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RATIONAL DESIGN OF VEHICLE BRAKING SYSTEMS WITH REDUCED WEAR OF FRICTION LINING

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The wear of the friction lining and break drum of a vehicle is uneven. It is, therefore, advisable to reduce wear where it matters most. Knowing the optimal microgeometry of the friction pair surface, this problem can be solved by design-technological methods at the design and manufacturing stages. This paper theoretically solves the problem of finding the friction surface microgeometry, which ensures the uniform wear of a friction lining. A model of a rough friction surface is adopted. To solve the optimization problem posed, the wear-contact problem of the indentation of the friction lining into the brake drum surface is first considered. Temperature functions, contact pressure, stresses, and displacements both in the friction lining and in the brake drum are sought in the form of expansions in a small parameter. For simplicity, the terms containing small parameter degrees that are higher than one are discarded. Each approximation satisfies the system of differential equations of plane thermoelasticity. The solution to the boundary-value problem of the theory of thermal conductivity in each approximation is found by the method of separation of variables. In each approximation, the thermoelastic displacement potential and the power series method are used to solve the thermoelasticity problem. Using the least-squares method, a closed system of algebraic equations is constructed, which allows one to obtain a solution to the problem of optimal design of the drum-lining friction pair, depending on the geometric and mechanical characteristics of both the brake drum and the friction lining. The found microgeometry of the friction surface provides an increase in the wear resistance of the friction lining.

Keywords: *friction pair, friction lining, drum, even wear, roughness, optimal microgeometry of the friction surface.*

Introduction

The safety, reliability, and durability of transport vehicles depends on the drum-lining friction pair of the brake mechanism. Numerous works have been devoted to the calculation and rational design of friction units (for example, [1–26], etc.). It is impossible to prevent the wear of drum-lining friction-pair elements during vehicle operation. To increase the life of the friction pair, various measures are required to reduce the wear of the friction lining and break drum [27–39]. In this regard, the optimal design of brake-mechanism friction-pair elements is of great importance.

It is known that the wear of a friction lining and a break drum occurs unevenly. The challenge is to reduce wear where it matters most. By changing the microgeometry of the friction pair surface, this problem can be solved by design and technological methods at the design stage. The problem of choosing the friction surface microgeometry, which ensures the even wear of the friction lining and maximum durability, has not yet been solved by a calculation method. The purpose of this article is to develop a mathematical model for the drum-lining friction pair, which allows one to calculate the optimal microgeometry of the friction surface for the given truck braking modes.

Problem Formulation

Consider the stress-strain state of the friction lining of a braking a car. In the intermittent braking mode, interaction occurs between the contacting surfaces of the friction lining and the break drum, and friction forces arise, which lead to the wear of the mating materials. To determine the contact pressure, it is necessary to consider the wear-contact problem of the indentation of the friction lining into the brake drum surface.

Let a friction lining with the mechanical characteristics G and μ be pressed against the brake-drum inner-surface with the mechanical characteristics G_1 (elastic modulus) and μ_1 (Poisson's ratio). In this case, the contact area occupies the entire width of the friction lining and is constant during braking.

We assume that the conditions of plane deformation are satisfied. We assign the friction lining to the polar coordinate system $r\theta$, the origin of which is in the center of concentric circles L_0 and L with radii R_0 and R, respectively. Imagine the unknown boundary of the outer contour of a friction lining L' in the form

$$r=\rho(\theta), \quad \rho(\theta)=R+\varepsilon H(\theta), \quad H(\theta)=\sum_{k=0}^{\infty}\left(a_k^0\cos k\theta+b_k^0\sin k\theta\right),$$

where $\varepsilon=R_{\max}/R$ is a small parameter; $R_{\max}$ is the maximum height of the depression (protrusion) of the unevenness of the friction lining profile; the function $H(\theta)$ is to be determined.

Similarly, the previously unknown inner contour of the break drum is close to circular and can be represented as

$$\rho_1(\theta)=R_1'+\varepsilon H_1(\theta), \qquad H_1(\theta)=\sum_{k=0}^{\infty}\left(a_k^1\cos k\theta+b_k^1\sin k\theta\right),$$

in which the function $H_1(\theta)$ is also to be determined during the solution of the optimization problem.

It is required that there be determined the friction surface microgeometry (the functions $H(\theta)$ and $H_1(\theta)$) at which an even wear will take place. To find the friction surface profile, it is necessary to supplement the statement of the problem with a criterion that allows us to determine the functions $H(\theta)$ and $H_1(\theta)$). To determine the expansion coefficients of the desired functions $H(\theta)$ and $H_1(\theta)$), we respectively use the principle of equal wear of the friction lining surface as an optimization criterion.

We carry out a theoretical analysis to determine the microgeometry of the drum-lining friction surface, which would provide an almost even distribution of wear. Therefore, by choosing the friction surface microgeometry, we will reduce the unevenness of wear of the friction lining and the break drum.

Solution Method

To solve the optimization problem posed, the wear-contact problem of the indentation of the friction lining into the surface of the brake drum is first considered [40].

The condition connecting the movement of the friction lining and the break drum has the form

$$v_1+v_2=\delta(\theta) \quad (|\theta|\le\theta_0), \tag{1}$$

where $\delta(\theta)$ is the settlement of points on the surfaces of the friction lining and the break drum, determined by the shapes of the friction-lining and break-drum surfaces, as well as the magnitude of the pressing force P; $2\theta_0$ is the angle contact of the friction lining. In the contact zone, in addition to the normal pressure $p(\theta, t)$, there is a tangential force associated with the contact pressure according to the Amonton-Coulomb law.

The friction forces $\tau_{r\theta}(\theta, t)$ contribute to heat release in the contact area. The total amount of heat per unit time is proportional to the power of the friction forces. The amount of heat generated per unit of time on a unit contact area with the coordinate θ will be $Q(\theta, t)=Vfp(\theta, t)$, where V is the speed of the vehicle at the moment of braking; f is the friction coefficient of the friction pair.

The total amount of heat, $Q(\theta, t)$, will be consumed as follows: on the heat flow into the friction lining, $Q_*(\theta, t)$, and on the heat flow to increase the temperature of the break drum, $Q_b(\theta, t)$.

For friction-lining and break-drum displacements, we have

$$v_1=v_{1e}+v_{1r}+v_{1w}, \qquad v_2=v_{2e}+v_{2r}+v_{2w}$$

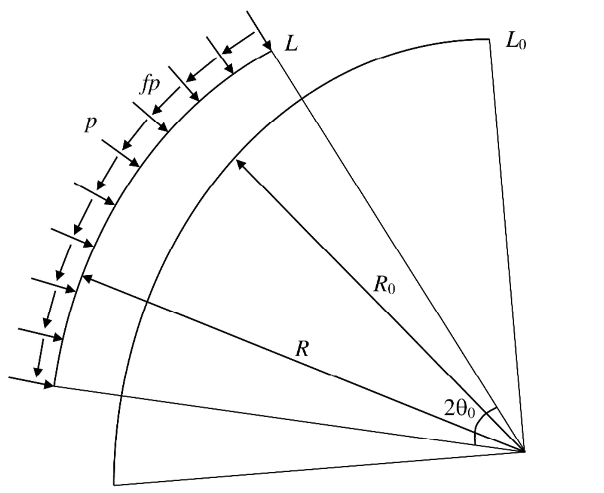
Here, v_{1e} are the thermoelastic displacements of the points of the friction-lining contact surface; v_{1r} are the displacements caused by the crushing of microprotrusions of the friction-lining surface, v_{1w} are the displacements caused by the wear of the friction-lining surface; v_{2e}, v_{2r}, v_{2w} are the displacements for the brake drum.

The rate of change of surface displacements during the wear of the friction lining and the break drum will be [41, 42]

$$\frac{dv_k}{dt}=K^{(k)}p(\theta,t), \tag{2}$$

where $K^{(k)}$ is the wear coefficient of the material (k=1 is the wear coefficient for the friction lining, and k=2 is the wear coefficient for the break drum).

To determine v_{1e}, v_{1r} and v_{2e}, v_{2r}, thermoelasticity problems are solved for the friction lining and the break drum, respectively. The thermal conductivity coefficients of the material in the axial, circumferential, and radial directions are assumed to be the same and independent of coordinates and temperature. The friction lining is modeled by a circular curved beam with a cross-section close to a narrow rectangle (see the figure).



A scheme for calculating the optimal microgeometry of the outer surface of the friction lining

For the friction lining,

$$\Delta T=0,$$

$$\text{with } r=\rho(\theta) \quad \lambda\frac{\partial T}{\partial n}=-Q_*,$$

$$T=T_c \quad \text{with } r=R_0,$$

$$T=T_c \quad \text{with } \theta=0; \qquad T=T_c \quad \text{with } \theta=2\theta_0,$$

$$\text{with } r=\rho(\theta) \quad \sigma_n=-p(\theta), \qquad \tau_{nt}=-fp(\theta),$$

$$\text{with } r=R_0 \quad v_r=0, \quad v_\theta=0.$$

Here, λ is the thermal conductivity coefficient of the friction lining material; Δ is the Laplace operator; T is a function of temperature; T_c is the ambient temperature; n, t are the normal and tangent to the outer surface of the friction lining; v_r, v_θ are the radial and tangent components of the displacement vector of points L, respectively.

At the straight ends of the friction lining, the boundary conditions are accepted as

$$\int_{R_0}^{R}\sigma_\theta dr=0, \qquad \int_{R_0}^{R}\tau_{r\theta} dr=0, \qquad \int_{R_0}^{R}\sigma_\theta r dr=0 \quad \text{with } \theta=\pm\theta_0,$$

where σ_r, σ_θ, $\tau_{r\theta}$ are the components of the stress tensor.

Similarly, the problem of thermoelasticity is posed to determine the displacements v_{2e}, v_{2r} of the contact surface of the break drum. To determine v_{1w} and v_{2w}, the kinetic equation of wear for the material of the friction lining and break drum is used (2).

We look for temperature functions, contact pressure, stresses, and displacements in the friction lining and break drum in the form of expansions in the small parameter ε, in which, for simplicity, we leave the terms containing degrees of ε that are not higher than one. We find the stress components for $r=\rho(\theta)$ by expanding in a series the expressions for stresses in the neighborhood $r=R$. Using the perturbation method for the boundary-value problem of thermoelasticity, we obtain a sequence of boundary-value problems for the friction lining with circular boundaries for the inner and outer surfaces [43]. Each approximation satisfies the system of differential equations of plane thermoelasticity. The solution to the boundary-value problem of the theory of heat conductivity in each approximation is sought by the method of separation of variables. In solving the thermoelasticity problem, in each approximation, the thermoelastic displacement potential and the power series method are used.

Based on the obtained solution to the problem of thermoelasticity and integration of the kinetic equation (2) of wear of the friction lining material in the zeroth approximation, we find the displacements v_1^0 of its contact surface. The thermoelasticity problem for the brake drum is solved in the same way. Based on the obtained solution of the thermoelasticity problem for the brake drum and the kinetic equation of wear of the brake drum material, the displacement v_2^0 of its contact surface is found in the zeroth approximation. The quantities v_1^0 and v_2^0 found are substituted into the main contact condition (1) in the zeroth approximation. Carrying out the algebraization of the main contact equation in a similar way [27–28], we obtain an infinite algebraic system with respect to α_k^0 (k=0, 1, 2, …), β_k^0 (k=1, 2, …) and α_k^1, β_k^1, etc. The obtained systems allow us to find the contact pressure in the zeroth approximation by numerical methods.

Then, in a similar way, the solution to the wear-contact problem is constructed in the first approximation. Repeating the procedure for constructing algebraic systems to find the desired coefficients, we obtain an infinite algebraic system with respect to $\alpha_{k,0}^1$ (k=0, 1, 2, …), $\beta_{k,0}^1$ (k=1, 2, …) and $\alpha_{k,1}^1$, $\beta_{k,1}^1$, etc.

The right-hand sides of infinite algebraic systems include the coefficients a_k^0, b_k^0, a_k^1, b_k^1 of expansions of the functions $H(\theta)$ and $H_1(\theta)$. With the known functions $H(\theta)$ and $H_1(\theta)$, the resulting systems make it possible to find the contact pressure $p(\theta, t)$. The resulting algebraic system of equations is not yet closed.

Let $\bar{h}$ be the desired value of material wear for the friction lining surface. The value $\bar{h}$ is initially unknown and requires determination in the process of solving the optimization problem. By solving the wear-contact problem of the indentation of the friction lining into the break-drum inner-surface, for the wear of the friction lining surface, we find

$$h(\theta,\tau) = K^{(1)}\left\{\left[p_0^0(\theta)+\varepsilon p_0^1(\theta)\right]\tau + \left[p_1^0(\theta)+\varepsilon p_1^1(\theta)\right]\frac{\tau^2}{2}+...\right\}.$$

The abrasive wear formula, which can be written as

$$h(\theta,t) = F(\theta,t,a_0^0,a_k^0,b_k^0,a_0^1,a_k^1,b_k^1) \qquad (k=1, 2, …, m),$$

shows that the wear linearly depends on the desired coefficients a_k^0, b_k^0, a_k^1, b_k^1 of Fourier series of the functions $H(\theta)$ and $H_1(\theta)$. To construct the missing algebraic equations for finding the coefficients a_k^0, b_k^0, a_k^1, b_k^1, we use the principle of least squares.

The wear h of the friction lining is a function of the independent variable θ and (4m+2) of parameters a_0^0, a_k^0, b_k^0, a_0^1, a_k^1, b_k^1. Time is considered a free parameter. The parameters a_0^0, a_k^0, b_k^0, a_0^1, a_k^1, b_k^1 are constant (in the general case, they depend on time), but they are unknown in advance and are be determined. To find the unknown parameters, we perform a number of calculations.

We divide the segment $[-\theta_0, \theta_0]$ of the change of θ into M parts, where M>4m+2.

$$\theta_i = -\theta_0 + i\Delta\theta, \quad \Delta\theta = 2\theta_0/M,$$

$$p(\theta_i,t) = F(\theta_i,t,a_0^0,a_k^0,b_k^0,a_0^1,a_k^1,b_k^1) \quad (i=1, 2, …, M). \tag{3}$$

Let us find the values of the unknown parameters that will provide for values of the abrasive wear function (3) a constant value in the best way

$$F(\theta_i,t,a_0^0,a_k^0,b_k^0,a_0^1,a_k^1,b_k^1) = \bar{h} \qquad (i=1, 2, …, M).$$

The most probable values of the parameters will be those for which the sum of squared deviations ε_i will be the smallest

$$U = \sum_{i=1}^{M}\left[F(\theta_i,t,a_0^0,a_k^0,b_k^0,a_0^1,a_k^1,b_k^1) - \bar{h}\right]^2 \to \min. \tag{4}$$

For any moment of time, we consider a_0^0, a_k^0, b_k^0, a_0^1, a_k^1, b_k^1 (k=1, 2, …, m) as independent variables. Equating to zero the partial derivatives of the left-hand side of (4) with respect to these variables and $\bar{h}$, we obtain (4m+3) equations with (4m+3) unknowns

$$\frac{\partial U}{\partial a_0^0}=0, \quad \frac{\partial U}{\partial a_k^0}=0, \quad \frac{\partial U}{\partial b_k^0}=0 \qquad (k=1, 2, …, m), \tag{5}$$

$$\frac{\partial U}{\partial a_0^1}=0, \quad \frac{\partial U}{\partial a_k^1}=0, \quad \frac{\partial U}{\partial b_k^1}=0, \quad \frac{\partial U}{\partial \bar{h}}=0.$$

The design and solution of system (5) is greatly simplified, since the function $F(\theta_i, t, a_0^0, a_k^0, b_k^0, a_0^1, a_k^1, b_k^1)$ is linear with respect to the unknown parameters. This system of equations closes the infinite algebraic system of the wear-contact problem and must be solved together with it for fixed values of time.

Analysis of Simulation Results

The joint solution of the obtained systems of equations allows us to find approximate values of the coefficients a_k^0, b_k^0, a_k^1, b_k^1, $\bar{h}$, α_k, β_k. The thermo-physical and mechanical characteristics of the friction- lining and brake-drum materials, their geometric dimensions, and vehicle speed are free parameters of the problem. For the numerical implementation of the proposed method, calculations were carried out as applied to the brake mechanisms of KamAZ-5320 trucks. The following parameters were taken to be constant: R=0.19 m; friction lining thickness $h_н$=0.016 m; friction lining width $b_н$=0.14 m; R_1'=0.2 m; outer radius of the break drum $R_1^í$=0.25 m; f=0.35; coefficient of wear resistance of the friction lining material $K^{(1)}$=1.5·10^{-6} and of the break drum $K^{(2)}$=2·10^{-8}; E=6.9·10^3 MPa; E_1=1.8·10^5 MPa; μ=0.4; μ_1=0.3 for gray iron material of the break drum.

The values of control parameters (coefficients a_0^0, a_k^0, b_k^0, a_0^1, a_k^1, b_k^1) were found depending on the physico-mechanical characteristics of the friction pair for different moments of time. In the expansion of the desired functions $H(\theta)$ and $H_1(\theta)$, limitations to k=5 terms were used. The calculation results for determining the microgeometry of the friction contact surface at the initial moment t=0 for different vehicle speeds during braking are given in the table where line 1 refers to the friction-lining surface roughness.

Values of Fourier coefficients for optimal roughness (μm)

Material	a_0	a_1	a_2	a_3	a_4	a_5	b_1	b_2	b_3	b_4	b_5
					V=50 km/h						
1	0.389	0.303	0.243	-0.185	0.086	0.042	0.411	0.278	0.115	-0.067	0.029
2	0.260	0.182	-0.163	0.154	0.077	0.035	0.228	0.191	0.089	0.039	0.028
					V=80 km/h						
1	0.456	0.393	-0.315	0.228	0.114	0.050	0.437	-0.278	0.185	0.096	0.044
2	0.193	0.176	0.108	0.089	-0.071	0.042	0.245	0.206	0.114	-0.068	0.039

With prolonged braking, temperature stresses become important for the brake mechanism of a vehicle. Our calculations show that the temperature rises when the wear of the friction lining increases.

Conclusion

The practice of operating the friction pairs of drum-lining brake mechanisms indicates that at the stage of developing new designs of truck brake mechanisms, it is necessary to take into account cases where uneven wear occurs. Knowing the coefficients a_k^0, b_k^0, a_k^1, b_k^1 of the functions $H(\theta)$ and $H_1(\theta)$ allows one to choose the roughness class of the machined outer surface of the friction lining and of the inner surface of the break drum, which provides increased wear resistance of the friction pair of a truck brake system.

References

1. Balakin, V. A., Sergiyenko, V. P., & Lysenok, Yu. V. (2004). *Optimizatsiya konstruktsiy ventiliruyemykh tormozov avtomobiley* [Design optimization for ventilated car brake]. *Treniye i iznos – Journal of Friction and Wear*, vol. 25, iss. 5, pp. 474–584 (in Russian).
2. Balakin, V. A., Sergiyenko, V. P., Chaus, V. P., & Ivanov, A. A. (2005). *Vliyaniye iznosa na teplovoy rezhim raboty tormoza* [Effect of wear on thermal regime of a brake]. *Treniye i iznos – Journal of Friction and Wear*, vol. 26, iss. 6, pp. 571–574 (in Russian).
3. Gao, C. H., Huang, J. M., Lin, X. Z., & Tang, X. S. (2007). Stress analysis of thermal fatigue fracture of brake disks based on thermomechanical coupling. *Journal of Tribology*, vol. 129, iss. 3, pp. 536–543. https://doi.org/10.1115/1.2736437.
4. Hwang, P., Wu, X., & Jeon, Y. B. (2009). Thermal–mechanical coupled simulation of a solid brake disc in repeated braking cycles. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 223, iss. 7, pp. 1041–1048. https://doi.org/10.1243/13506501JET587.
5. Valetov, V. A. & Ivanov, A. Yu. (2010). *Mikrogeometriya poverkhnostey detaley i ikh funktsionalnyye svoystva*

[Microgeometry and functional characteristics of machine parts surfaces]. *Izvestiya vysshikh uchebnykh zavedeniy. Priborostroyeniye – Journal of Instrument Engineering*, vol. 53, iss. 8, pp. 7–11 (in Russian).

6. Baranovskiy, D. M. (2010). *Pidvyshchennia dovhovichnosti dyzeliv iz zastosuvanniam optymalnoi mikroheometrii trybosystemy «hilza-kiltse»* [Rise of longevity of diesels with application of optimum microgeometry of "shell-ring" tribosystems]. *Avtomobilnyi transport – Automobile Transport,* iss. 26, pp. 81–84 (in Ukrainian).
7. Andreyev, Yu. S. & Medunetskiy, V. V. (2012). *Issledovaniye izmeneniya mikrorelyefa poverkhnostey v protsesse ikh treniya skolzheniya* [Study of surface micro-relief change under sliding friction]. *Izvestiya vysshikh uchebnykh zavedeniy. Priborostroeniye – Journal of Instrument Engineering*, vol. 55, iss. 9, pp. 30–34 (in Russian).
8. Sergienko, V. P., Tseluev, M. Yu., Kolesnikov, V. I., Sychev, A. P., Savonchik, V. A., & Yanuchkovskii, V. I. (2013). Studying thermal state of friction pairs of multidisc brake. *Journal of Friction and Wear*, vol. 34, pp. 421–428. https://doi.org/10.3103/S106836661306010X.
9. Rashid, A. & Strömberg, N. (2013). Sequential simulation of thermal stresses in disc brakes for repeated braking. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 227, iss. 8, pp. 919–929. https://doi.org/10.1177/1350650113481701.
10. Collignon, M., Cristol, A.-L., Dufrénoy, P., Desplanques, Y., & Balloy, D. (2013). Failure of truck brake discs: A coupled numerical-experimental approach to identifying critical thermomechanical loadings. *Tribology International*, vol. 59. pp. 114–120. https://doi.org/10.1016/j.triboint.2012.01.001.
11. Bezazychnyy, V. F., Sutyagin, A. N. (2014). *K voprosu raschetnogo opredeleniya intensivnosti iznashivaniya kontaktiruyemykh poverkhnostey s uchetom uprochneniya poverkhnostnogo sloya detaley* [The problem of calculation of the wear rate of the contacted surfaces, taking into account hardening of the surface layer of parts]. *Uprochnyayushchiye tekhnologii i pokrytiya – Strengthening technologies and coatings*, no. 1, pp. 3–6 (in Russian).
12. Volchenko, A. I., Kindrachuk, M. V., Bekish, I. O., Malyk, V. Ya., & Snurnikov, V. I. (2015). *Termicheskiye napryazheniya v obodakh tormoznykh barabanov avtotransportnykh sredstv* [Thermal stresses of the rims of brake drums in vehicles]. *Problemy treniya i iznashivaniya – Problems of Friction and Wear*, no. 4 (69), pp. 28–37 (in Russian). https://doi.org/10.18372/0370-2197.4(69).9992.
13. Valetov, V. A. (2015). *Problemy optimizatsii mikrogeometrii poverkhnostey detaley dlya obespecheniya ikh konkretnykh funktsionalnykh svoystv* [Problems of optimization of workpiece surface microgeometry to ensure specific functional properties]. *Izvestiya vysshikh uchebnykh zavedeniy. Priborostroenie – Journal of Instrument Engineering*, vol. 58, iss. 4, pp. 250–267 (in Russian). https://doi.org/10.17586/0021-3454-2015-58-4-250-267.
14. Ostapchuk, A. K., Mikhalishchev, A. G., & Kuznetsova, E. M. (2015). *Tekhnologicheskoye obespecheniye parametrov sherokhovatosti poverkhnosti kataniya kolesnoy pary posle mekhanicheskoy obrabotki* [Technological support of the roughness parameters of the rolling surface of the pair of wheels after machining]. *Transport. Transportnyye sooruzheniya. Ekologiya – Transport. Transport facilities. Ecology*, no. 2, pp. 73–86 (in Russian).
15. Yevtushenko, A. A., Grzes, P., & Adamowicz, A. (2015). Numerical analysis of thermal stresses in disk brakes and clutches (a review). *Numerical Heat Transfer, Part A: Applications*, vol. 67, iss. 2, pp. 170–188. https://doi.org/10.1080/10407782.2014.923221.
16. Yevtushenko, A., Kuciej, M., Och, E., & Yevtushenko, O. (2016). Effect of the thermal sensitivity in modeling of the frictional heating during braking. *Advances in Mechanical Engineering*, vol. 8, iss. 12, pp. 10. https://doi.org/10.1177/1687814016681744.
17. Belhocine, A., Abu Bakar, A., & Bouchetara, M. (2016). Thermal and structural analysis of disc brake assembly during single stop braking event. *Australian Journal of Mechanical Engineering*, vol. 14, iss, 1, pp. 26–38. https://doi.org/10.1080/14484846.2015.1093213.
18. Polyakov, P. A., Fedotov Ye. S., & Polyakova, Ye. A. (2017). *Metod proyektirovaniya sovremennykh tormoznykh mekhanizmov s servousileniyem* [Design method of modern servo-assisted brake mechanisms]. *Vestnik Irkutskogo gosudarstvennogo tekhnicheskogo universiteta – Proceedings of Irkutsk State Technical University*, vol. 21, no. 7, pp. 39–50 (in Russian). https://doi.org/10.21285/1814-3520-2017-7-39-50.
19. Le Gigan, G. (2017). Improvement in the brake disc design for heavy vehicles by parametric evaluation. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 231, iss. 14, pp. 1989–2004. https://doi.org/10.1177/0954407016688421.
20. Djafri, M., Bouchetara, M., Busch, C., Khatir, S., Khatir, T., Weber, S., Shbaita, K., & Abdel Wahab, M. (2018). Influence of thermal fatigue on the wear behavior of brake discs sliding against organic and semimetallic friction materials. *Tribology Transactions*, vol. 61, iss. 5, pp. 861–868. https://doi.org/10.1080/10402004.2018.1437491.
21. Yevtushenko, A., Kuciej, M., & Och, E. (2018). Modeling of the temperature regime and stress state in the thermal sensitive pad-disk brake system. *Advances in Mechanical Engineering*, vol. 10, iss. 6, pp. 12. https://doi.org/10.1177/1687814018781285.

22. Yevtushenko, A., Kuciej, M., & Topczewska, K. (2018). Analytical model to investigate distributions of the thermal stresses in the pad and disk for different temporal profiles of friction power. *Advances in Mechanical Engineering*, vol. 10, iss. 10, pp. 10. https://doi.org/10.1177/1687814018806670.
23. Bilgic Istoc, S. & Winner, H. (2019). Heat cracks in brake discs for heavy-duty vehicles: Influences, interactions and prediction potential. Proceedings of XXXVIII. International μ-Symposium 2019 Bremsen-Fachtagung, October 25th 2019. Berlin – Heidelberg: Springer Vieweg, pp. 55–69. https://doi.org/10.1007/978-3-662-59825-2_7.
24. Afzal, A. & Abdul Mujeebu, M. (2019). Thermo-mechanical and structural performances of automobile disc brakes: A review of numerical and experimental studies. *Archives of Computational Methods in Engineering*, vol. 26, pp. 1489–1513. https://doi.org/10.1007/s11831-018-9279-y.
25. Modanloo, A. & Talaee, M. R. (2020). Analytical thermal analysis of advanced disk brake in high speed vehicles. *Mechanics of Advanced Materials and Structures*, vol. 27, iss. 3, pp. 209–217. https://doi.org/10.1080/15376494.2018.1472340.
26. Subel, J. & Kienhöfer, F. W. (2020). Thermal comparison of heavy vehicle wheel assemblies under alpine braking. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 234, iss. 1, pp. 28–38. https://doi.org/10.1177/0954407019844359.
27. Mirsalimov, V. M. (2005). *Optimalnoye proyektirovaniye uzla treniya* [Optimal design of friction unit]. *Izvestiya TulGU, seriya: Matematika, Mekhanika, Informatika – Bulletin of the Tula State University. Series: Mathematics, Mechanics, Informatics*, iss. 2, pp. 161–172 (in Russian).
28. Mirsalimov, V. M. (2008). An inverse wear contact problem for a friction couple. *Journal of Machinery Manufacture and Reliability*, vol. 37, pp. 53–59. https://doi.org/10.1007/s12001-008-1011-2.
29. Mirsalimov, V. M. & Aknundova, P. E. (2015). Minimization of contact pressure for hub–shaft friction pair. *Journal of Friction and Wear*, vol. 36, pp. 404–408. https://doi.org/10.3103/S1068366615050116.
30. Mirsalimov, V. M. & Aknundova, P. E. (2016). Minimization of abrasive wear for the internal surface of the hub of a friction pair. *Journal of Friction and Wear*, vol. 37, pp. 424–429. https://doi.org/10.3103/S1068366616050135.
31. Mirsalimov, V. M. & Akhundova, P. E. (2017). Optimal design of a frictional pair of a hub–plunger. *Journal of Friction and Wear*, vol. 38, pp. 384–389. https://doi.org/10.3103/S1068366617050075.
32. Mirsalimov, V. M. & Aknundova, P. E. (2018). Inverse problems of damage mechanics for a hub of a friction pair. *International Journal of Damage Mechanics*, vol. 27, iss. 1, pp. 82–96. https://doi.org/10.1177/1056789516662698.
33. Mirsalimov, V. M. & Aknundova, P. E. (2018). Minimization of the thermal state of the hub of a frictional pair using the criterion of uniform temperature distribution on a friction surface. *Journal of Friction and Wear*, vol. 39, pp. 405–411. https://doi.org/10.3103/S1068366618050112.
34. Mirsalimov, V. M. & Aknundova, P. E. (2018). Minimization of the thermal state of the hub of a friction pair. *Engineering Optimization*, vol. 50, pp. 651–670. https://doi.org/10.1080/0305215X.2017.1328062.
35. Mirsalimov, V. M. & Aknundova, P. E. (2018). Minimization of stress state of a hub of friction pair. *Advances in Mathematical Physics*, vol. 2018, Article ID 8242614, 10 p. https://doi.org/10.1155/2018/8242614.
36. Mirsalimov, V. M. & Aknundova, P. E. (2018). Inverse wear contact problem of the friction unit. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 232, iss. 22, pp. 4216–4226. https://doi.org/10.1177/0954406217749267.
37. Mirsalimov, V. M. & Akhundova, P. E. (2019). The optimal design of a friction unit with uniform contact pressure. *Journal of Friction and Wear*, vol. 40, pp. 562–568. https://doi.org/10.3103/S1068366619060187.
38. Mirsalimov, V. M. & Aknundova, P. E. (2019). Inverse problem of contact fracture mechanics for a hub of friction pair taking into account thermal stresses. *Mathematics and Mechanics of Solids*, vol. 24, iss. 6, pp. 1763–1781. https://doi.org/10.1177/1081286518805525.
39. Mirsalimov, V. M. & Aknundova, P. E. (2020). Optimum problem on wear decrease for a hub of friction pair. *Mechanics of Advanced Materials and Structures*, vol. 27, iss. 5, pp. 353–363. https://doi.org/10.1080/15376494.2018.1472827.
40. Mirsalimov, V. M., Hasanov, Sh. G., & Heidarov, Sh. G. (2018). *Iznosokontaktnaya zadacha o vdavlivanii kolodki s friktsionnoy nakladkoy v poverkhnost barabana* [Wear-contact problem of pressing brake shoe with friction lining into drum surface]. Proceedings of XII International scientific conference "Tribology for Mechanical Engineering" dedicated to the 80th anniversary of IMASH RAS (Moscow, November 19–21 2018). Moscow – Izhevsk: Institute of Computer Research, pp. 341–344 (in Russian).
41. Goryacheva, I. G. & Dobychin, M. N. (1988). *Kontaktnyye zadachi v tribologii* [Contact problems in tribology]. Moscow: Mashinostroyeniye, 256 p. (in Russian).
42. Goryacheva, I. G. (2001). *Mekhanika friktsionnogo vzaimodeystviya* [Mechanics of frictional interaction]. Moscow: Nauka, 478 p. (in Russian).
43. Muskhelishvili, N. I. (1977). Some basic problems of mathematical theory of elasticity. Dordrecht: Springer, 732 p. https://doi.org/10.1007/978-94-017-3034-1.

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Розв'язання задачі зниження зношення фрикційної накладки гальмівної системи автомобіля

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Зношування накладки і барабана гальмівного механізму автомобіля відбувається нерівномірно, тому доцільно зменшувати знос там, де він має найбільше значення. Знаючи оптимальну мікрогеометрію поверхні тертя фрикційної пари, цю задачу можна розв'язувати конструкторсько-технологічними методами на етапах проектування і виготовлення. В роботі теоретично розв'язується задача зі знаходження мікрогеометрії поверхні тертя, що забезпечує рівномірний знос фрикційної накладки. Прийнята модель шорсткої поверхні тертя. Для розв'язання поставленої задачі оптимізації спочатку розглядається зносоконтактна задача щодо вдавлювання накладки в поверхню гальмівного барабана. Температурні функції, контактний тиск, напруження і переміщення в накладці і барабані шукаються у вигляді розкладів по малому параметру. Для спрощення члени, що мають ступінь малого параметра вище першого, відкидаються. Кожне наближення задовольняє систему диференціальних рівнянь плоскої термопружності. Розв'язок крайової задачі теорії теплопровідності в кожному наближенні знаходиться методом розділення змінних. У кожному наближенні для розв'язання задачі термопружності використовуються термопружний потенціал переміщень і метод степеневих рядів. За допомогою методу найменших квадратів побудована замкнута система алгебраїчних рівнянь, що дозволяє отримати розв'язок задачі оптимального проектування пари тертя «барабан-накладка» в залежності від геометричних і механічних характеристик гальмівного барабана і накладки. Знайдена геометрія поверхні тертя забезпечує підвищення зносостійкості фрикційної накладки.

***Ключові слова**: фрикційна пара, накладка, барабан, рівномірне зношування, шорсткість, оптимальна геометрія поверхні тертя.*

Література

1. Балакин В. А., Сергиенко В. П., Лысенок Ю. В. Оптимизация конструкций вентилируемых тормозов автомобилей. *Трение и износ*. 2004. Т. 25. № 5. С. 474–584.
2. Балакин В. А., Сергиенко В. П., Чаус В. П., Иванов А. А. Влияние износа на тепловой режим работы тормоза. *Трение и износ*. 2005. Т. 26. № 6. С. 571–574.
3. Gao C. H., Huang J. M., Lin X. Z., Tang X. S. Stress analysis of thermal fatigue fracture of brake disks based on thermomechanical coupling. *J. Tribology*. 2007. Vol. 129. Iss. 3. P. 536–543. https://doi.org/10.1115/1.2736437.
4. Hwang P., Wu X., Jeon Y. B. Thermal–mechanical coupled simulation of a solid brake disc in repeated braking cycles. *Proc. Institution Mech. Eng., Part J: J. Eng. Tribology*. 2009. Vol. 223. Iss. 7. P. 1041–1048. https://doi.org/10.1243/13506501JET587.
5. Валетов В. А., Иванов А. Ю. Микрогеометрия поверхностей деталей и их функциональные свойства. *Изв. вузов. Приборостроение*. 2010. Т. 53. № 8. С. 7–11.
6. Барановський Д. М. Підвищення довговічності дизелів із застосуванням оптимальної мікрогеометрії трибосистеми «гільза-кільце». *Автомоб. трансп.*. 2010. Вип. 26. С. 81–84.
7. Андреев Ю. С., Медунецкий В. В. Исследование изменения микрорельефа поверхностей в процессе их трения скольжения. *Изв. вузов. Приборостроение*. 2012. Т. 55. № 9. С. 30–34.
8. Сергиенко В. П., Целуев М. Ю., Колесников В. И., Сычев А. П., Савочник В. А., Янучковский В. И. Исследование теплового режима пар трения многодискового тормоза. *Трение и износ*. 2013. Т. 34. № 6. С. 555–564.
9. Rashid A., Strömberg N. Sequential simulation of thermal stresses in disc brakes for repeated braking. *Proc. Institution Mech. Eng. Part J: J. Eng. Tribology*. 2013. Vol 227. Iss. 8. P. 919–929. https://doi.org/10.1177/1350650113481701.
10. Collignon M., Cristol A.-L., Dufrénoy P., Desplanques Y., Balloy D. Failure of truck brake discs: A coupled numerical–experimental approach to identifying critical thermomechanical loadings. *Tribology Intern*. 2013. Vol. 59. P. 114–120. https://doi.org/10.1016/j.triboint.2012.01.001.
11. Безъязычный В. Ф., Сутягин А. Н. К вопросу расчетного определения интенсивности изнашивания контактируемых поверхностей с учетом упрочнения поверхностного слоя деталей. *Упрочняющие технологии и покрытия*. 2014. № 1. С. 3–6.
12. Вольченко А. И., Киндрачук М. В., Бекиш И. О., Малык В. Я., Снурников В. И. Термические напряжения в ободах тормозных барабанов автотранспортных средств. *Пробл. трения и изнашивания*. 2015. № 4 (69). С. 28–37. https://doi.org/10.18372/0370-2197.4(69).9992.
13. Валетов В. А. Проблемы оптимизации микрогеометрии поверхностей деталей для обеспечения их конкретных функциональных свойств. *Изв. вузов. Приборостроение*. 2015. Т. 58. № 4. С. 250–267. https://doi.org/10.17586/0021-3454-2015-58-4-250-267.

14. Остапчук А. К., Михалищев А. Г., Кузнецова Е. М. Технологическое обеспечение параметров шероховатости поверхности катания колесной пары после механической обработки. *Трансп. Трансп. сооружения. Экология*. 2015. № 2. С. 73–86.
15. Yevtushenko A. A., Grzes P., Adamowicz A. Numerical analysis of thermal stresses in disk brakes and clutches (a review). *Numerical Heat Transfer, Part A: Applications*. 2015. Vol. 67. Iss. 2. P. 170–188. https://doi.org/10.1080/10407782.2014.923221.
16. Yevtushenko A., Kuciej M., Och E., Yevtushenko O. Effect of the thermal sensitivity in modeling of the frictional heating during braking. *Advances in Mech. Eng*. 2016. Vol. 8. Iss. 12. P. 10. https://doi.org/10.1177/1687814016681744.
17. Belhocine A., Abu Bakar A., Bouchetara M. Thermal and structural analysis of disc brake assembly during single stop braking event. *Australian J. Mech. Eng*. 2016. Vol. 14. Iss. 1. P. 26–38. https://doi.org/10.1080/14484846.2015.1093213.
18. Поляков П. А., Федотов Е. С., Полякова Е. А. Метод проектирования современных тормозных механизмов с сервоусилением. *Вестн. Иркут. техн. ун-та*. 2017. Т. 21. № 7. С. 39–50. https://doi.org/10.21285/1814-3520-2017-7-39-50.
19. Le Gigan G. Improvement in the brake disc design for heavy vehicles by parametric evaluation. *Proc. Institution Mech. Eng. Part D: J. Automobile Eng*. 2017. Vol. 231. Iss. 14. P. 1989–2004. https://doi.org/10.1177/0954407016688421.
20. Djafri M., Bouchetara M., Busch C., Khatir S., Khatir T., Weber S., Shbaita K., Abdel Wahab M. Influence of thermal fatigue on the wear behavior of brake discs sliding against organic and semimetallic friction materials. *Tribology Transactions*. 2018. Vol. 61. Iss. 5. P. 861–868. https://doi.org/10.1080/10402004.2018.1437491.
21. Yevtushenko A., Kuciej M., Och E. Modeling of the temperature regime and stress state in the thermal sensitive pad-disk brake system. *Advances in Mech. Eng*. 2018. Vol. 10. Iss. 6. P. 12. https://doi.org/10.1177/1687814018781285.
22. Yevtushenko A., Kuciej M., Topczewska K. Analytical model to investigate distributions of the thermal stresses in the pad and disk for different temporal profiles of friction power. *Advances in Mech. Eng.* 2018. Vol. 10. Iss. 10. P. 10. https://doi.org/10.1177/1687814018806670.
23. Bilgic Istoc S., Winner H. Heat cracks in brake discs for heavy-duty vehicles: influences, interactions and prediction potential. *XXXVIII Intern. μ-Symposium 2019 Brake Conf.*: proc. XXXVIII Intern. μ-Symposium 2019 Bremsen-Fachtagung. October 25th 2019. Berlin – Heidelberg: Springer Vieweg, 2019. P. 55–69. https://doi.org/10.1007/978-3-662-59825-2_7.
24. Afzal A., Abdul Mujeebu M. Thermo-mechanical and structural performances of automobile disc brakes: A review of numerical and experimental studies. *Archives Computational Methods in Eng*. 2019. Vol. 26. P. 1489–1513. https://doi.org/10.1007/s11831-018-9279-y.
25. Modanloo A., Talaee M. R. Analytical thermal analysis of advanced disk brake in high speed vehicles. *Mechanics of Advanced Materials and Structures*. 2020. Vol. 27. Iss. 3. P. 209–217. https://doi.org/10.1080/15376494.2018.1472340.
26. Subel J., Kienhöfer F. W. Thermal comparison of heavy vehicle wheel assemblies under alpine braking. *Proc. Institution Mech. Eng. Part D. J. Automobile Eng.* 2020. Vol. 234. Iss. 1. P. 28–38. https://doi.org/10.1177/0954407019844359.
27. Мирсалимов В. М. Оптимальное проектирование узла трения. *Изв. Тул. гос. ун-та. Сер. Математика, Механика, Информатика*. 2005. Вып. 2. С. 161–172.
28. Мирсалимов В. М. Обратная износоконтактная задача для фрикционной пары. *Пробл. машиностроения и надежности машин*. 2008. Т. 37. № 1. С. 62–69.
29. Мирсалимов В. М., Ахундова П. Э. Минимизация контактного давления для фрикционной пары «втулка-вал». *Трение и износ*. 2015. Т. 36. № 5. С. 529–535.
30. Мирсалимов В. М., Ахундова П. Э. Минимизация абразивного износа внутренней поверхности втулки фрикционной пары. *Трение и износ*. 2016. Т. 37. № 5. С. 551–557.
31. Мирсалимов В. М., Ахундова П. Э. Оптимальное проектирование фрикционной пары втулка-плунжер. *Трение и износ*. 2017. Т. 38. № 5. С. 454–460.
32. Mirsalimov V. M., Aknundova P. E. Inverse problems of damage mechanics for a hub of a friction pair. *Intern. J. Damage Mechanics*. 2018. Vol. 27. Iss. 1. P. 82–96. https://doi.org/10.1177/1056789516662698.
33. Мирсалимов В. М., Ахундова П. Э. Минимизация теплового состояния втулки фрикционной пары с помощью критерия равномерного распределения температуры на поверхности трения. *Трение и износ*. 2018. Т. 39. № 5. С. 514–522.
34. Mirsalimov V. M., Aknundova P. E. Minimization of the thermal state of the hub of a friction pair. *Eng. Optimization*. 2018. Vol. 50, Iss. 4. P. 651–670. https://doi.org/10.1080/0305215X.2017.1328062.
35. Mirsalimov V. M., Aknundova P. E. Minimization of stress state of a hub of friction pair. *Advances in Math. Physics*. 2018. Vol. 2018. Article ID 8242614. https://doi.org/10.1155/2018/8242614.
36. Mirsalimov V. M., Aknundova P. E. Inverse wear contact problem of the friction unit. *Proc. Institution Mech. Eng. Part C: J. Mech. Eng. Sci*. 2018. Vol. 232. Iss. 22. P. 4216–4226. https://doi.org/10.1177/0954406217749267.
37. Мирсалимов В. М., Ахундова П. Э. Оптимальное проектирование узла трения с равномерным контактным давлением. *Трение и износ*. 2019. Т. 40. № 6. С. 740–749.

38. Mirsalimov V. M., Aknundova P. E. Inverse problem of contact fracture mechanics for a hub of friction pair taking into account thermal stresses. *Mathematics and Mechanics of Solids*. 2019. Vol. 24. Iss. 6. P. 1763–1781. https://doi.org/10.1177/1081286518805525.
39. Mirsalimov V. M., Aknundova P. E. Optimum problem on wear decrease for a hub of friction pair. *Mechanics of Advanced Materials and Structures*. 2020. Vol. 27. Iss. 5. P. 353–363. https://doi.org/10.1080/15376494.2018.1472827.
40. Мирсалимов В. М., Гасанов Ш. Г., Гейдаров Ш. Г. Износоконтактная задача о вдавливании колодки с фрикционной накладкой в поверхность барабана. *Трибология – машиностроению*: тр. XII междунар. науч.-техн. конф., посвященной 80-летию ИМАШ РАН (М., 19-21 нояб. 2018 г.). М. – Ижевск: Ин-т компьютер. исслед., 2018. С. 341–344.
41. Горячева И. Г., Добычин М. Н. Контактные задачи в трибологии. М.: Машиностроение, 1988. 256 с.
42. Горячева И. Г. Механика фрикционного взаимодействия. М.: Наука, 2001. 478 с.
43. Мусхелишвили Н. И. Некоторые основные задачи математической теории упругости. М.: Наука, 1966. 707 с.

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USING THE R-FUNCTIONS THEORY APPARATUS TO MATHEMATICALLY MODEL THE SURFACE OF THE SOYUZ-APPOLO SPACECRAFT MOCK-UP FOR 3D PRINTING

Creation of mathematical models of objects to be 3D printed is of considerable interest, which is associated with the active introduction of 3D printing in various industries. The advantages of using modern 3D printers are: lower production costs and shorter periods of time for their appearance on the market, modeling objects of any shape and complexity, speed and high precision of manufacturing, their ability to use various materials. One of the methods for solving the problem of creating a mathematical and computer model of the object being designed is the application of the R-functions theory, with the help of which it is possible to describe geometric objects of complex shapes in a single analytical expression. The use of alphabetic parameters, when one specifies geometric information in analytical form, allows one to quickly change the size and shape of the object being designed, which helps to spend less time on building computational models. The proposed method can significantly reduce the complexity of work in CAD systems in those cases when one needs to view a large number of design options in search of an optimal solution. This gives a great effect on reducing labor intensity in the construction of computational models to determine aero-gas-dynamic and strength characteristics. Characterization is also often associated with the need to account for changes in aircraft shape. This leads to the fact that the determination of aerodynamic characteristics only due to the need to build a large number of computational models increases the duration of work by months. With parametric assignments, computational regions change almost instantly. In this paper, on the basis of the basic apparatus of the theory of R-functions as well as cylindrical, spherical, ellipsoidal, and conusoidal support functions, a multiparametric equation for the surface of a Soyuz-Apollo spacecraft model is constructed. A number of support functions were normalized according to a general formula, which made it possible to illustrate a new approach to constructing three-dimensional equations for surfaces of a given thickness.

Keywords: *R-functions, alphabetic parameters, standard primitives, Soyuz-Apollo spacecraft model.*