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COMPUTATIONAL MODEL FOR DURABILITY ANALYSIS OF STRUCTURE ELEMENTS WITH DEFECTS

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A methodology for the determination of number of cycles before the destruction of structure elements exposed to cyclic loading (tension-compression) has been developed. The analysis of the structure element static and dynamic stress-strain state with a usage of numerical methods of finite and boundary elements in order to determine the stress concentration zones is carried out. Model cracks that are placed in the highest stress concentration zones are selected. A database of model cracks is proposed. The initial length at which crack development begins is determined with a usage of the stress intensity factor threshold value. For each crack from the database, a critical number of cycles during which the crack grows to unacceptable sizes, is found based on the Paris criterion. A method for determining stress intensity factors for a structure element with cracks is proposed. The problem is reduced to solving singular integral equations. To obtain a numerical solution of these equations, the boundary element method is used. Densities, which appear as unknown functions in the considered integral equations, are used to calculate stress intensity factors. The analytical and numerical solutions of singular equations are compared. The critical number of cycles for plates with isolated cracks and cracks chains, cracks located at the elements holes and boundaries is determined. It was established that at the same loading level, a smaller critical number of cycles corresponds to a structure element with cracks that are in close proximity to the technological hole. An analysis of the fatigue crack development at holes in an elastic-plastic statement in order to determine the number of cycles before destruction is made, estimated number of cycles before the fatigue crack appearance is given.

Keywords: durability, crack, stress intensity factor, singular integral equations, Paris criterion.

Introduction

Due to the production of the hydro turbine, petrochemical, and power equipment resource at many industrial enterprises in Ukraine, the problem of the possibility of the individual elements service time extension and the need to replace morally and physically obsolete components and parts arises. The solution to this problem will ensure the units' operational reliability during further operation upon fulfilling guarantees in terms of power and efficiency. Since the said equipment works for a long time, usually its elements and components can be weakened by various kinds of microdefects. The development of these defects under operational loadings can lead to the failure of individual parts or to complete structure disruption. There are two main approaches to the structure elements durability estimation. The first approach lies in the defects development modeling (pores, cracks, etc.) in the framework of the fracture mechanics. The second approach lies in the structure elements durability estimation when explicit defects are not considered, and the time between the fatigue cracks formation can be predicted. In this regard, the problem of determining the time for the structure elements with defects destruction under the action of cyclic loadings is relevant for the equipment residual life estimation. A similar problem arises while studying the possibility of safe long-term transportation of structure elements.

The objective of the structure with cracks durability estimation is to determine the time (number of cycles) after which the crack grows to a critical size and disruption occurs. Usually it is not known in advance what exactly defects are present in the studied structure element, since microdefects cannot always be detected even by ultrasound scanning methods. Therefore, it is advisable to study the defects of various configurations and sizes effect on the structure durability in order to find out the most dangerous cracks and de-

termine the shortest time to destruction. This time can serve both to estimate the residual life and to determine the timing of the equipment overhaul period.

It is known that small cracks, which are not possible to detect during the structure visual inspection upon repair work, lead to partial or complete destruction of the structure element due to gradual developing under the influence of alternating loadings. Fatigue cracks are usually located near bearing surfaces. They occur due to corrosion, intense temperature and power loadings. The formed crack begins to grow slowly, even if the applied loadings do not exceed the rated values. Then, if a critical crack length is reached, the bearing element suddenly receives a huge amount of kinetic energy, which leads to a catastrophic destruction of the entire structure.

The problem is relevant due to the great interest in the fatigue cracks study in the last decade. By now, a significant amount of experimental and theoretical researches of the patterns of crack growth under the influence of variable cyclic loadings has been accumulated. We note the fundamental monographs of Andreikiv A., Darchuk A., [1] Makhutov N. [2], Panasyuk V., Andreikiv A., Kovchik S. [3], articles [4–6], review [7], in which this problem was reviewed in detail. A regulatory document [8] on the hydraulic turbines elements flowing part residual life calculation, including the presence of cracks, has also been developed. The crack resistance of oil and gas pipelines is the subject of [9]. In [10], a database and classification of cracks in pipelines are presented; work [11] is devoted to the aircraft equipment residual life estimation. The latest studies on the theory of cracks and inclusions that form clusters and chains serve as the foundation for the development of modern technologies for the residual resource determination. In [12], interacting surface cracks are studied; the effect of a pore chain on the weld strength is studied in [13]; in [14], an analysis of the strength of structure element with a cracks chain at a temperature loading was made; work [15] was devoted to studying the durability of a hydraulic turbine shaft with surface cracks.

This work's objective is to develop a methodology for the structure elements with defects of various kinds durability estimation with the usage of modern methods of boundary and finite elements.

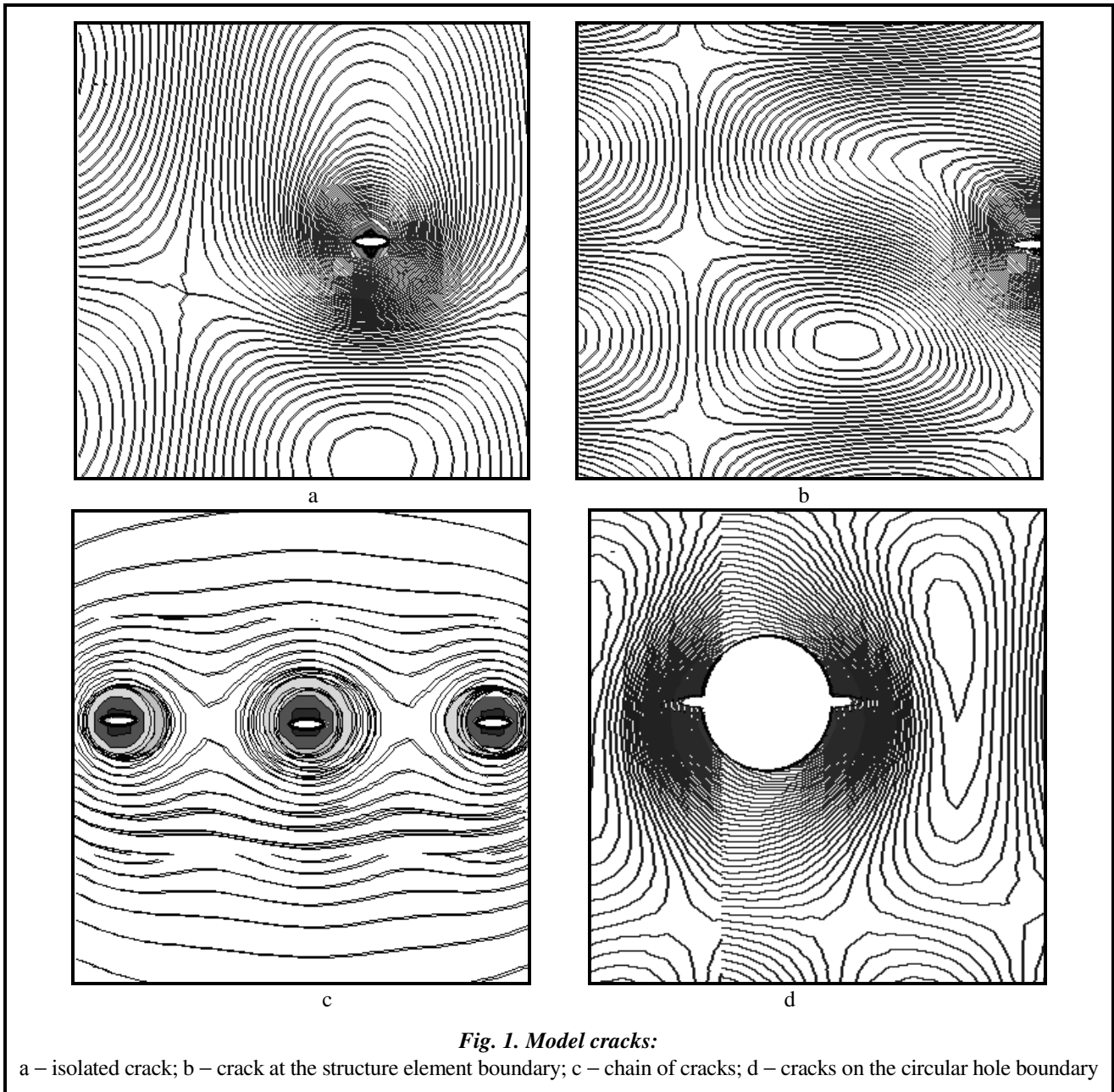
Methodology for the structure elements durability determination

Based on the available experimental and theoretical data, the following methodology for the structure elements durability estimation is proposed. At the first stage, an analysis of the static and dynamic stress-strain state is carried out, the frequencies of the structures free vibrations are determined with a usage of experimental or numerical methods [16–18]. These studies make it possible to find out unwanted vibration frequencies during transportation, to determine the highest stress concentration zones in the structure element. Such zones are usually located near the holes, elements boundaries and welds.

After the analysis, model cracks, which are placed in the highest stress concentration zones, are selected, Fig. 1.

The well-known approach [19] described further is used next. First, the stress-strain state of a structure element without cracks under the action of the given loading is considered. In this case, stresses σ arise in the vicinity of the model crack. Further, it is assumed that predetermined loadings act on the crack edges, $p = -\sigma$, and the element contour is free of loadings, and the stress-strain state of the element with a crack is determined. Then, the stress-strain state of the structure element with a crack under the action of given loading is determined as the sum of the solutions of the two problems described above.

We consider flat structure elements that are under the action of an alternating tensile extension-compression in the perpendicular to the crack line direction. It is assumed that the model crack is located along the contour L_0 . Let $L_i (i = 1, 2, \dots, K)$ be the contours bounding the considered element. We use the integral equations method [20]. Denote the unknown densities on the contour L_0 as α_1, α_2 , and on the contours $L_i (i = 1, 2, \dots, K)$ as $\beta_i, \gamma_i (i = 1, 2, \dots, K)$. Let $n_1^j, n_2^j (j = 0, 1, 2, \dots, K)$ be the components of the internal unit normals to the considered contours. We set that the structure element is under the action of predetermined external loadings S_{11}, S_{12}, S_{22} . In [20, 21], it was indicated that unknown functions α_1, α_2 and $\beta_i, \gamma_i (i = 1, 2, \dots, K)$ are determined from a system $2K + 2$ of singular integral equations regarding $2K + 2$ of unknown functions $\alpha_1, \alpha_2, \beta_i, \gamma_i (i = 1, 2, \dots, K)$



$$M \left\{ \int_{L_0} \sum_{i=1}^2 H_i(\xi_i, x_j) \alpha_i(\xi_0) dL_0 + \sum_{i=1}^K \int_{L_i} [P_1(\xi_i, x_j) \beta_i(\xi_i) + Q_1(\xi_i, x_j) \gamma_i(\xi_i)] dL_i \right\} = n_1^j S_{11} + n_2^j S_{12},$$

$$M \left\{ \int_{L_0} \sum_{i=1}^2 H_i(\xi_i, x_j) \alpha_i(\xi_0) dL_0 + \sum_{i=1}^K \int_{L_i} [P_2(\xi_i, x_j) \beta_i(\xi_i) + Q_2(\xi_i, x_j) \gamma_i(\xi_i)] dL_i \right\} = n_1^j S_{12} + n_2^j S_{22}, \quad (1)$$

where $j = 0, 1, 2, \dots, K$, $M = -\frac{\mu(\lambda + \mu)}{\pi(\lambda + 2\mu)}$, λ, μ are Lamé parameters.

The integrals with kernels $H_i(\xi_i, x_j)$ in the equations of system (1) are hypersingular [21], and in the kernels $P_1(\xi_i, x_j)$, $Q_1(\xi_i, x_j)$, $P_2(\xi_i, x_j)$, $Q_2(\xi_i, x_j)$ there are logarithmic singularities or singularities of Cauchy

type if the points ξ_j and x_j coincide. The numerical solution of the integral equations system (1) is carried out by the boundary element method [18], [22].

To determine the critical number of cycles, the Paris dependence [23] is used, and it is expressed by the formula

$$\frac{dl}{dN} = \begin{cases} 10^{-20} & \Delta K < \Delta K_{th} \\ C(\Delta K)^m & \Delta K_{th} < \Delta K < K_{1C} \\ 10^3 & \Delta K > K_{1C} \end{cases} \quad (2)$$

Here l is the characteristic defect size; N – critical number of cycles; ΔK_{th} – threshold value of the stress intensity factor (SIF); K_{1C} – critical SIF value; the value ΔK is determined by the formula

$$\Delta K = K_{max} - K_{min},$$

where K_{max} , K_{min} – maximum and minimum SIF for one loading cycle; m – fatigue curve exponent; C – fatigue curve characteristic constant.

It is assumed that the loading cycle is symmetrical, i.e.

$$\frac{\sigma_{max}}{\sigma_{min}} = -1.$$

Then $\Delta K = K_{max} - K_{min} = 2K_{max}$. The number of cycles to destruction is determined by relation (2) integrating

$$dN = \frac{1}{C} (\Delta K)^{-m} dl; \quad \Delta K_{th} < \Delta K < K_{1C}.$$

We note that cracks do not develop at $\Delta K < \Delta K_{th}$, and at $\Delta K > K_{1C}$ an avalanche-like crack development that leads to the structure element destruction occurs.

Thus, we have the following formula for the cycles critical number calculation:

$$N = \frac{1}{C} \int_{l_0}^{l_1} (\Delta K)^{-m} dl, \quad \Delta K = 2K_{max}, \quad (3)$$

where l_0, l_1 – initial and final crack lengths.

In order to apply the formula (3), it is necessary to know the initial and final crack dimensions and have an analytical or algorithmic expression for SIF depending on the crack length.

With a usage of the integral equations system solutions (1), we determine SIF by the formulas [19], [21]

$$k_1 = \frac{G}{2\pi(1-\nu)} (n_1^0 S_{12} + n_2^0 S_{22}) \lim_{r \rightarrow 0} \frac{\alpha_2(x)}{\sqrt{r}}, \quad k_2 = \frac{G}{2\pi(1-\nu)} (n_1^0 S_{11} + n_2^0 S_{12}) \lim_{r \rightarrow 0} \frac{\alpha_1(x)}{\sqrt{r}}, \quad (4)$$

where r – the distance between the crack tip and the observation point, G – shear modulus, ν – Poisson's ratio.

The development of rectilinear cracks located perpendicular to the loading is researched. Then only the coefficient k_1 will be nonzero. To determine the length l_0 with which crack development begins, the following relation is used

$$\Delta K = \Delta K_{th}. \quad (5)$$

Cracks with an initial size less than the calculated value l_0 , according to the Paris criterion, do not develop.

We note that cracks with dimensions more than 0.005 m are usually considered unacceptable, i.e., after scheduled inspections, structure elements in the presence of such cracks are subject to repair or modernization [8]. Thus, it is of interest to find the number of cycles during which the crack grows from the initial size calculated by the formula (5) to the final size $l_1=0.005$ m. We also note that the value l_1 must satisfy the condition

$$\Delta K < K_{1C}, \quad (6)$$

otherwise, an avalanche-like crack growth occurs.

The following model cracks were considered: *defect 1* – an isolated crack in the elastic plane (Fig. 1, a), *defect 2* – a crack that extends to the half-plane boundary (Fig. 1, b), *defect 3* – a cracks chain in the weld vicinity (Fig. 1, c) *defect 4* – two symmetrical cracks located in the hole area, (Fig. 1,d).

We give singular integral equations for the unknown densities α_1, α_2 determination.

For an isolated crack with a length $2l$ (Fig. 1, a), we have the following system of hypersingular equations [20]:

$$M \int_{-l}^l \frac{\alpha_1(\xi) d\xi}{(x-\xi)^2} = S_{12}, \quad M \int_{-l}^l \frac{\alpha_2(\xi) d\xi}{(x-\xi)^2} = S_{22}. \quad (7)$$

For a crack that is perpendicular to the half-plane boundary (Fig. 1, b), a hypersingular integral equation is obtained in the form

$$\int_a^b \alpha_2(\xi) \left[-\frac{1}{(\xi-x)^2} + \frac{1}{(\xi+x)^2} - \frac{12\xi}{(\xi-x)^3} + \frac{12\xi^2}{(\xi-x)^4} \right] d\xi = p(x); \quad p(x) = S_{22} / M. \quad (8)$$

For a cracks chain under the action of loading perpendicular to the cracks location line (Fig. 1, c), the corresponding hypersingular equation takes the form

$$\int_{-l}^l \alpha_2(\xi) \sum_{k=-\infty}^{\infty} \frac{1}{(\xi-x-kd)^2} d\xi = p(x); \quad p(x) = S_{22} / M, \quad (9)$$

where l – the crack half length; d – the distance between the cracks centers in the chain.

The problem of determining the stress concentration in a structure element with a technological hole of radius R and two symmetric cracks with a length l (Fig. 1, d) that extend to the hole boundary is reduced to the singular equation [19]

$$\int_0^l K(x, \xi) \alpha_2(\xi) d\xi = \pi S_{22}, \quad (10)$$

where $K(x, \xi) = P(x, \xi) + S(x, \xi)$,

$$P(x, \xi) = \frac{1}{x-\xi} + \frac{\lambda}{1+\lambda\xi} + \frac{2(\xi^2 + 4x\xi - x^2) + 2\lambda\xi(\xi^2 + 9x\xi + 6x^2)}{2(1+\lambda\xi)^2(1+\lambda x)(x+\xi+\lambda x\xi)^3} +$$

$$+ \frac{2\lambda^2 x\xi(4\xi^2 + 12x\xi + 3x^2) + 9\lambda^3 x^2\xi^2(\xi+x) + 3\lambda^4 x^3\xi^3}{2(1+\lambda\xi)^2(1+\lambda x)(x+\xi+\lambda x\xi)^3},$$

$$S(x, \xi) = \frac{\lambda}{2(1+\lambda\xi)} + \frac{\lambda[3x^2 - 2x\xi - \xi^2 + \lambda(x^3 - x^2\xi - 7x\xi^2 - \xi^3)]}{2(1+\lambda\xi)^2(1+\lambda x)(x+\xi+\lambda\xi)^2} -$$

$$- \frac{\lambda[\lambda^2 x\xi(3\xi^2 + 8x\xi + x^2) + 3\lambda^3 x^2\xi^2(\xi+x) + \lambda^4 x^3\xi^3]}{2(1+\lambda\xi)^2(1+\lambda x)(x+\xi+\lambda\xi)^2}.$$

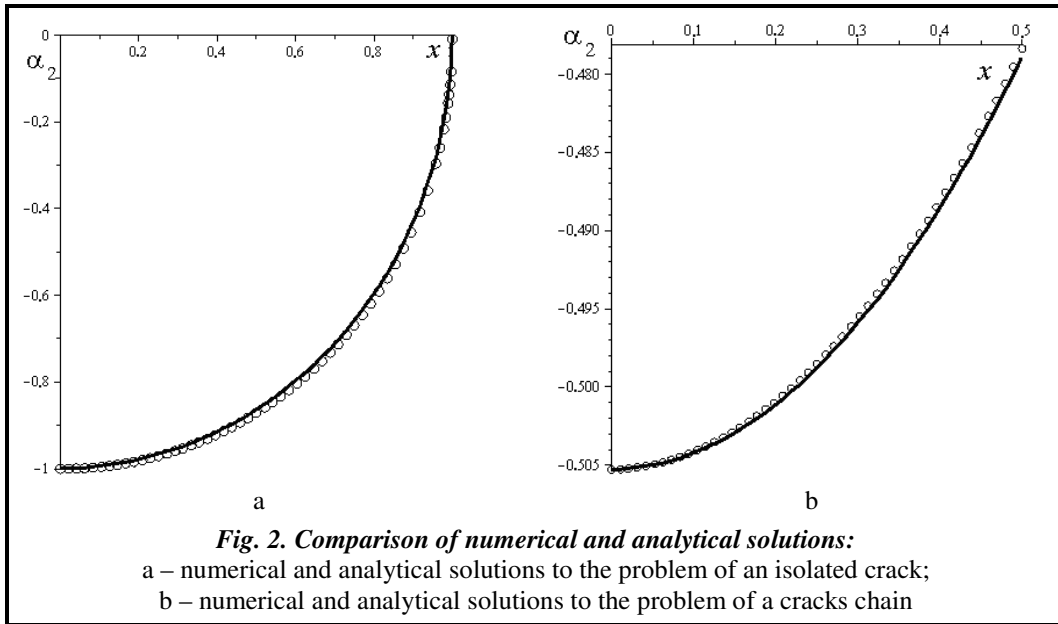
Here $\lambda = l/R$.

Analytical solutions for equations (7) and (9), [19], [20] are known. Numerical solutions of the singular equations (7)–(10) were obtained with a usage of 80 boundary elements with a constant approximation of the density inside the elements [21], [22].

Figure 2 shows the numerical and analytical solutions of hypersingular equations (7), Fig. 2, a, and (9), Fig. 2, b.

The solid lines in Fig. 2 correspond to analytical solutions, dots indicate numerical solutions, accuracy $\epsilon=10^{-3}$ is achieved in the calculations.

Having determined the unknown functions α_1, α_2 , we find the stress intensity factors by the formulas (4).



Structure elements crack resistance analysis

We calculate the cycles number for which cracks of different types grow to unacceptable sizes. In this case, we will consider the same loading conditions. According to the regulatory documents [8], the permissible stresses of the base metal are $\sigma_0=147$ MPa, and for the weld they are $\sigma_w=95$ MPa. We assume that the applied loading is $\sigma=\alpha\sigma_0$ if the crack is located in the base metal, and $\sigma=\alpha\sigma_w$ for cracks located in the weld zone. A material with the following characteristics is considered: $C=3.2 \cdot 10^{-11}$ MPa \sqrt{m} – fatigue curve characteristic constant; $m=3.09$ – the exponent of this curve; $K_{1C}=49$ MPa \sqrt{m} – critical stress intensity factor; $\Delta K_{th}=12$ MPa \sqrt{m} – SIF threshold value. Values K_{1C} , ΔK_{th} are determined by JSC Turboatom by calculation and experimental method that includes the metal quality study [24]. Figure 3 shows the SIF ΔK , threshold values ΔK_{th} , critical values K_{1C} .

The numbers 1 on all the diagrams show the dependences ΔK on the crack length l at $\alpha=1$, the numbers 2 correspond to the loading factor $\alpha=0.1$, the numbers 3 – to the coefficient $\alpha=0.75$. The numbers 4 denote the values ΔK_{th} , the numbers 5 – the critical values K_{1C} .

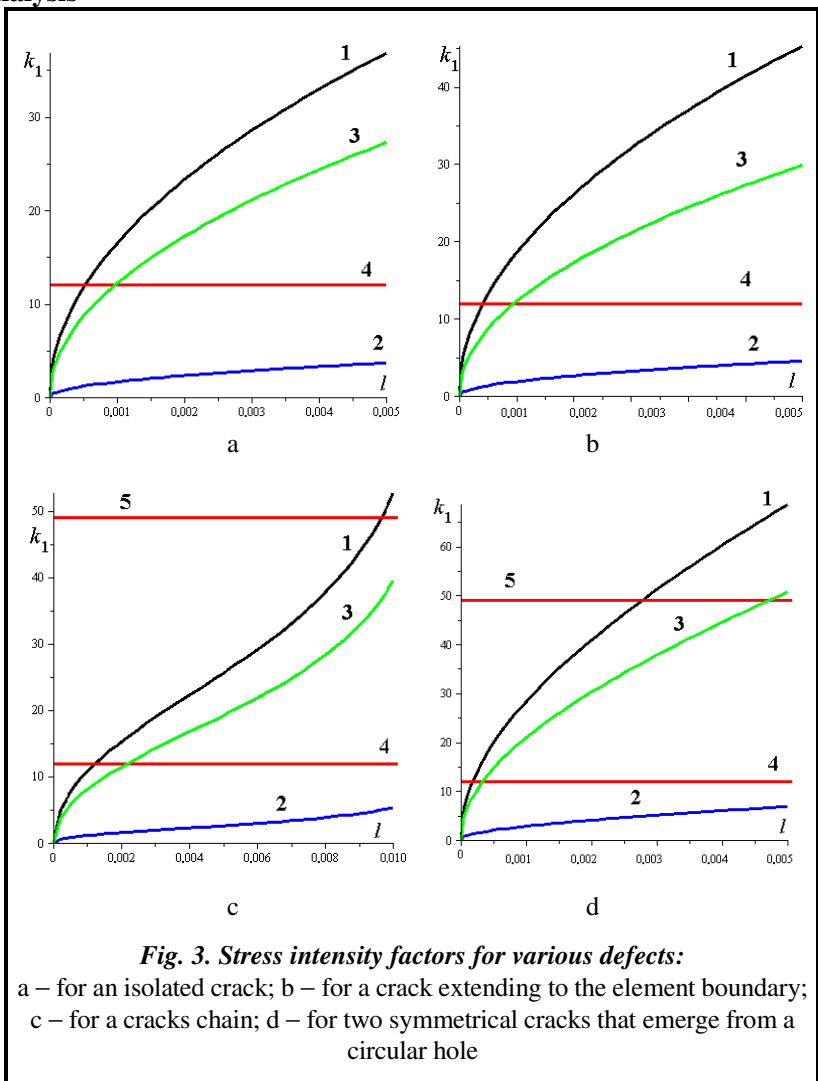


Figure 3, a shows SIF for an isolated crack in the elastic plane, defect 1; Figure 3, b – SIF for a crack extending to the half-plane boundary, defect 2; Figure 3, c – SIF for a cracks chain, $d=0.025$ m, defect 3; Figure 3, d shows the change in SIF for two symmetrical cracks that emerge from a circular hole of radius $R=0.1$ m, defect 4.

Analyzing the diagrams shown in Fig. 3, we establish that at $\sigma=0.1\sigma_0=14.7$ MPa for the base metal and $\sigma=0.1\sigma_w=9.5$ MPa for the welds, none of the considered defects develops.

Next, we calculate the initial crack sizes for loadings $\sigma=0.75\sigma_0$ for the base metal, and $\sigma=0.75\sigma_w$ for cracks located in the weld zone. We obtain the following values: $l_0=0.0005$ m for an isolated crack and a crack extending to the boundary in the base metal zone, $l_0=0.0012$ m for a cracks chain located in the weld zone, $l_0=0.00025$ m for cracks near the technological hole in the base metal. Cracks with an initial size of less than calculated values, according to the Paris criterion, do not develop.

We calculate the number of loading cycles according to the formula (3) for each of the indicated defects if the crack length changes from the initial value l_0 to the final value $l_1=0.005$ m or to the value that is unacceptable from the point of view of the K_{1C} criterion (formula (6) at $\alpha=0.75$). The results are shown in the table.

Number of loading cycles

Defect N	l_0 , m	l_1 , m	Number of cycles
1	0.0005	0.0050	21563
2	0.0005	0.0050	15130
3	0.0012	0.0050	6726
4	0.0002	0.0047	1102

The presented results show that if the loading amplitude is 75% of the permissible, there is a rapid destruction of the structure element with cracks. Defects located near the technological hole turned out to be the most dangerous; in this case, the number of cycles until the crack grows to unacceptable sizes is essentially smaller than for isolated cracks and for cracks chains.

We note that the obtained data do not include cyclic operating time indicators before the appearance of initial fatigue cracks. Therefore, it is of interest to estimate the number of cycles before the appearance of fatigue cracks for the most dangerous defect (symmetrical cracks extending to the technological hole boundary, defect 4, Fig. 1, a). For this purpose, the calculation of the cyclic strength in the system ANSYS, [25] was conducted.

Calculation of a structure element with a technological hole for cyclic strength

We considered a steel plate with a central hole with the following mechanical and geometric characteristics: density $\rho=7850$ kg/m³, Young's modulus $E=200$ GPa, Poisson's ratio $\nu=0.3$, plate length $L=1$ m, width $b=0.4$ m, thickness $h=0.01$ m, hole radius $R=0.1$ m, Fig. 4.

The main material characteristic used to describe its ability to resist the process of multi-cycle fatigue is the Weller's curve (fatigue curve). For this material, this curve in the logarithmic scale is shown in Fig. 5.

We considered plate extension in the horizontal direction by forces of 10^6 N. The distribution of the von Mises equivalent stresses in the plate material is shown in Fig. 6. The maximum stress was 108.7 MPa. The highest stresses were observed at the circular hole boundary

The calculation for multi-cycle fatigue showed that the smallest number of cycles until destruction is observed at the circular hole boundary, in the same areas where the highest stresses are reached, as shown by the static calculation (Fig. 7).

When calculating cyclic fatigue, the structure was loaded according to a sinusoidal law with amplitudes of ± 108.7 MPa and a frequency of 1 Hz. The calculation showed that the minimum number of cycles until destruction is $N=271300$. It was shown above that for the considered structure element, the development of a fatigue crack from the initial crack size $l_0=0.0002$ m to the maximum allowable size $l_1=0.0047$ m occurs in 1102 cycles, which is significantly less than the number of loading cycles during the multi-cycle fatigue calculation. Thus, the main contribution to the resource estimation is made by the cyclic operating time indicator, i.e., the number of loading cycles until the appearance of fatigue cracks. Knowing the total number of cycles to destruction (in the considered case 272402) and the frequency of loading, we determine the time to the structure element destruction. The obtained data make it possible to estimate the structure element transportation safety if there are real transportation cyclograms.

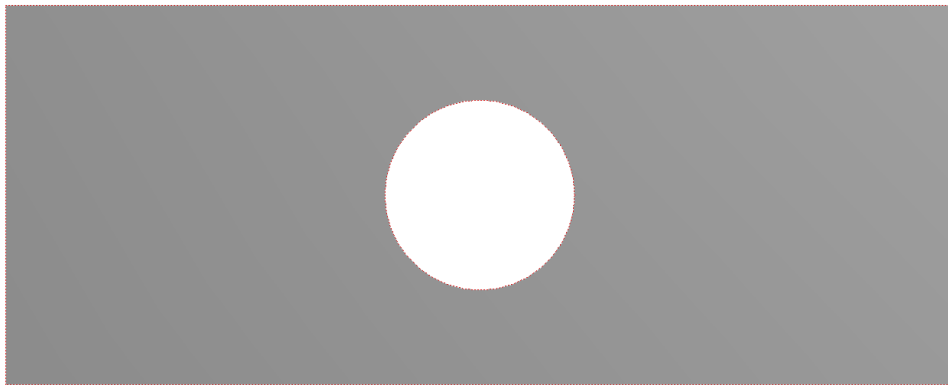


Fig. 4. Plate with hole

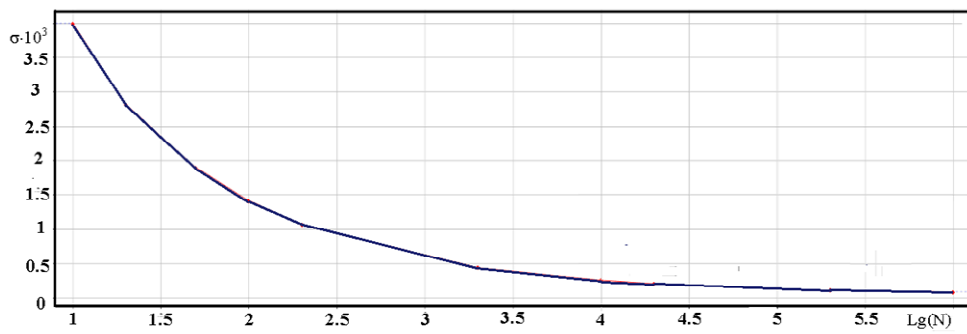


Fig. 5. σ -N fatigue curve

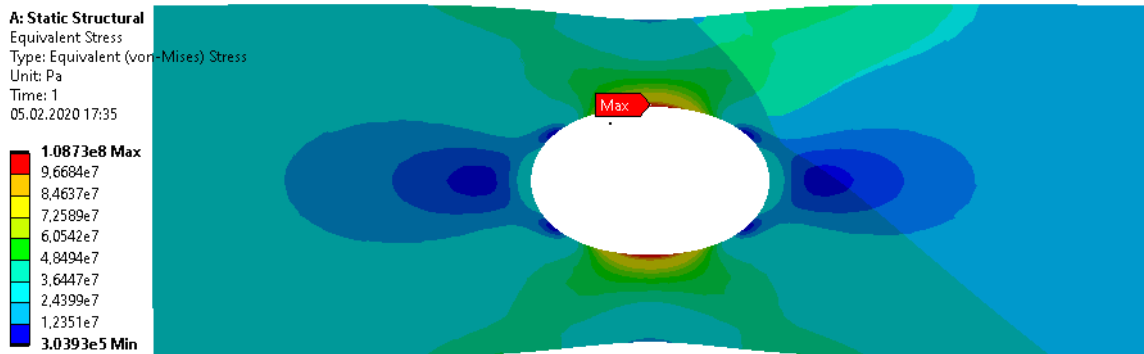


Fig. 6. Von Mises equivalent stresses in the plate

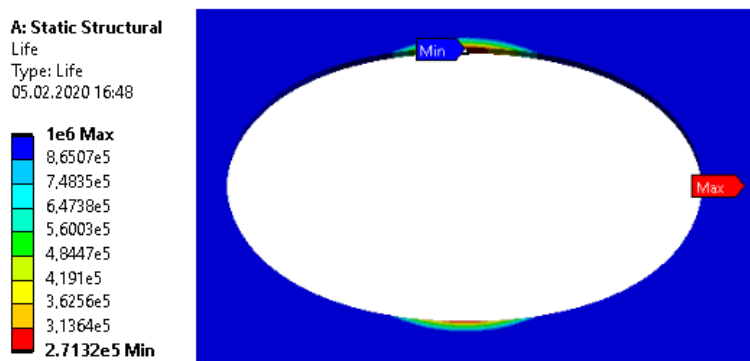


Fig. 7. The number of cycles until the structure element destruction

Conclusion

A method for the calculation of critical number of structure elements loading cycles under conditions of alternating cyclic loading has been developed. It is assumed that in the highest stress concentration zones such defects as cracks may occur. The number of stress cycles, which leads to the growth of cracks to unacceptable sizes, is determined according to the Paris criterion. It is proposed to use the model crack base, which contains the isolated crack, the cracks chain, the crack extending to the structure element boundary, and symmetrical cracks extending to the structure hole boundary. Formulas based on the application of numerical solutions of singular and hypersingular integral equations are used to calculate stress intensity factors. It is established that cracks that are in the vicinity of the holes are the most dangerous. An estimation of the number of loading cycles until the appearance of fatigue cracks is given for this case precisely.

This methodology does not take into account the effect of material properties degradation due to damage accumulation in the considered structure elements durability estimation. This factor accounting will be the objective of further research.

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Розрахункова модель для аналізу довговічності елементів конструкцій з дефектами

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Побудовано методику визначення кількості циклів до руйнування елементів конструкцій, які зазнають впливу циклічного навантаження (розтягнення-стиснення). Проведено аналіз статичного та динамічного напружено-деформованого стану за допомогою числових методів скінченних та граничних елементів з метою з'ясування зон концентрації напружень в конструктивному елементі. Вибираються модельні тріщини, які поміщають в зони найбільшої концентрації напружень. Запропоновано базу даних щодо модельних тріщин. З використанням порогового значення коефіцієнта інтенсивності напружень визначається початкова довжина, за якої починається розвиток тріщин. Для кожної тріщини з бази даних на підставі критерію Періса знаходиться критична кількість циклів, за яку тріщина підрастає до неприпустимих розмірів. Розроблено методику визначення коефіцієнтів інтенсивності напружень для елемента конструкції з модельними тріщинами. Цю задачу зведено до розв'язання сингулярних

інтегральних рівнянь. Для отримання числового розв'язку цих рівнянь використано метод граничних елементів. Густина, які фігурують як невідомі функції в розглянутих інтегральних рівняннях, використовуються для обчислення коефіцієнтів інтенсивності напружень. Проведено порівняння аналітичних і числових розв'язків сингулярних рівнянь. Визначено критичне число циклів для пластин з поодинокими ізольованими тріщинами й з ланцюжками тріщин, тріщинами біля отворів та границь елементів. Встановлено, що за однакового рівня навантаження найменше критичне число циклів відповідає елементу конструкції з тріщинами в безпосередній близькості від технологічного отвору. Проведено аналіз розвинення втомної тріщини біля отворів в пружно-пластичному формулюванні з метою визначення кількості циклів до руйнування та надано оцінку кількості циклів до появи втомної тріщини.

Ключові слова: довговічність, тріщина, коефіцієнт інтенсивності напружень, сингулярні інтегральні рівняння, критерій Періса.

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METHOD TO STUDY THE CREEP OF COMPLEX-SHAPED FUNCTIONALLY- GRADED BODIES

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The creep problem of complex-shaped functionally-graded bodies of revolution is considered. For the variational statement of the problem, the Lagrange functional is used, defined at kinematically possible displacement rates. A numerical-analytical method is developed for solving a non-linear initial-boundary creep problem. It is based on the combined use of the R-functions, Ritz and Runge-Kutta-Merson methods. The advantages of the proposed method include: exact consideration of the geometric information about the boundary-value problem at the analytical level, without any approximation thereof; representation of an approximate solution to the problem in an analytical form; exact satisfaction of boundary conditions; automatic time step selection. Solved are the problems of creep both for a hollow straight cylinder and a complex-shaped body of revolution (a cylinder with a rectangular cut-out on the outer surface), both cylinders being loaded with a constant inner pressure, made of the functionally graded material (FGM) based on SiC particle-reinforced aluminium. The creep of the material is described by Norton' law. Both Young's modulus and creep characteristics of the material depend on the volume part of the reinforcing material. Both ends of the cylinder are free of external load, and are fixed in such a way that the radial displacements are equal to zero. A corresponding partial solution structure is constructed that satisfies the boundary conditions for displacement rates. The calculations were performed for cylinders of two different composite materials: a material with a uniform distribution of SiC particles and an FGM with a difference in the volume content of reinforcing particles along the radius, with the average volumetric content of reinforcing SiC particles in the two cases being the same. The influence of both the gradient properties of the material and geometric shape on the stress-strain state (SSS) under creep conditions was investigated. The presence of a rectangular cut-out on the outer surface of a cylinder in all cases leads to an increase in displacements and stresses. Moreover, the degree of influence of the geometric shape on the SSS during creep substantially depends on the gradient properties of the material. For a cut-out cylinder made of the material with a uniform distribution of SiC particles, there is a significant increase in displacements and stresses after 100 hours of creep compared with a straight cylinder. For bodies of revolution made of a functionally graded material, the cut-out effect on the SSS is less pronounced.

Keywords: functionally graded material, body of revolution, creep, R-functions method.

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