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UDC 624.074.4:681.3

OPTIMAL DESIGN OF SMOOTH SHELLS BOTH WITH AND WITHOUT TAKING INTO ACCOUNT INITIAL IMPERFECTIONS

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This paper considers the application of the random search method for the optimal design of both axially-compressed smooth cylindrical ideal thin-walled shells and a shell with initial imperfections. In stating a mathematical programming problem, the objective function is the minimum weight of the shell. As constraints imposed on the region of permissible solutions, the following constraints are adopted: on the critical load of local buckling, on the critical load of buckling of the shell axis; strength condition, and condition for constraining the dimensions of a shell (radius and wall thickness of a shell). With the optimal design of a shell with initial imperfections, the statement of the mathematical programming problem remains the same as for an ideal shell, with only local buckling constraint changing. The aim of this paper is both to study the zone of influence of the optimum shell weight on the value of compressive force and to determine the range of the external compressive loads at which the general and local buckling shell constraints are decisive. A numerical experiment was carried out. Dependences of the weight, wall thickness, radius of the middle surface, and the ratio of the middle surface radius to the wall thickness on the magnitude of the compressive load both for an ideal shell and a shell with initial imperfections were investigated. As a result of the numerical experiment, it was established that the presence of initial imperfections in an axially-compressed smooth cylindrical shell leads to an increase in its weight compared to that of an ideal shell. The weight does not increase over the entire range of compressive loads, but only with the loads at which both local and general buckling constraints are decisive. If the optimal solution pertains to the strength constraint, which is typical for large compressive loads, there is no influence of initial imperfections on the optimal design. The weight of an ideal shell and that of a shell with initial imperfections in the optimal design turn out to be the same.

Keywords: *thin-walled cylindrical shell, initial imperfections, optimal design, random search.*

Introduction

Thin-walled constructions in the form of shells are used in many branches of engineering and construction. The variety of types of shell structures, different loading and operating conditions, the complexity of stress and strain state analysis led to the creation of both specific techniques and computational methods, often mathematically quite complex and therefore almost unavailable to wide circles of engineers.

Even more complicated is the computation of optimally designed shells. With the development of mathematical programming methods, it became possible to find the optimal parameters of shells. However, not all mathematical programming methods successfully cope with this task. Whenever shell-type structures are designed, physical, technological, operational, and geometric constraints are usually imposed onto them. These constraints are usually written as inequalities. Thus, in the space of optimization parameters, there is a certain stationary region within which the optimal solution is located. As noted above, problems of this type are successfully solved only by modern non-linear programming methods, in particular, computer-aided random search methods [1].

In addition, in the optimal design of compressed reinforced cylindrical shells, critical stresses become a function of not only the casing and reinforcement parameters, but also the number of circumferential and meridional half-waves, which are formed when buckling is lost. The number of half-waves, in turn, also depends on variable shell parameters. Therefore, the search area becomes unsteady. The constraints that are imposed on the non-stationary region "breathe". To solve this problem, special algorithms are needed that take into account the specifics of the problem. Such algorithms exist, and are given in the specialized literature [2, 3].

Parametric Optimization of Cylindrical Shells Without Initial Imperfections

Let us consider a smooth isotropic circular cylindrical shell of a given length L simply-supported at the ends and loaded with an axial compressive load N (Fig. 1). The characteristics of the shell material are known: E is the elastic modulus, $\sigma_т$ is the yield strength, γ is the specific gravity, and μ is Poisson's ratio. It is required to find such values of the wall thickness δ and radius R of the middle surface of the shell R on this continuous set of parameter values that at a given load N the shell has a minimum weight G.

The problem formulated above mathematically reduces to finding the minimum value of the weight function

$$G = 2\pi\gamma LR\delta \tag{1}$$

at fulfilling the constraints

$$\frac{2\pi E\delta^2}{\sqrt{3(1-\mu^2)}} \geq N ; \tag{2}$$

$$\frac{\pi^3 E\delta R^3}{L^2} \geq N ; \tag{3}$$

$$2\pi R\delta\sigma_T \geq N ; \tag{4}$$

$$\delta^{\min} \leq \delta \leq \delta^{\max} ; \; R^{\min} \leq R \leq R^{\max} , \; (\delta > 0; \quad R > 0) . \tag{5}$$

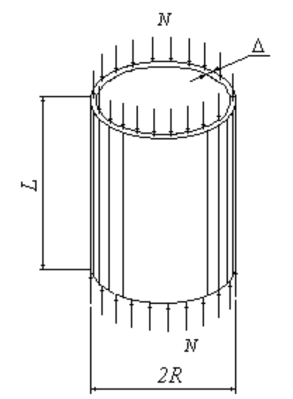

Fig. 1. The design scheme of a shell

Condition (2) is the constraint on the critical load of the local buckling of an ideal circular cylindrical shell; condition (3) is the constraint on the critical load of the buckling of the shell axis; condition (4) is the strength condition and, finally, condition (5) constrains the dimensions and wall thickness of the shell.

Using the notations

$$\left.\begin{aligned} &A = 2\pi\gamma L; \quad B = \frac{\pi^3 E}{L^2}; \quad C = 2\pi\sigma_T; \\ &D = \frac{2\pi E}{\sqrt{3(1-\mu^2)}}; \quad x_1 = \delta; \quad x_2 = R \end{aligned}\right\} \tag{6}$$

and substituting them into equations (1) to (5), we obtain the following non-linear programming problem: to find non-negative values of x_1 and x_2 which minimize the function:

$$\Phi = Ax_1x_2 \tag{7}$$

and satisfy the constraints

$$\left.\begin{aligned} &Dx_1^2 \ge N;\quad Bx_1x_2^3 \ge N;\quad Cx_1x_2 \ge N \\ &x_1^{\min} \le x_1 \le x_1^{\max};\quad x_2^{\min} \le x_2 \le x_2^{\max} \end{aligned}\right\}. \qquad (8)$$

The initial data for solving the problem were taken as follows: E=8.16·10^4 MPa; $\sigma_т$=162 MPa; μ=0.3; γ=78.55 kN/m^3; L=3 m. Shell loads: N=1000 kN and N=10 kN. Geometric restrictions: 0.01≤δ≤2.0 cm; 1≤R≤100 cm.

Problem (7)–(8) was solved by using the random search algorithm with coordinate-wise self-learning with forgetting [1]. The descent was carried out from two points with coordinates: x_1=2 cm, x_2=100 cm and x_1=1.5 cm, x_2=50 cm. With a axial load of N=1000 kN on the shell, the values of the optimal parameters δ and R turned out to be different, but the value of the objective function was the same (G=1459.4 N). The analysis of the solution to the problem showed that at a large external load for a given shell length in the example, dominant for such a shell is the strength condition, which graphically, over a sufficiently significant length segment, coincides with the level line of the objective function. In Fig. 2, on the level line of the weight function, which corresponds to a value of G=1459.4 N, distinguished is the interval of possible optimal solutions for δ and R, which deliver the minimum objective function (7). In solving the problem that was formulated in [4], where the parameters δ and R were found for the optimal shell loaded with a longitudinal load of N=1000 kN, the interval of identical minimum values of the objective function was determined analytically by using the Lagrange multiplier method. The minimum value of the objective function was G=1459.4 N, which completely coincides with the solution obtained by the random search method.

The shell loaded with a longitudinal compressive load of N=10 kN has the only optimal solution G=15.5 N, at δ=0.018 cm and R=5.72 cm (according to [4]). The search results are shown in Table 1. The search path is shown in Fig. 3. It can be seen from Table 1 that the problem solution results presented in work [4] and obtained with the algorithm with coordinate-wise self-learning with forgetting coincide completely.

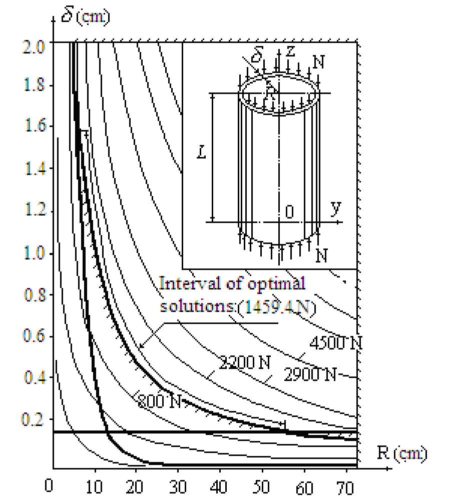

Fig. 2. Illustration for the optimal design of a shell with a large compressive load

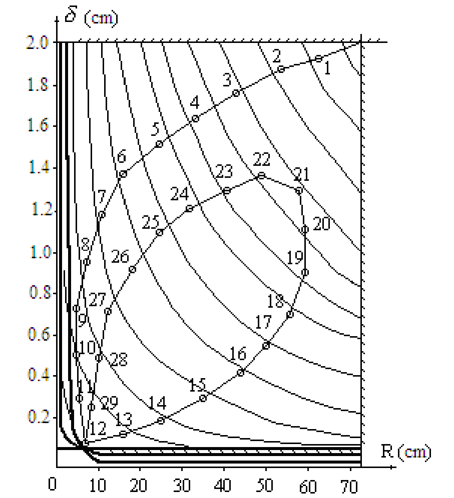

Fig. 3. The search path for the optimal solution for a shell with a low value of compressive load

Table 1. Results of the optimal design of an ideal cylindrical shell

Method	Shell	Weight G, N	Wall thickness δ, cm	Radius R, cm
of Lagrange Multipliers [4]	Ideal	15.5	0.018	5.72
of Coordinate-wise Self-learning with Forgetting	Ideal	15.5	0.018	5.72

Influence of Initial Imperfections on the Optimal Parameters of a Smooth Cylindrical Shell

The modern theory of designing smooth shells has various methods for estimating the minimum critical loads. Among them are classical approaches [5] based on the use of the Euler static criterion with the account taken of the homogeneous momentless subcritical state of a shell, as well as non-linear approaches that take into account both the inhomogeneity and momentness of the subcritical state [6, 7]. Extensive use has been made of methods of calculating thin-walled shells with initial imperfections. These methods use both classical approaches [8] and non-linear methods [9].

Consider a cylindrical smooth isotropic circular cylindrical shell of a given length L, the shell being simply-supported at the ends, loaded with an axial compressive load P, and having initial imperfections. The characteristics of the shell material are known: E is the elastic modulus, $\sigma_т$ is the yield strength, γ is the specific gravity, and μ is Poisson's ratio. It is required to find such values of the wall thickness δ and radius R of the middle surface of the shell that at a given load N it would have a minimum weight G. It is known that in the presence of imperfections, the values of critical axial compression forces change significantly, the change being the greater, the greater the ratio R/δ. In this regard, the following question is of considerable interest: how, all else being equal, the optimal (in the weight sense) parameter values are affected by the shell surface. In other words, it is necessary to find such parameters δ and R that would deliver the minimum of the weight function of the shell with initial imperfections. The formulated problem in many respects coincides with the previous one. The only difference is in the expression for the critical load of local buckling. Given the influence of initial imperfections, this expression has the form [10]

$$\frac{2\pi E\delta^2}{\sqrt{3(1-\mu^2)}}\left[\left(1+k\frac{R}{\delta}\right)^{\frac{1}{2}}-\left(k\frac{R}{\delta}\right)^{\frac{1}{2}}\right]\ge N\ , \tag{9}$$

where the coefficient k takes into account the quality of the shell surface and, as shown in [10], at k=0.005, the curve constructed in coordinates N^* (critical force) R/δ is the lower enveloping curve for the numerous experiments that were conducted by different authors.

Introducing notation (6), we obtain the non-linear programming problem: to find non-negative values of x_1 and x_2 that minimize the function

$$\Phi = Ax_1x_2 \tag{10}$$

in satisfying the constraints

$$\left.\begin{array}{l} Dx_1^2\left[\left(1+k\dfrac{x_2}{x_1}\right)^{\frac{1}{2}}-\left(k\dfrac{x_2}{x_1}\right)^{\frac{1}{2}}\right]\ge N;\quad Bx_1x_2^3\ge N;\quad Cx_1x_2\ge N; \\ x_1^{\min}\le x_1\le x_1^{\max};\qquad x_2^{\min}\le x_2\le x_2^{\max} \end{array}\right\} \tag{11}$$

To assess the influence of initial imperfections on the optimal design and to establish the weight efficiency zone of an ideal shell with respect to a shell with initial imperfections, we performed weight optimization of an axially-compressed smooth cylindrical shell without initial imperfections (ideal shell). The results of solving this problem are given in Table 2.

The formulated problem (10)−(11) was solved using the continuous learning algorithm with a director cone [1] at the following initial data: N=10 kN; $E=8.16\cdot10^4$ MPa; $\sigma_т$=162 MPa; μ=0.3; L=3 m; γ=78.55 kN/m^3; k=0.005; 0.01≤δ≤1.5, 1≤R≤50 cm.

As a starting point, the point with coordinates x_1=1.5 cm ; x_2=50 cm was chosen. The initial step value equaled a=0.4, with subsequent crushing. The cone opening angle was accepted equal to 0.5 rad and halved in tracking a new direction near the border of the search area. The results of the solution are given in Table 2.

Table 2. Results of the optimal design of the ideal cylindrical shell and the shell with initial imperfections

Method	Shell	Weight G, N	Wall thickness δ, cm	Radius R, cm
Guide cone	Ideal	15.60	0.0181	5.820
Guide cone	With initially imperfections	20.64	0.0276	5.042

As expected, the weight of an optimal shell with initial imperfections was greater than that of an ideal shell.

The influence of initial imperfections on the optimal design was investigated in a wider range of changes in compressive loads. To solve this problem, a random search algorithm with a density-controlled sampling distribution was used [11].

The numerical experiment was carried out with a compressive load P varying from 0 to 1000 kN. Fig. 4 shows graphs of the dependence of the weight of an optimal shell on the magnitude of a compressive load in the form of a peculiar loop.

Three zones can be distinguished in this loop. The first zone of shell weight values lies below section A. With load values of P≤10 kN, the constraints that determine the optimal design are geometric constraints on the shell wall thickness and the total buckling of the shell axis (3). The effect of initial imperfections on the value of the shell weight in this zone is absent. The weight of an ideal shell and the weight of a shell with initial imperfections are the same. The second zone is located between sections *A* and *B*. The optimal solutions for an ideal shell and a shell with initial imperfections belong to the zone under consideration, and are characteristic in that the determining constraints for them are the local and total buckling constraints (9) and (3). In this second zone, the influence of initial imperfections on the increase in shell weight is observed, the shell wall thickness turns out to be greater than that of an ideal shell (Fig. 5), and the radius of the middle surface, smaller (Fig. 6).

The ratio of the middle surface radius to the wall thickness of an ideal shell turned out to be larger within the second zone than that of a shell with initial imperfections (Fig. 7).

The largest relative weight increase in a shell with initial imperfections relative to the weight of an ideal shell

$$\xi = \frac{G_{\text{ii}} - G_{\text{i}}}{G_{\text{i}}} \cdot 100\%$$

was observed within the second zone, and amounted to 36.71%. With an increase in compressive load, the relative increase in weight decreased, and at P=26 kN it turned out to be zero.

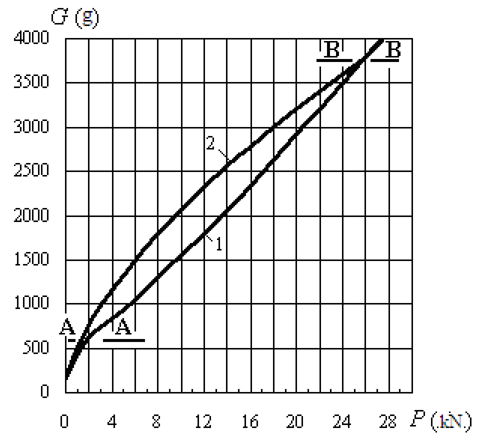

Fig. 4. The weight dependence of an ideal shell (1) and a shell with initial imperfections (2) on the amount of the compressive force P

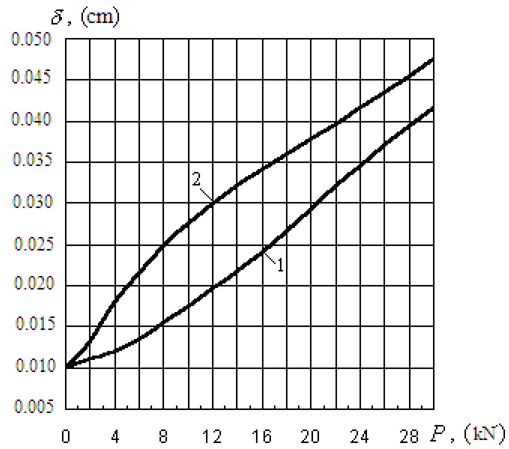

Fig. 5. The dependence of the wall thickness of an ideal shell (1) and a shell with initial imperfections (2) on the amount of the compressive force P

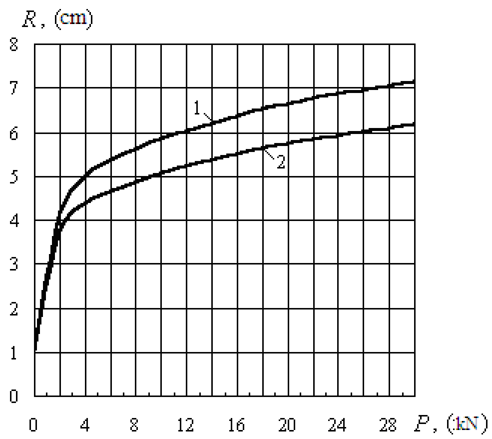

Fig. 6. The dependence of the radius of the middle surface of an ideal shell (1) and a shell with initial imperfections (2) on the compressive strength P

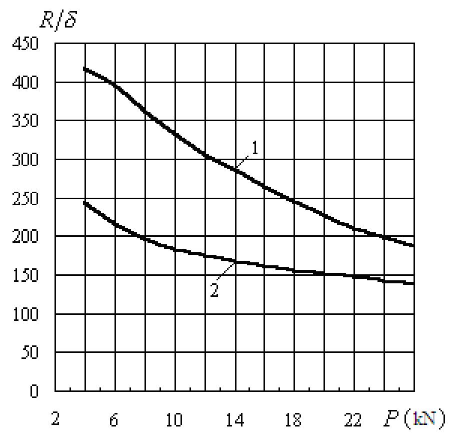

Fig. 7. The dependence of the ratio of the middle surface radius to the wall thickness R/δ of an ideal shell (1) and a shell with initial imperfections (2) on the amount of the compressive force P

The third zone in Fig. 4 lies above section B, and is characterized by the equality of values of the objective function both for an ideal shell and a shell with initial imperfections.

The determining constraint for the third zone is the strength constraint (4). The influence of initial imperfections on the optimal design in this area is absent.

From other observations, it can be noted that for shells with initial imperfections lying in the second zone, the total buckling constraint begins to work much earlier than for ideal shells. So, if for a smooth shell the local buckling constraint begins to work at P=12 kN, then for shells with initial imperfections, this constraint begins to work at the compressive force P=24 kN. For an ideal shell, the optimal solution will be found on the strength constraint with significantly lower compressive loads (P=12 kN). This means that, starting from P=12 kN, the optimal parameters for an ideal shell can be determined taking into account only the strength conditions, which greatly simplifies the solution. With the optimal design of shells with initial imperfections, such a calculation can be carried out only after the compressive force reaches a value of P=24 kN

Conclusions

The numerical experiment allows us to draw the following conclusions:

1. The presence of initial imperfections in an axially-compressed smooth cylindrical shell leads to an increase in its weight as compared with an ideal shell optimally designed.

2. An increase in shell weight does not occur over the entire range of compressive loads. It occurs only under those loads, where both the local and general bucklings are the determining constraints. If the optimal solution pertains to the strength constraint, which is typical for large compressive loads, the influence of initial imperfections on the optimal design is not observed, the weight of both an ideal shell and a shell with initial imperfections in the optimal design is the same.

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Received 25 November 2019

Оптимальне проектування гладких оболонок без урахування та з урахуванням початкових недосконалостей

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У статті розглядається застосування методу випадкового пошуку до оптимального проектування стиснутих в осьовому напрямку гладких циліндричних ідеальних тонкостінних оболонок і оболонок з початковими недосконалостями. При постановці задачі математичного програмування як цільова функція розглядається мінімальна вага оболонки. Як обмеження, що накладаються на зону допустимих розв'язків, приймаються обмеження: з критичного навантаження місцевої втрати стійкості, з критичного навантаження втрати стійкості осі оболонки; умова міцності і умова щодо обмеження габаритів оболонки (радіуса і товщини стінки оболонки). При оптимальному проектуванні оболонки з початковими недосконалостями постановка задачі математичного програмування залишається такою ж, як і для ідеальної оболонки, змінюється тільки обмеження з місцевої втрати стійкості. Метою цієї роботи є дослідження зони впливу оптимальної ваги оболонок на величину стискальної сили і визначення діапазону зовнішніх стискальних навантажень, за яких визначальними є обмеження з загальної та місцевої втрати стійкості оболонки. Проведено числовий експеримент. Досліджувалися залежності ваги, товщини стінки, радіуса серединної поверхні і відношення радіуса серединної поверхні до товщини стінки від величини стискального навантаження для ідеальної оболонки і оболонки з початковими недосконалостями. В результаті проведеного числового експерименту встановлено, що наявність початкових недосконалостей у гладкої циліндричної оболонки, стиснутої в осьовому напрямку, призводить до збільшення її ваги у порівнянні з ідеальною оболонкою. Збільшення ваги відбувається не на всьому діапазоні стискальних навантажень, а тільки при навантаженнях, коли визначальними є обмеження з місцевої та загальної втрати стійкості. Якщо оптимальний розв'язок належить обмеженню з міцності, що характерно для великих стискальних навантажень, впливу початкових недосконалостей на оптимальний проект не спостерігається. Вага ідеальної оболонки і оболонки з початковими недосконалостями в оптимальному проекті виявляється однаковою.

Ключові слова: *тонкостінна циліндрична оболонка, початкові недосконалості, оптимальне проектування, випадковий пошук.*

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