches to the analytical identification of UAV surfaces have been developed. Both well-known methods of standard primitives and various options for frame blending are used. Multi-parameter equations of UAV surfaces of various shapes and purposes have been constructed and visualized [5].

The goal of this paper is to create a mathematical and computer model of the surface of a flying wing UAV on the basis of the theory of R-functions.

Main Part

In this paper, both the R-operations $fk \wedge_0 fl = fk + fl - \sqrt{fk^2 + fl^2}$; $fk \vee_0 fl = fk + fl + \sqrt{fk^2 + fl^2}$ and standard primitives [6, 7] are used. The equation of a conical surface with a vertex at a point $A(x_0, y_0, z_0)$, for which the guide is described by the equation $\omega(x, y) = 0$, is obtained by replacing the variables $\begin{cases} x \Leftarrow x_0 - z_0 \dfrac{x - x_0}{z - z_0} \\ y \Leftarrow y_0 - z_0 \dfrac{y - y_0}{z - z_0} \end{cases}$ in the equation $\omega(x, y) = 0$, namely $\omega\left(x_0 - z_0 \dfrac{x - x_0}{z - z_0}, y_0 - z_0 \dfrac{y - y_0}{z - z_0}\right) = 0$.

In addition, to construct an object of a given thickness δ, its normalized equation $\omega(x, y, z) \geq 0, \left.\dfrac{\partial \omega}{\partial n}\right|_{\partial\Omega} = 1$ was used. It is well known that the function $\omega(x, y, z)$ in the vicinity of the boundary $\partial\Omega$ behaves like the distance to $\partial\Omega$. Then the function $\omega 1(x, y, z) = \dfrac{\delta}{2} - |\omega(x, y, z)| \geq 0$ will be positive only in the band width δ [5, 8]. The use of this approach in some cases can significantly reduce the number of R-operations, and, consequently, the computational process.

By using the planes $f1, f2, f3, fc1, fc2, fc3, fc4$, R-operations and by replacing the variables $x_1 \Leftarrow x, y_1 \Leftarrow y$, we construct the flying wing base equation with the four alphabetic parameters: α – the cone angle, (x_0, y_0, z_0) – the coordinates of the cone point

$$f1 = y_1 - x_1 tg\alpha \geq 0;\ \ f2 = y_1 + x_1 tg\alpha \geq 0;\ \ f3 = -y_1 + \frac{8.8}{tg\dfrac{\pi}{5}} \geq 0;\ \ f = (f1 \wedge_0 f2) \wedge_0 f3 \geq 0;$$

$$fc1 = y_1 - 9 - (x_1 - 2.5) \geq 0;\ \ fc2 = y_1 - 9 + (x_1 - 2.5) \geq 0;$$

$$fc3 = y_1 - 9 - (x_1 + 2.5) \geq 0;\ \ fc4 = y_1 - 9 + (x_1 + 2.5) \geq 0;$$

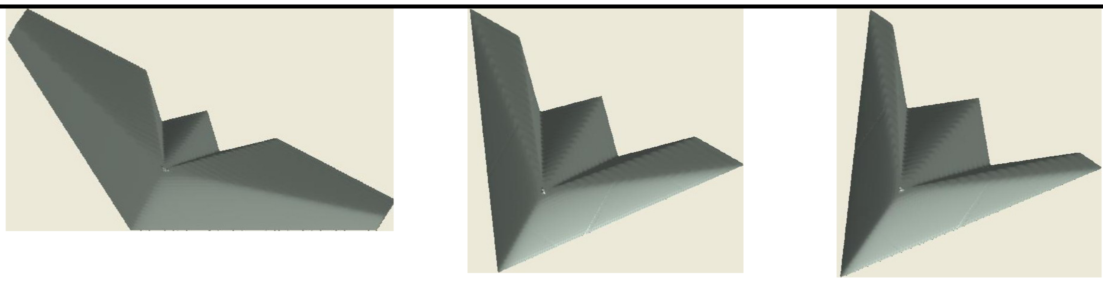
$$x_1 = x_0 - z_0 \frac{x - x_0}{z - z_0};\ \ y_1 = y_0 - z_0 \frac{y - y_0}{z - z_0};$$

$$fcc1 = fc1 \wedge_0 fc2 \geq 0;\ \ fcc2 = fc3 \wedge_0 fc4 \geq 0;\ \ fcc = \overline{fcc1} \wedge_0 \overline{fcc2} \geq 0$$

$$fff = f \wedge_0 fcc \geq 0;\ \ WK = fff \wedge_0 z(1.5 - z) \geq 0.$$

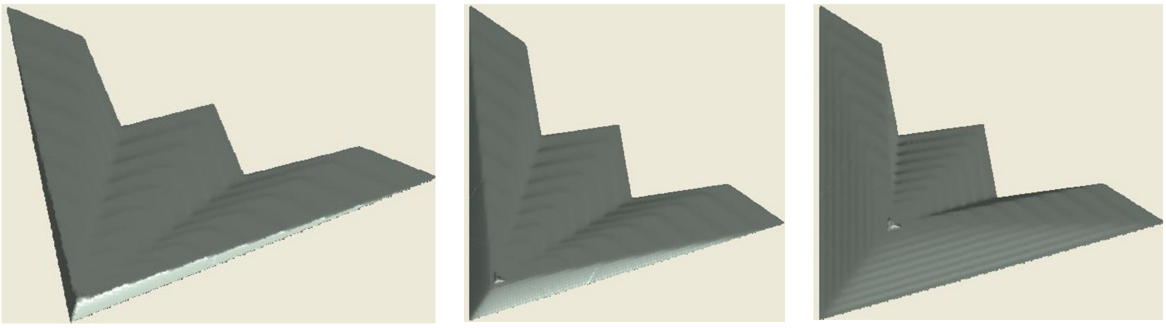
By changing the values of the parameter α, at $x_0 = 0, y_0 = 5, z_0 = 1.5$, we quickly obtain the results presented in Fig. 1.

By changing the position and magnitude (x_0, y_0, z_0) of the coordinates of the cone point at $\alpha = \frac{3\pi}{10}$, we quickly obtain the results presented in Fig. 2.



$\alpha = \frac{\pi}{6}$ $\alpha = \frac{3\pi}{10}$ $\alpha = \frac{\pi}{3}$

Fig. 1. UAV surface view with the change in the parameter α



$x_0 = 0, y_0 = 0.5, z_0 = 0.8$ $x_0 = 0, y_0 = 2, z_0 = 0.8$ $x_0 = 0, y_0 = 5, z_0 = 0.8$

Fig. 2. UAV surface view with the change in the position of the cone point

We construct the UAV equation with the working compartments of various shapes (Fig. 3): in the form of an elliptical cylinder (Fig. 3, a)

$$e1 = \frac{\delta}{2} - \left| 1 - \frac{x^2}{1.5^2} - \frac{(z-0.2)^2}{1.01^2} \right|; \ fe1 = \left(e1 \wedge_0 (y-2.5)(10-y)\right) \wedge_0 z \geq 0;$$

$$fe = fe1 \wedge_0 \left(10^2 - x^2 - y^2 - (z-7)^2\right) \geq 0; \ W = WK \vee_0 fe \geq 0$$

and in the form of a cone with an elliptical guide (Fig. 3, b, c)

$$x_{12} = x_{02} - y_{02}\frac{x - x_{02}}{y - y_{02}}; \ z_{12} = z_{02} - y_{02}\frac{z - z_{02}}{y - y_{02}};$$

$$fe = \left(\left(1 - \frac{x12^2}{1.5^2} - \frac{(z12-0.3)^2}{1.01^2} \right) \wedge_0 (y-2.5)(9-y) \right) \wedge_0 z \geq 0; \ W = WK \vee_0 fe \geq 0 .$$

Here, for clarity, the influence of only four parameters on the UAV surface is studied. In the general case, the UAV equation with the alphabetic parameters will have the following form:

$$f1 = y_1 - x_1 \mathrm{tg}\alpha \geq 0; \ f2 = y_1 + x_1 \mathrm{tg}\alpha \geq 0; \ f3 = -y_1 + \frac{a}{\mathrm{tg}\beta} \geq 0; \ f = (f1 \wedge_0 f2) \wedge_0 f3 \geq 0;$$

$$fc1 = y_1 - c - (x_1 - k) \geq 0; \ fc2 = y_1 - c + (x_1 - k) \geq 0;$$

$$fc3 = y_1 - c - (x_1 + k) \geq 0; \ fc4 = y_1 - c + (x_1 + k) \geq 0;$$

$$x_1 = x_0 - z_0 \frac{x - x_0}{z - z_0}; \;\; y_1 = y_0 - z_0 \frac{y - y_0}{z - z_0};$$

$$fcc1 = fc1 \wedge_0 fc2 \geq 0; \;\; fcc2 = fc3 \wedge_0 fc4 \geq 0; \;\; fcc = \overline{fcc1} \wedge_0 \overline{fcc2} \geq 0;$$

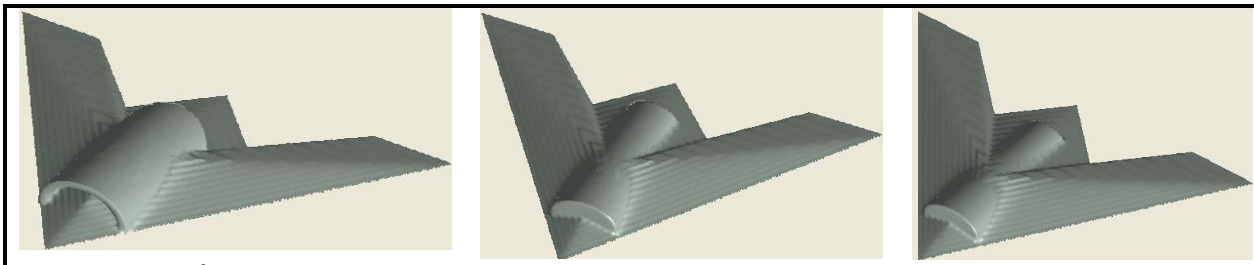
$$fff = f \wedge_0 fcc \geq 0; \;\; WK = fff \wedge_0 z(h - z) \geq 0;$$

$$x_{12} = x_{02} - y_{02} \frac{x - x_{02}}{y - y_{02}}; \;\; z_{12} = z_{02} - y_{02} \frac{z - z_{02}}{y - y_{02}};$$

$$fe = \left(\left(1 - \frac{x12^2}{r_x^{\,2}} - \frac{(z12 - z_c)^2}{r_y^{\,2}} \right) \wedge_0 (y - y_d)(y_h - y) \right) \wedge_0 z \geq 0; \;\; W = WK \vee_0 fe \geq 0$$

$$a = 8.8, c = 9, k = 2.5, \beta = \frac{\pi}{5}, x_0 = 0, \; y_0 = 0.5, \; z_0 = 0.8, \; h = 1.5, \; x_{02} = 0, \; y_{02} = 25, \; z_{02} = 0, \; z_c = 0.3,$$

$$r_x = 1.5, \; r_y = 1.01, \; y_d = 2.5, \; y_h = 9.$$



$\delta = 0.4$ $\qquad$ $x_{02} = 0, y_{02} = 25, z_{02} = 0$ $\qquad$ $x_{02} = 0, y_{02} = 20, z_{02} = 0$

a $\qquad$ b $\qquad$ c

Fig. 3. Surface view of a UAV with a working compartment in the form of:
a – an elliptical cylinder; b – a cone with a vertex at the point (0, 25, 0) and an elliptical guide;
c – a cone with a vertex at the point (0, 20, 0) and an elliptical guide

Thus, we obtain a 16-parameter family of surfaces. However, taking into account symmetry, x_0=0 and x_{02}=0 should not change, which is why, as a result, we have a 14-parameter family. By changing the values of the alphabetic parameters, it is possible to quickly investigate the various forms of UAV surfaces.

The obtained UAV surface equations are visualized using the RFPreview program [9].

In this paper, we did not set the goal to obtain ideal, honed surface forms. The work demonstrates the possibilities of constructing mathematical models of such surfaces by the R-function method.

Conclusions

The construction of a mathematical model is the central stage in the study or design of any system. The quality of the model determines the entire subsequent analysis of the system. The model should be sufficiently accurate, adequate, and convenient to use.

To summarize, it should be noted that for the first time, thanks to the theory of R-functions, the surface equation of a flying wing UAV is constructed in the form of a single analytical expression with alphabetic parameters, and this expression can be used both in solving strength and aero-hydrodynamic problems, as well as designing and manufacturing the system itself, for example, on a 3D printer. An analytical recording of the objects being designed makes it possible to use alphabetic geometric parameters, complex superpositions of functions, which, in turn, allows us to quickly change their structural elements.

The proposed method for specifying the shapes of products by using a limited number of parameters can significantly reduce the complexity of work in CAD systems in cases where one needs to view a large number of design options in search of an optimal solution.

This can significantly reduce complexity in constructing computational models for determining aerodynamic and strength characteristics, and the process of constructing a perturbed flow region near an aircraft of complex geometry can take from several working days to weeks. At the same time, during the design process, it is required to successively review a large number of design options in order to optimize its characteristics. After deciding on the aircraft geometry, the determination of characteristics is also often associated with the need to take into account changes in its shape, for example, if the aircraft has controls that change their geometry during the flight. This leads to the fact that the determination of aerodynamic characteristics only due to the need to build a large number of computational models to take this factor into account increases the duration of work by months. With a parametric task, the change in the calculation areas is carried out almost instantly.

With using R-functions, an algorithm for the phased construction of UAV equations has been developed and implemented, which allows us to check the model and make adjustments to it at each stage of its construction.

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R-функції в аналітичному описанні поверхні безпілотного літального апарата, який виконано за схемою «літаюче крило»

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Безпілотні літальні апарати (БПЛА) стають все більш затребуваними в усьому світі. Область їх потенційного застосування досить велика. Вони використовуються в військових цілях, при доставці вантажів, моніторингу навколишнього середовища, патрулюванні кордонів, повітряній розвідці і картографуванні, контролі дорожнього руху та ін. Ряд важливих переваг БПЛА перед пілотованою авіацією привів до більш активного розвитку цієї галузі, серед яких відносно невелика вартість при великій тривалості і дальності польоту, малі витрати на їх експлуатацію, можливість виконувати маневри з перевантаженнями, що перевищують фізичні можливості людини. Проектування БПЛА і системи керування неможливо уявити без математичного моделювання БПЛА. Для побудови математичних моделей створено швидкодіючі ЕОМ і сучасні програмні засоби, наприклад, такі, як програмні комплекси Solid Works, Ansys CFX, POLYE і ін. Виникає проблема задання та оперативного змінювання геометричної інформації для створення математичної та комп'ютерної моделі проектованого БПЛА. На етапі проектування може бути вирішено багато завдань, які ставляться перед дослідниками при використанні БПЛА. При цьому параметричному заданню поверхонь літальних апаратів приділяється недостатньо уваги. Розширення сфери застосування апарату теорії R-функцій для моделювання поверхонь БПЛА є актуальною науково-технічною задачею. У даній роботі вперше, за допомогою теорії R-функцій, побудовано рівняння поверхні БПЛА, виконаного за схемою «літаюче крило» у вигляді єдиного аналітичного виразу з буквеними параметрами. Таке рівняння може бути використане як під час розв'язання різноманітних практичних задач, так і під час проектування та виготовлення самого виробу, наприклад, на 3D-принтері. Запропонований метод задання форми виробів за допомогою обмеженого числа параметрів може істотно скоротити трудомісткість робіт в CAD-системах в тих випадках, коли потрібно переглянути велику кількість варіантів конструкції в пошуках оптимального розв'язку. В роботі побудовано 14-параметрична сім'я поверхонь БПЛА, виконаних за схемою «літаюче крило». Змінюючи значення буквених параметрів, можна оперативно дослідити різні форми.

Ключові слова: *безпілотний літальний апарат, R-функції, буквені параметри, стандартні примітиви.*

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