38. Mirsalimov V. M., Aknundova P. E. Inverse problem of contact fracture mechanics for a hub of friction pair taking into account thermal stresses. *Mathematics and Mechanics of Solids*. 2019. Vol. 24. Iss. 6. P. 1763–1781. https://doi.org/10.1177/1081286518805525.
39. Mirsalimov V. M., Aknundova P. E. Optimum problem on wear decrease for a hub of friction pair. *Mechanics of Advanced Materials and Structures*. 2020. Vol. 27. Iss. 5. P. 353–363. https://doi.org/10.1080/15376494.2018.1472827.
40. Мирсалимов В. М., Гасанов Ш. Г., Гейдаров Ш. Г. Износоконтактная задача о вдавливании колодки с фрикционной накладкой в поверхность барабана. *Трибология – машиностроению*: тр. XII междунар. науч.-техн. конф., посвященной 80-летию ИМАШ РАН (М., 19-21 нояб. 2018 г.). М. – Ижевск: Ин-т компьютер. исслед., 2018. С. 341–344.
41. Горячева И. Г., Добычин М. Н. Контактные задачи в трибологии. М.: Машиностроение, 1988. 256 с.
42. Горячева И. Г. Механика фрикционного взаимодействия. М.: Наука, 2001. 478 с.
43. Мусхелишвили Н. И. Некоторые основные задачи математической теории упругости. М.: Наука, 1966. 707 с.

DOI: https://doi.org/10.15407/pmach2020.03.055

UDC 517.95+518.517+629.735.33-519

USING THE R-FUNCTIONS THEORY APPARATUS TO MATHEMATICALLY MODEL THE SURFACE OF THE SOYUZ-APPOLO SPACECRAFT MOCK-UP FOR 3D PRINTING

[1] **Tetiana I. Sheiko**,
sheyko@ipmach.kharkov.ua
ORCID: 0000-0003-3295-5998

[1, 2] **Kyrylo V. Maksymenko-Sheiko**,
m-sh@ipmach.kharkov.ua
ORCID: 0000-0002-7064-2442

[3] **Anna I. Morozova**
ORCID: 0000-0002-7082-4115

[1] A. Pidhornyi Institute of Mechanical Engineering Problems of NASU,
2/10, Pozharskyi St., Kharkiv, 61046, Ukraine

[2] V. N. Karazin Kharkiv National University,
4, Svobody Sq., Kharkiv, 61022, Ukraine

[3] Kharkiv National University of Radio Electronics,
14, Nauky Ave., Kharkiv, 61166, Ukraine

Creation of mathematical models of objects to be 3D printed is of considerable interest, which is associated with the active introduction of 3D printing in various industries. The advantages of using modern 3D printers are: lower production costs and shorter periods of time for their appearance on the market, modeling objects of any shape and complexity, speed and high precision of manufacturing, their ability to use various materials. One of the methods for solving the problem of creating a mathematical and computer model of the object being designed is the application of the R-functions theory, with the help of which it is possible to describe geometric objects of complex shapes in a single analytical expression. The use of alphabetic parameters, when one specifies geometric information in analytical form, allows one to quickly change the size and shape of the object being designed, which helps to spend less time on building computational models. The proposed method can significantly reduce the complexity of work in CAD systems in those cases when one needs to view a large number of design options in search of an optimal solution. This gives a great effect on reducing labor intensity in the construction of computational models to determine aero-gas-dynamic and strength characteristics. Characterization is also often associated with the need to account for changes in aircraft shape. This leads to the fact that the determination of aerodynamic characteristics only due to the need to build a large number of computational models increases the duration of work by months. With parametric assignments, computational regions change almost instantly. In this paper, on the basis of the basic apparatus of the theory of R-functions as well as cylindrical, spherical, ellipsoidal, and conusoidal support functions, a multiparametric equation for the surface of a Soyuz-Apollo spacecraft model is constructed. A number of support functions were normalized according to a general formula, which made it possible to illustrate a new approach to constructing three-dimensional equations for surfaces of a given thickness.

Keywords: *R-functions, alphabetic parameters, standard primitives, Soyuz-Apollo spacecraft model.*

Introduction

One of the new technologies, that has recently been gaining increasing popularity, is 3D printing, which allows one to create three-dimensional models of any objects by using special 3D printing equipment. This is a high-tech equipment, requiring that one has certain knowledge and skills in order to use it. The heyday of the era of 3D printing began in the 21st century, although the principle of operation itself was formulated back in 1986. 3D printing in this century has already become one of the main achievements of mankind in the development of technology. Scientists have mastered the methods of creating three-dimensional models of any shape and content.

The 3D printing process consists of several stages. The first stage is preparatory. In the process of preparation, one creates a computer 3D model of the object to be printed. The creation of such a model is possible, for example, using a 3D modeling program. After the virtual model has been created, one needs to convert it to a set of instructions for the printer, i.e. generate a G-code, and then start the printing process. The potential and capabilities of 3D printers are enormous, most often they are used in mechanical engineering, architecture, film industry, and medicine. The main advantage of 3D printing is that it is a completely robotic process. Using a 3D modeling program, one can look at the prototype from all angles, see the real dimensions, proportions, and also correct errors and improve the product at the modeling stage. Experts from the Economist, one of the most influential magazines, have called 3D printing technology the third industrial revolution, which is now taking place quietly and insensibly right outside our windows. And while some Eastern European countries are investing billions in "traditional production", many have already realized the scale of changes ahead. For example, the United States plans to return production to its territory in the nearest future, but it will be mainly high-tech and versatile technologies based on 3D printing, for which it is planned to provide large-scale government subsidies for this process, similar to the current American breakthrough in the production of its own shale gas and oil. It turned out that it is economically feasible to 3D print entire rocket engines. 3D Systems Corporation (USA) announced a successful zero-gravity test of the BFB-3000 3D printer. The tests indicate the possibility of producing complex assemblies and parts in outer space. 3D printing will change the way we approach space exploration as well as how spacecraft operate. At present, all the items and parts necessary in space are manufactured on Earth, carefully adjusted and matched to spacecraft models for testing. Then they wait for the next spacecraft to fly, and only after docking in orbit, the astronauts receive the required items [1]. 3D printers make it easier to create and arrange living space in orbit, reducing the need to deliver a wide range of cargo from Earth. NASA engineers are even building a rocket from special plastic through using a 3D printer with selective laser sintering technology that can reach other planets in the solar system.

However, there is a problem of setting the print information, i.e. creating a mathematical and computer model of the object being designed. One of the methods for solving this problem is the application of the R-functions theory, which allows one to describe geometric objects of complex shapes in a single analytical expression. In [2], based on the R-functions theory, new approaches were developed to the analytical identification of the surfaces of unmanned aerial vehicles to be 3D printed. We used both well-known methods of standard primitives and various frame blending options. Multiparameter equations for the surfaces of unmanned aerial vehicles of various shapes and purposes have been constructed and visualized. The adequacy of the results obtained to the objects being designed is confirmed by visualization both in the operating conditions of the RFPreview program and by 3D printing. The use of alphabetic parameters, when one specifies geometric information in analytical form, allows one to quickly change the size and shape of the objects being designed, which helps to spend less time when building computational models. The proposed method can significantly reduce the complexity of work in CAD systems in those cases when one needs to view a large number of design options in search of an optimal solution.

This can give a great effect on reducing labor intensity in the construction of computational models to determine aero-gas-dynamic and strength characteristics. Characterization is also often associated with the need to account for changes in aircraft shape. This leads to the fact that the determination of aerodynamic characteristics only due to the need to build a large number of computational models to take this factor into account increases the duration of work by months. With parametric assignments, computational regions change almost instantly.

In [3], on the basis of the theory of R-functions, equations of the surfaces of the models of the "Angara" launch vehicle, the "Buran" type spacecraft, and missiles for the delivery of objects to their destination points are constructed. Thus, at present, a certain experience has been accumulated in constructing equations for the surfaces of aerospace objects. However, each new spacecraft design requires, as a rule, the improvement of the basic apparatus of R-functions.

The purpose of this work is to create a mathematical and computer model of the surface of the Soyuz-Apollo type spacecraft mock-up.

Main part

In this work, we used the R-operations $fk \wedge_0 fl = fk + fl - \sqrt{fk^2 + fl^2}$; $fk \vee_0 fl = fk + fl + \sqrt{fk^2 + fl^2}$ [4, 5], as well as cylindrical, spherical, ellipsoidal, and conusoidal support functions. A number of support functions were normalized according to the general formula $\omega n = \frac{\omega}{\sqrt{\omega^2 + (\nabla\omega)^2}}$ [4, 5], which made it possible to illustrate a new approach to constructing three-dimensional equations for surfaces of a given thickness $W = \delta - |\omega| \geq 0$, where 2δ is the wall thickness [6].

Let us construct the equation for the surface of the main part of the Soyuz spacecraft (see the figure), using as support functions the normalized cylindrical surfaces $f1, f2$, an ellipsoid fe, and a sphere $f31$, truncated by the corresponding planes

$$f1 = \left(r1^2 - x^2 - y^2\right)/2r1 \wedge_0 z(25 - z)/25 \geq 0;\ \ f2 = \left(r2^2 - x^2 - y^2\right)/2r2 \wedge_0 z(4 - z)/4 \geq 0;$$

$$fe = 1 - \left(\frac{x}{r1}\right)^2 - \left(\frac{y}{r1}\right)^2 - \left(\frac{z - 25}{30}\right)^2 \geq 0;$$

$$fen = \frac{fe}{\sqrt{fe^2 + gfe}};\ \ f12 = \left(\left(f1 \vee_0 f2\right) \vee_0 fen\right) \wedge_0 z(50 - z)/50 \geq 0;$$

$$f31 = \left(r3^2 - x^2 - y^2 - (z - 60)^2\right)/2r3 \wedge_0 65 - z \geq 0;\ \ f32 = f12 \vee_0 f31 \geq 0 \text{ (figure, a).}$$

Let us construct the equation of the surface of the inner part of the Soyuz spacecraft, using the normalized support functions

$$ff = \left(f1 \vee_0 1 - \left(\frac{x}{r1}\right)^2 - \left(\frac{y}{r1}\right)^2 - \left(\frac{z - 25}{30}\right)^2\right) \wedge_0 z(50 - z)/50 \geq 0;$$

$$ff32 = ff \vee_0 f31 \geq 0;\ fp = \left(\left(1 - abs(ff32)\right) \vee_0 f2\right) \wedge_0 y \geq 0 \text{ (figure, b).}$$

Let us construct the equation for the surface of the Soyuz spacecraft with a conical docking block $f33$, turning into a cylindrical one $f4$

$$x3 = -\frac{85x}{z - 85};\ \ y3 = -\frac{85y}{z - 85};\ \ f33 = \left(r4^2 - x3^2 - y3^2\right)/2r4 \wedge_0 (75 - z)(z - 65)/10 \geq 0;$$

$$f3 = f32 \vee_0 f33 \geq 0;\ \ f4 = \left(r5^2 - x^2 - y^2\right)/2r5 \wedge_0 (100 - z)(z - 66)/34 \geq 0;$$

$$f34 = f3 \vee_0 f4 \geq 0 \text{ (figure, c).}$$

Let us write down the equation for the surface of solar cells.

$$fa = \left(\left(1 - |y|\right) \wedge_0 (25 - z)(z - 10)/15\right) \wedge_0 \left(45^2 - x^2\right)/90 \geq 0;\ \ f5 = fa \vee_0 f34 \geq 0 \text{ (figure, d).}$$

Let us construct the main part of the cylindrical surface block of the Apollo spacecraft

$$f6 = \left(r6^2 - x^2 - y^2\right)/2r6 \wedge_0 (150 - z)(z - 115)/35 \geq 0;\ \ f56 = f5 \vee_0 f6 \geq 0 \text{ (figure, e).}$$

Let us construct the general equation for the surface of the Soyuz-Apollo spacecraft, using two conical surfaces and two half-planes

$$x1 = -\frac{145x}{z - 145};\ \ y1 = -\frac{145y}{z - 145};\ \ fk1 = \left(r7^2 - x1^2 - y1^2\right)/2r7 \wedge_0 (175 - z)(z - 150)/25 \geq 0;$$

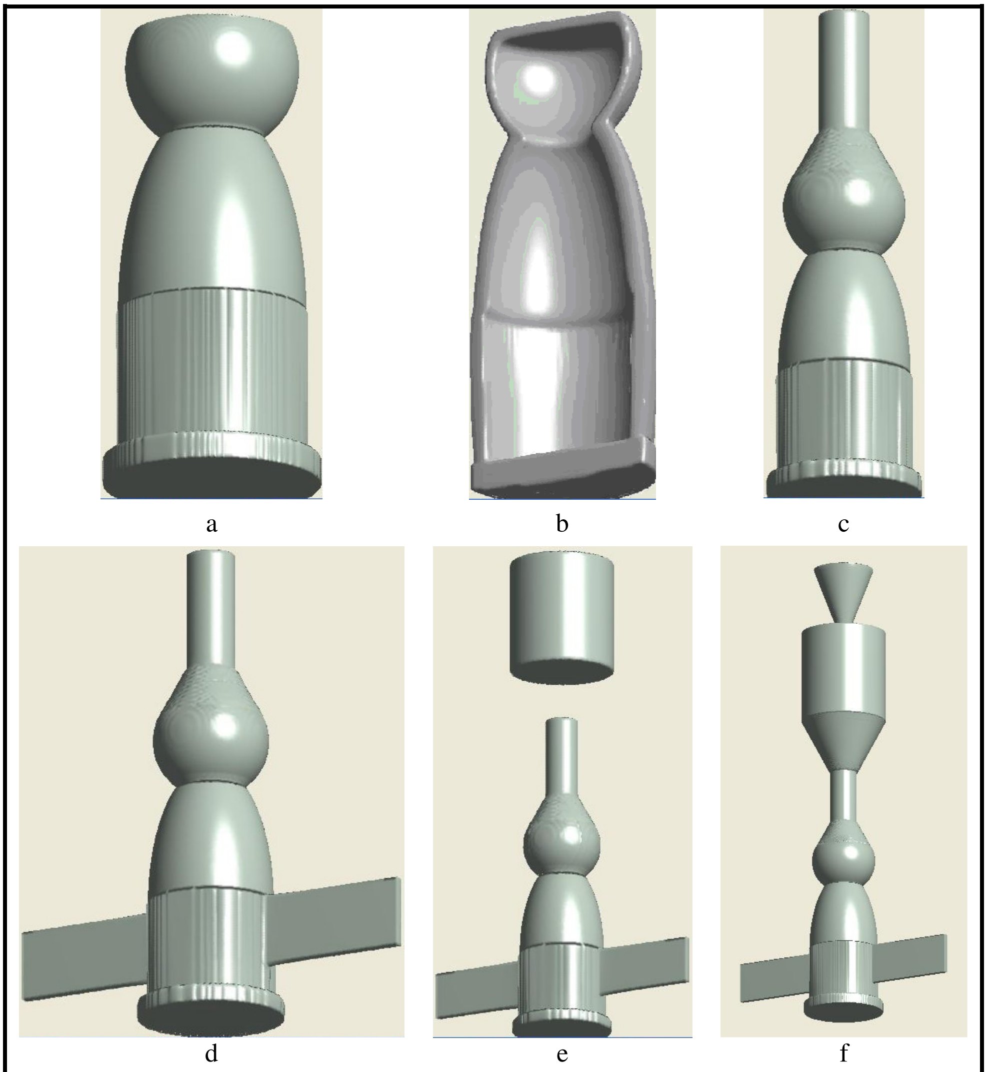
$$x2 = -\frac{85x}{z-85};\ \ y2 = -\frac{85y}{z-85};\ \ fk2 = \left(r8^2 - x2^2 - y2^2\right)/2r8 \wedge_0 (115-z)(z-95)/20 \geq 0;$$

$fk = fk1 \vee_0 fk2 \geq 0;\ WSA = f56 \vee_0 fk \geq 0$ (figure, f).

The alphabetic parameter values are:

$r1 = 13; r2 = 15; r3 = 12; r4 = 45; r5 = 5; r6 = 16; r7 = 55; r8 = 45.$

It should be noted that with a change in any of the presented alphabetic parameters, the shape of the corresponding fragments of the spacecraft will automatically change.



Phased construction of the surface of the Soyuz-Apollo type spacecraft mock-up

Conclusions

Creation of mathematical models of objects to be 3D printed is of considerable interest, which is associated with the active introduction of 3D printing in various industries. The advantages of using 3D printers are obvious: they make it possible to manufacture non-standard models, reduce time for creating new prototypes, simplify and significantly reduce cost of production even if modern ultra-strong materials are used. In this paper, based on the R-functions theory, methods are formulated for creating mathematical and computer models of 3D objects. Algorithms have been developed for the step-by-step construction of mathematical models of complex cross-sections, three-dimensional equations of surfaces, including those with the wall of a given thickness.

The reliability of the results obtained, their adequacy to the objects being designed is confirmed by visualization in the operating conditions of the RFPreview program. The analytical recording of the objects being designed makes it possible to use alphabetic geometric parameters, complex superposition of functions, which, in turn, allows one to quickly change their structural elements. The positivity property of the constructed functions at the internal points of the object is very convenient for 3D printing.

The results of this article were partially obtained within the framework of the program to support the state's priority scientific researches as well as scientific and technical (experimental) developments of the Department of Physical and Technical Problems of Power Engineering of the National Academy of Sciences of Ukraine (KPKVK 6541230).

References

1. (2020). *3D-printer budet "pechatat'" detali pryamo v kosmose* [The 3D printer will "print" parts right in space.]. Nano News Net: Official site, 2020. URL: http://www.nanonewsnet.ru/news/2011/3d-printer-budet-pechatat-detali-pryamo-v-kosmose.
2. Sheyko, T., Maksymenko-Sheyko, K., Sirenko, V., Morozova, A., & Petrova, R. (2019). Analytical identification of the unmanned aerial vehicles' surfaces for the implementation at a 3D printer. *Eastern-European Journal of Enterprise Technologies*, vol. 1, no. 2 (97), pp. 48–56. https://doi.org/10.15587/1729-4061.2019.155548.
3. Sheyko, T. I., Maksimenko-Sheyko, K. V., Tolok, A. V., & Morozova, A. I. (2019). *Matematicheskoye i kompyuternoye modelirovaniye aerokosmicheskikh obyektov dlya realizatsii tekhnologii 3D-pechati* [Mathematical and computer modeling of aerospace objects for the implementation of 3D printing technology]. *Informatsionnyye tekhnologii v proyektirovanii i proizvodstve – Information technology of CAD/CAM/CAE*, iss. 2 (174), pp. 16–20 (in Russian).
4. Rvachev, V. L. (1982). *Teoriya R-funktsiy i nekotoryye yeye prilozheniya* [R-functions theory and some of its applications]. Kiyev: Naukova dumka, 552 p. (in Russian).
5. Rvachev, V. L. & Sheiko, T. I. (1995). R-functions in boundary value problems in mechanics. *Applied Mechanics Reviews*, vol. 48, no. 4, pp. 151–188. https://doi.org/10.1115/1.3005099.
6. Maksimenko-Sheyko, K. V. (2009). *R-funktsii v matematicheskom modelirovanii geometricheskikh obyektov i fizicheskikh poley* [R-functions in mathematical modeling of geometric objects and physical fields]. Kharkov: IPMash NAN Ukrainy, 306 p. (in Russian).

Received 11 May 2020

Математичне моделювання поверхні макета космічного корабля типу «Союз-Аполлон» за допомогою базового інструментарію R-функцій для реалізації на 3D-принтері

[1] Т. І. Шейко, [1,2] К. В. Максименко-Шейко, [3] А. І. Морозова

[1] Інститут проблем машинобудування ім. А. М. Підгорного НАН України,
61046, Україна, м. Харків, вул. Пожарського, 2/10

[2] Харківський національний університет імені В. Н. Каразіна,
61022, Україна, м. Харків, майдан Свободи, 4

[3] Харківський національний університет радіоелектроніки, 61166, Україна, м. Харків, пр. Науки, 14

Створення математичних моделей об'єктів для 3D-друку становить значний інтерес, який пов'язаний з активним впровадженням 3D-друку в різні галузі промисловості. Переваги використання сучасних 3D-принтерів: зниження собівартості виготовлення продукції і скорочення термінів її появи на ринку, моделювання об'єктів будь-якої форми і складності, швидкість і висока точність виготовлення, можливість використання різних матеріалів. Одним з методів вирішення проблеми створення математичної та комп'ютерної моделі проектованого об'єкта є застосування теорії R-функцій, яка дозволяє описувати геометричні об'єкти складної форми єдиним аналітичним виразом. Використання буквених параметрів під час задання геометричної інформації в аналітичному вигляді дозволяє оперативно змінювати розміри і форму проектованих об'єктів, що допомагає скоротити витрати часу під час побудови розрахункових моделей. Запропонований метод може істотно скоротити трудомісткість робіт в CAD-системах в тих випадках, коли потрібно переглянути велику кількість варіантів конструкції в пошуках оптимального рішення. Це може зумовити значний ефект щодо зниження трудомісткості під час побудови розрахункових моделей для визначення аерогазодинамічних і міцнісних характеристик. Визначення характеристик також часто пов'язано з необхідністю врахування зміни форми літального апарата. Це призводить до того, що визначення аеродинамічних характеристик тільки за рахунок необхідності побудови великого числа розрахункових моделей для

врахування цього фактора збільшує тривалість робіт на місяці. За параметричного задання зміна розрахункових областей проводиться практично миттєво. У роботі на основі базового інструментарію теорії R-функцій і циліндричних, сферичних, еліпсоїдальних, конусоїдальних опорних функцій побудовано багатопараметричне рівняння поверхні макета космічного корабля типу «Союз-Аполлон». Ряд опорних функцій був нормалізований за загальною формулою, що дало можливість проілюструвати новий підхід до побудови тривимірних рівнянь поверхонь заданої товщини.

Ключові слова: *R-функції, буквені параметри, стандартні примітиви, макет космічного корабля «Союз-Аполлон».*

Література

1. 3D-принтер будет "печатать" детали прямо в космосе. Nano News Net: Official site, 2020. URL: http://www.nanonewsnet.ru/news/2011/3d-printer-budet-pechatat-detali-pryamo-v-kosmose.
2. Sheyko T., Maksymenko-Sheyko K., Sirenko V., Morozova A., Petrova R. Analytical identification of the unmanned aerial vehicles' surfaces for the implementation at a 3D printer. *Eastern-European J. Enterprise Techn.* 2019. Vol. 1. No. 2 (97). P. 48–56. https://doi.org/10.15587/1729-4061.2019.155548.
3. Шейко Т. И., Максименко-Шейко К. В., Толок А. В., Морозова А. И. Математическое и компьютерное моделирование аэрокосмических объектов для реализации технологии 3D-печати. *Информ. технологии в проектировании и пр-ве*. 2019. № 2 (174). С. 16–20.
4. Рвачев В. Л. Теория R-функций и некоторые ее приложения. Киев: Наук. думка, 1982. 552 с.
5. Rvachev V. L., Sheiko T. I. R-functions in boundary value problems in mechanics. *Appl. Mech. Reviews*. 1995. Vol. 48. No. 4. P. 151–188. https://doi.org/10.1115/1.3005099.
6. Максименко-Шейко К. В. R-функции в математическом моделировании геометрических объектов и физических полей. Харьков: ИПМаш НАН Украины, 2009. 306 с.

DOI: https://doi.org/10.15407/pmach2020.03.060

UDC 624.04

OPTIMIZATION OF BENDABLE I-SECTION ELEMENTS UNDER CONDITIONS OF CORROSION AND MATERIAL DAMAGE

Mark M. Fridman,
mark17@i.ua
ORCID: 0000-0003-3819-2776

Kryvyi Rih Metallurgical Institute of the National Metallurgical Academy of Ukraine
5, Stephan Tilha St.,
Kryvyi Rih, 5006, Ukraine

Operation of structures in high temperature conditions and aggressive environments leads to such phenomena as corrosion and material damage. Corrosion leads to a reduction in structural cross-section and, consequently, an increase in stresses. As to material damage, namely, the appearance of micro-cracks and voids resulting from inelastic creep strain, it leads to a deterioration of physical characteristics (for example, the elastic modulus) and a sharp decrease in the stress values at which structural failure occurs. This paper is a continuation of the research in the field of optimal design of structures operating under conditions of corrosion and material damage (high temperature, aggressive environment, etc.). A first paper in this field was devoted to the optimization of bendable rectangular cross-section elements. This paper considers the optimization of the lengthwise thickness of flanges of bendable I-section elements, using the same principle of equal damage, which was applied to optimize the bendable rectangular cross-section elements. It is assumed that the flange width and web height of an I-section element are fixed. Since, during bending, mainly I-beam flanges work (their moment of inertia is 85% of the moment of inertia of the entire cross-section), the web is not taken into account in the calculation. As an equation of corrosion, V. M. Dolinsky's model is adopted, taking into account the effect of tension on the corrosion wear of structures. In the model of the kinetic equation that describes the change in material damage, Yu. N. Rabonov's model is adopted, where the value of damage ω varying from 0 to 1 is taken to be a variable parameter. As the criterion of optimality, the minimum weight of structures is adopted. In conclusion, presented is an algorithm for solving a more complete problem of optimizing the parameters of bendable I-section elements, namely, the web height and the flange width, using the obtained analytical expressions that determine the optimal distribution of the thickness of flanges along the length of the structure.

Keywords: *corrosion, material damage, optimization.*