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THE EFFICIENCY INCREASE OF THE STEAM TURBINE LOW PRESSURE CYLINDER LAST STAGE BY THE BLADES SPATIAL PROFILING

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The paper presents an option of the steam condensing turbine K-325-23.5 (K-300 series) low pressure cylinder flow part improvement due to the last stage modernization. The K-325-23.5 turbine is designed to replace the outdated K-300 series turbines, which together with the K-200 series turbines form the basis of Ukraine's thermal energy. In the modernized flow part, new last stage guide apparatus blades with a complex circular lean near the hub are used. The purpose of the modernization was to increase the low-pressure cylinder efficiency in the "bad" condenser vacuum to ensure that it did not "decrease" its efficiency at rated operating modes. The modernized low-pressure cylinder flow part is developed with the usage of modern methods of the viscous three-dimensional flow calculation based on the numerical integration of the Reynolds-averaged Navier-Stoks equations. For the turbulent effects, a two-parameter differential SST Menter turbulence model is applied, and for the hydraulic fluid real properties, the IAPWS-95 state equation is used. To construct the axial blades three-dimensional geometry, the original method, the initial data for which was the limited number of parameterized quantities, was used. The applied methods of gas-dynamic calculations and design of flow turbomachines are implemented in the ***IPMFlow*** *software package, which is the development of the* ***FlowER*** *and* ***FlowER-U*** *software packages. The researched low-pressure cylinder flow part is limited by the last two stages (4th and 5th). A difference grid with a total element volume of more than 3 million is used to construct the calculation area. The research examined more than 20 options of the last stage stator blades. In the modernized flow part of the low-pressure cylinder last stage at rated operating mode, the gain of the efficiency coefficient (efficiency) is 0.9% and power – 0.61 MW. In the mode of "bad" condenser vacuum (with high pressure) a significant increase is achieved: efficiency – by 11.5%, power increased by almost 2 MW.*

Keywords: *spatial profiling, numerical modeling, spatial flow, gas-dynamic efficiency, steam turbine, last stage.*

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Introduction

In Ukraine, despite the high growth of "green" energy rate, in the overall balance of electricity generation, a large proportion of it falls on TPP and TPS steam turbines (about 30%). In addition, due to the lack of a sufficient number of HPPs and PSPSs, thermal power plants power units perform uncharacteristic functions of regulating capacities. In the future, according to the global trends, a significant increase in the renewable energy sources share (sun, wind, etc.) is planned for Ukraine [New Green Deal]. It is known that renewable energy sources are unstable, and it is necessary to have a sufficient amount of not only maneuverability but also compensating power to integrate them into a unified energy system [1]. In Ukraine, this function will be assigned to thermal power plants power units.

Most of the existing thermal power plants power units in Ukraine have overworked specified and renewed resources; they need to be replaced with new ones or need a radical reconstruction. First of all, those are 200 and 300 MW power units. The purpose of such work is not only to ensure the power equipment reliability, but also to increase its efficiency and reduce the negative impact on the environment [2].

One of the most common ways to improve turbine efficiency is to use so-called spatial profiling (complex blade lean). Researches and proposals for its application have emerged a long time ago [3, 4, etc.], but their implementation became possible after the widespread application of the computational fluid dynamics three-dimensional methods (3D CFD) in design processes [5, 6, 7, etc.].

The usage of complex blade lean during the turbomachines flow parts designing occurs in two main ways. The first way is used for relatively short blades. Its purpose is to increase gas-dynamic efficiency by reducing final losses. This area is quite popular, it is engaged in by many researchers in scientific and design organizations [8, 9, etc.]. Nevertheless, the authors believe that the benefits of using this approach are not obvious due to the fact that measures aimed at reducing final losses usually cause an increase in other losses (such as profile or edge losses) [10]. The authors experience also shows that the usage of modern approaches makes it possible to ensure a very high level of efficiency without spatial profiling when creating blade profiles individually for each flow part [11]. The second way is used for relatively long blades in order to align the flow gas-dynamic characteristics by the channel height. It is most appropriate to use this approach in the high-power condensing steam turbines last stages. There are examples when this approach has yielded positive results in modes with "bad" condenser vacuum (increased pressure values) [12], but the problem of providing original efficiency values at rated operating modes remains unsolved.

Possibilities of K-300 series (modification K-325) condensing steam turbine low pressure cylinder (LPC) improvement are considered in the article due to the last stage modernization with the usage of modern methods of viscous three-dimensional flows calculation and flow parts design. The option of the last stage modernization which the usage stator blades complex lean, is presented.

Object of research

Figure 1 shows the meridional cross-section of the LPC steam turbine K–325–23.5 flow part produced by JSC Turboatom. Figure 2 shows the sets of profiles that define the fifth stage stator and rotor blades, as well as the stator blades isometric view.

Calculations methodology

The calculation area is limited by the last two stages, which are described by a finite-difference H-type grid. The calculations were performed with the assumption that the flow in all inter-blade channels of one crown is the same (in the cylindrical coordinate system), and therefore the grid was constructed accordingly. The total grid size is over 3 million elemental volumes (90×72×108+90×72×108+90×72×108+90×72×148).

The research of two operating modes, which differ from each other by the pressure values on the last stage (in the condenser) P_{out}=3.8 kPa (mode No. 1 – rated) and P_{out}=8 kPa (mode No. 2) is carried out. Theoretically, the condenser may have higher pressure values, and turbine is calculated at the maximum value P_{out}=12 kPa, but according to the guidelines, the turbine operation with outlet pressure of more than 8 kPa is not allowed. The parameters at the calculation area inlet (before the fourth stage) were set equal – total pressure P_{in}=36.7 kPa, total temperature T_{in}=346.8 K.

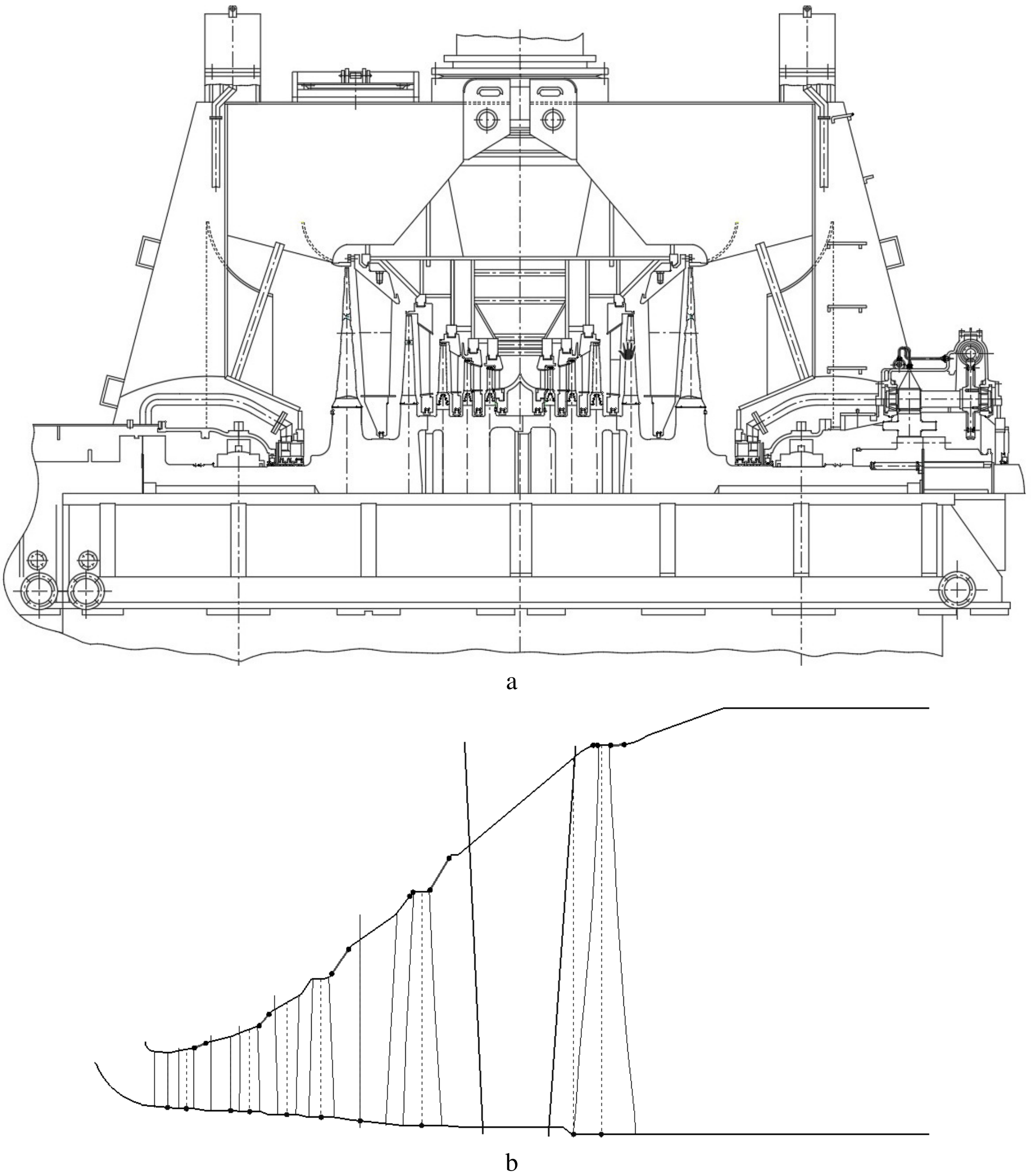


Fig. 1. The LPC steam turbine K-325-23,5 meridional cross-section:
a – sketch drawing; b – computer model in the ***IPMFlow*** software complex

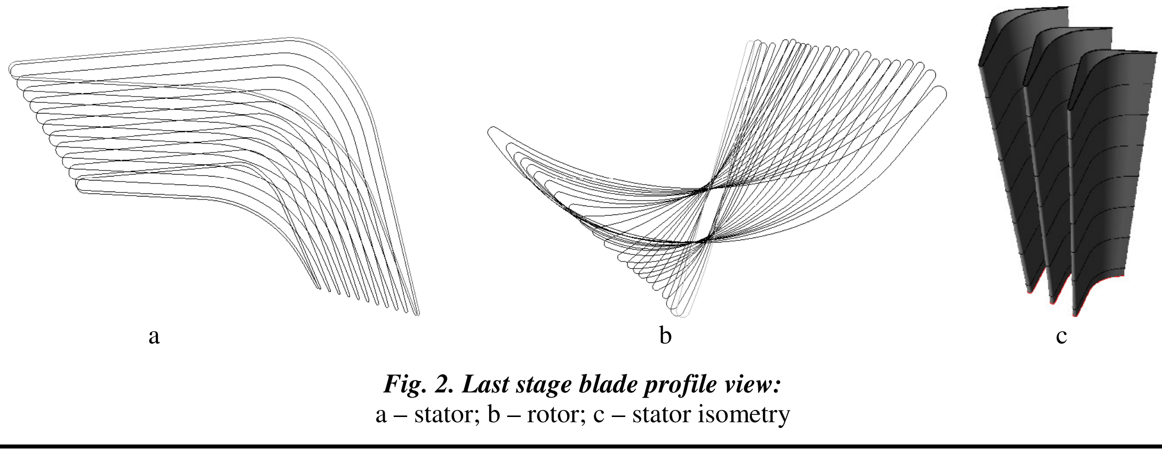


Fig. 2. Last stage blade profile view:
a – stator; b – rotor; c – stator isometry

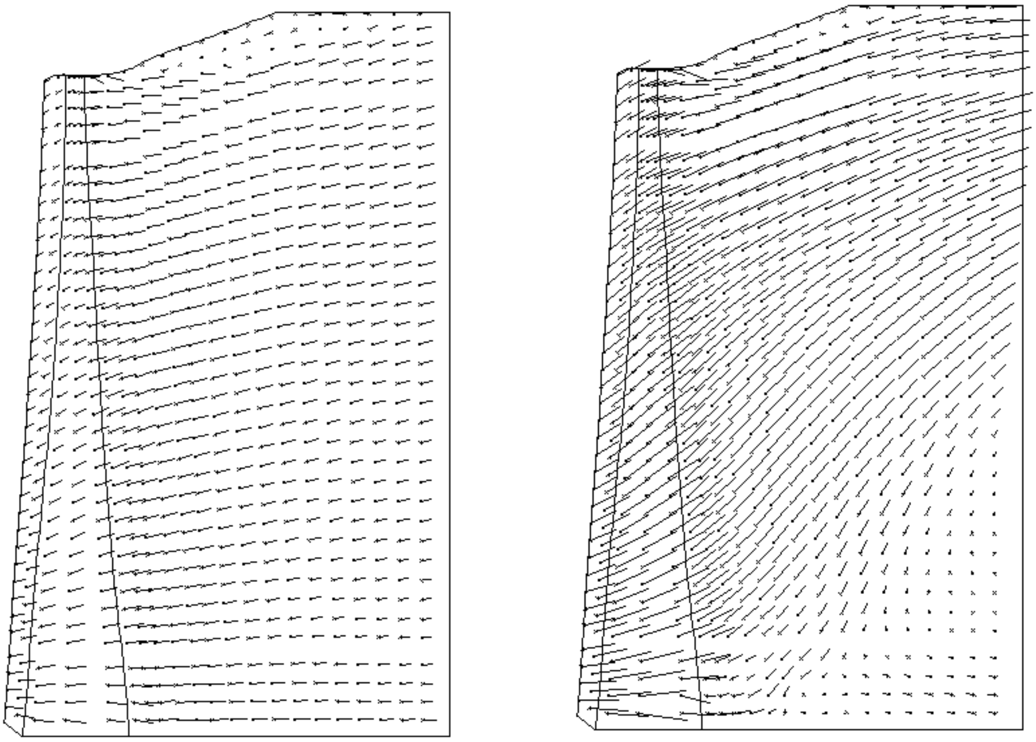
The modeling includes over-flows and steam selections. As the last two stages operate in the wet vapor zone, the calculations were performed using the equation of state IAPWS-95 [13] and its interpolation-analytical approximation method [14].

The spatial viscous flows modeling in the LPC flow part was performed using the ***IPMFlow*** software complex, which is the development of the ***FlowER*** and ***FlowER-U*** software packages [15, 10]. Turbulence was taken into account with the usage of two-parameter differential SST Menter turbulence model [16].

Flow analysis in the last stage of LPC original flow part

Figure 3 shows the flow visualization in two operating modes in the last stage rotor. It can be seen that, in mode No. 1, the flow has almost no flow separations (Fig. 3, a). A small localized flow separation is present on the periphery closer to the outlet of calculation area. Its appearance is caused by a significant "opening" of the peripheral contour. In mode No. 2, this separation narrowed, but a significant strong separation in the hub meridional contour (Fig. 3, b) appeared. It extends to almost half of the channel height at the calculation area outlet. This flow pattern is due to the fact that during the pressure increase in the condenser the heat load per stage decreases and its reactivity decreases as well (Fig. 4). Wherein, due to the fact that stages with relatively long blades (small D/L values) are characterized by a significant reactivity gradient along the channel height, with its total decrease, the reactivity at the hub becomes negative (Fig. 4).

Negative reactivity and, consequently, a significant flow separation near the hub cause the flow efficiency to fall. Thus, in the mode No. 1 the last stage efficiency that includes mass flow with the outlet speed is 78.8%, without it – 88.8%, and in mode No. 2 59.9% and 81.2% respectively. Stage power for modes No. 1 and 2 is 8.6 and 3.4 MW, respectively.



a b

Fig. 3. Velocity vectors in the rotor of LPC original flow part last stage:
a – mode No. 1 – P_{out}=3.8 kPa, b – mode No. 2 – P_{out}=8 kPa

Modernized flow part

For the turbines axial blades design, a complex methodology developed by A. Podgorny Institute of Mechanical Engineering Problems NAS of Ukraine is used. The said methodology includes methods of different levels of complexity – from one-dimensional to spatial viscous flows calculation models and analytical methods for describing the flow parts spatial geometries based on a limited number of parameterized values [7]. The complex design methodology is implemented in the ***IPMFlow*** software package.

Based on the experience of other authors and our own [12, 17], a new last stage stator with a blade complex circular lean near the hub contour was designed (Fig. 5). To develop the modernized flow part, it was necessary to consider about 20 options of stator blades.

This blade shape provided an increase in the last stage reactivity, especially near the hub contour (Fig. 6).

The decrease in negative reactivity led to a significant decrease in the flow separation at the flow part outlet in mode No. 2 (Fig. 7).

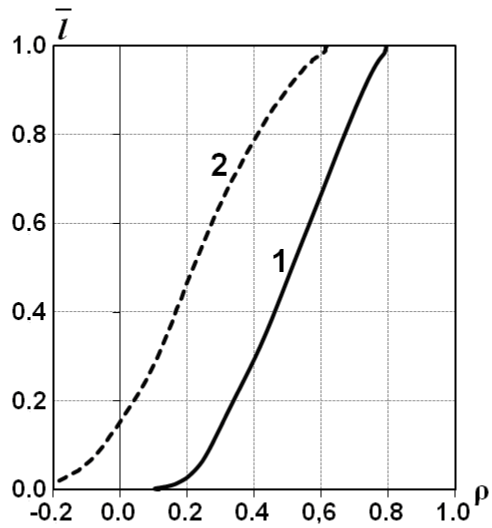


Fig. 4. LPC original flow part last stage reactivity:
1 – mode No. 1 – P_{out}=3.8 kPa,
2 –mode No. 2 – P_{out}==8 kPa

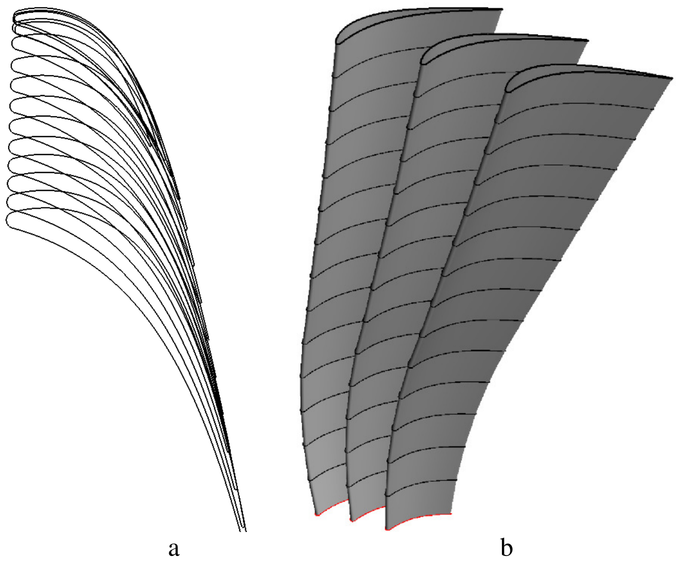


Fig. 5. The last stage modernized stator blade:
a – profiles; b – isometry

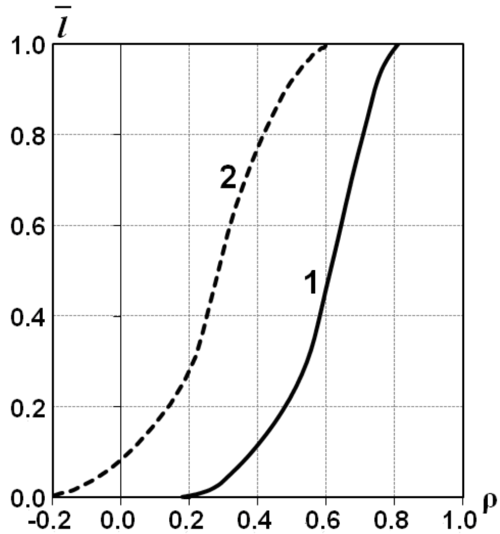
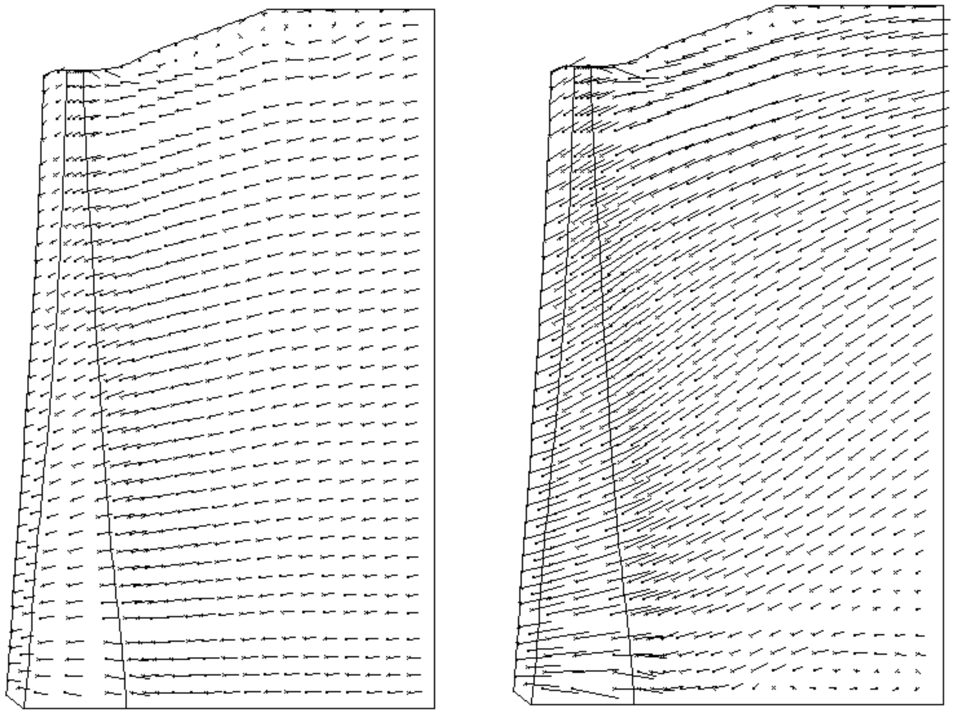


Fig. 6. LPC original flow part last stage reactivity:
1 – mode No. 1 – P_{out}=3,8 kPa,
2 – mode No. 2 – P_{out}=8 kPa



a b

Fig. 7. Velocity vectors in the last stage rotor with the new stator:
a – mode No. 1 – P_{out}=3,8 kPa, b – mode No. 2 – P_{out}=8 kPa

The table shows the integral characteristics of the original and modernized last stages for the two operating modes.

Integral characteristics

Mode No.	Original flow part			Modernized flow part		
	Efficiency, %	Efficiency, % with outlet velocity losses	*N*, MW (on three flows)	Efficiency, %	Efficiency, % with outlet velocity losses	*N*, MW (on three flows)
1	88.8	78.8	25.83	89.0	79.7	26.44
2	81.2	59.9	10.36	87.0	71.4	12.34

The results in the table show that in the mode No. 1 in the modernized flow part, the efficiency increase was 0.9% and power – 0.61 MW. Much more was achieved in mode No. 2, with the efficiency of 11.5% and power of almost 2 MW. The total power values for the three flows in LPC are given.

Conclusions

With the usage of modern methods of viscous three-dimensional flows calculation and design, the modernized flow part of the steam turbine K–325–23.5 LPC is developed. In the modernized flow part, a new last stage stator blade with a complex circular lean near the meridional hub contour is implemented.

The modernization corresponds to the paper's main objective, namely, to increase the last stage efficiency in modes with increased pressure in the condenser ("bad" vacuum mode), ensuring "non-decrease" of the efficiency initial level at the rated operating mode. Thus, in rated mode (mode No. 1) the efficiency increase was 0.9%, power – 0.61 MW, and in the mode with high pressure in the condenser (mode No. 2) much higher indicators increase is achieved – 11.5% and almost 2 MW in efficiency and power respectively.

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Підвищення ефективності останнього ступеня циліндра низького тиску парової турбіни за рахунок просторового профілювання лопаток

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В роботі наведено варіант удосконалення проточної частини циліндра низького тиску (ЦНТ) парової конденсаційної турбіни К-325-23,5 (серія К-300) за рахунок модернізації останнього ступеня. Турбіна К-325-23,5 розроблена для заміни застарілих турбін серії К-300, які разом з турбінами серії К-200 складають основу теплової енергетики України. В модернізованій проточній частині застосовані нові лопатки направляючого апарата останнього ступеня зі складним коловим навалом біля кореня. Метою модернізації було збільшення ефективності ЦНТ на режимах «поганого» вакууму в конденсаторі з забезпеченням «незниження» його ефективності на номінальних режимах праці. Удосконалена проточна частина ЦНТ розроблена з використанням сучасних методів розрахунку в'язкої тривимірної течії, що ґрунтуються на числовому інтегруванні осереднених рівнянь Нав'є-Стокса. Для турбулентних ефектів застосовано двопараметричну модель турбулентності SST Ментера, а для врахування реальних властивостей робочого тіла – рівняння стану IAPWS-95. Для побудови тривимірної геометрії осьових лопаток використовувався оригінальний метод, вихідними даними для якого була обмежена кількість параметризованих величин. Застосовані методи газодинамічних розрахунків та проектування проточних турбомашин реалізовані в програмному комплексі IPMFlow, який є розвитком програмних комплексів FlowER і FlowER–U. Досліджувану проточну частину ЦНТ обмежено двома останніми ступенями (4-м та 5-м). Для побудови розрахункової області використано різницеву сітку з загальною кількістю елементарних об'ємів понад 3 млн. В процесі дослідження розглянуто більше 20 варіантів лопаток направляючого апарата останнього ступеня. У модернізованій проточній частині останнього ступеня ЦНТ на номінальному режимі роботи приріст коефіцієнта корисної дії (ККД) склав 0,9 % й потужності – 0,61 МВт. На режимі з «поганим» вакуумом (з підвищеним тиском) у конденсаторі досягнуто значнішого приросту: ККД – на 11,5 %, потужність зросла майже на 2 МВт.

Ключові слова: *просторове профілювання, числове моделювання, просторова течія, газодинамічна ефективність, парова турбіна, останній ступінь.*

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NONLOCAL ANISOTROPIC SHELL MODEL OF LINEAR VIBRATIONS OF MULTI-WALLED CARBON NANOTUBES

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A simply-supported multi-walled carbon nanotube (MWCNT) is considered. Its vibrations will be studied in a cylindrical coordinate system. The elastic constants in Hooke's law depend on the CNT wall diameter, which is why each wall has its own elastic constants. CNT vibrations are described by the Sanders-Koiter shell theory. To derive the partial differential equations (PDE) describing self-induced variations, a variational approach is used. The PDEs of vibrations are derived with respect to three projections of displacements. The model takes into account the van der Waals forces between CNT walls. The three projections of displacements are decomposed into basis functions. It was not possible to select the basis functions satisfying both geometric and natural boundary conditions. Therefore, selected are the basis functions that satisfy only geometric boundary conditions. To obtain a linear dynamic system with a finite number of degrees of freedom, the method of weighted residuals is used. To derive the basic relations of the method of weighted residuals, methods of variational calculus are used. The vibrational eigenfrequencies of single-walled (SW) CNTs are analyzed depending on the number of waves in the circumferential direction. With the number of waves in the circumferential direction from 2 to 4, the vibrational eigenfrequencies of CNTs are minimal. These numbers are smaller than those for the vibrational eigenfrequencies of engineering shells. Anisotropic models of triple-walled (TW) CNTs were investigated. In their eigenforms, there is interaction between the basis functions and different numbers of waves in the longitudinal direction. This phenomenon was not observed in the isotropic CNT model. The appearance of such vibrations is a consequence of structural anisotropy.

Keywords: *nanotube, Sanders-Koiter shell model, Van der Waals forces, nonlocal elasticity.*

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