врахування цього фактора збільшує тривалість робіт на місяці. За параметричного задання зміна розрахункових областей проводиться практично миттєво. У роботі на основі базового інструментарію теорії R-функцій і циліндричних, сферичних, еліпсоїдальних, конусоїдальних опорних функцій побудовано багатопараметричне рівняння поверхні макета космічного корабля типу «Союз-Аполлон». Ряд опорних функцій був нормалізований за загальною формулою, що дало можливість проілюструвати новий підхід до побудови тривимірних рівнянь поверхонь заданої товщини.

Ключові слова: *R-функції, буквені параметри, стандартні примітиви, макет космічного корабля «Союз-Аполлон».*

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DOI: https://doi.org/10.15407/pmach2020.03.060

UDC 624.04

OPTIMIZATION OF BENDABLE I-SECTION ELEMENTS UNDER CONDITIONS OF CORROSION AND MATERIAL DAMAGE

Mark M. Fridman,
mark17@i.ua
ORCID: 0000-0003-3819-2776

Kryvyi Rih Metallurgical Institute of the National Metallurgical Academy of Ukraine
5, Stephan Tilha St., Kryvyi Rih, 5006, Ukraine

Operation of structures in high temperature conditions and aggressive environments leads to such phenomena as corrosion and material damage. Corrosion leads to a reduction in structural cross-section and, consequently, an increase in stresses. As to material damage, namely, the appearance of micro-cracks and voids resulting from inelastic creep strain, it leads to a deterioration of physical characteristics (for example, the elastic modulus) and a sharp decrease in the stress values at which structural failure occurs. This paper is a continuation of the research in the field of optimal design of structures operating under conditions of corrosion and material damage (high temperature, aggressive environment, etc.). A first paper in this field was devoted to the optimization of bendable rectangular cross-section elements. This paper considers the optimization of the lengthwise thickness of flanges of bendable I-section elements, using the same principle of equal damage, which was applied to optimize the bendable rectangular cross-section elements. It is assumed that the flange width and web height of an I-section element are fixed. Since, during bending, mainly I-beam flanges work (their moment of inertia is 85% of the moment of inertia of the entire cross-section), the web is not taken into account in the calculation. As an equation of corrosion, V. M. Dolinsky's model is adopted, taking into account the effect of tension on the corrosion wear of structures. In the model of the kinetic equation that describes the change in material damage, Yu. N. Rabonov's model is adopted, where the value of damage ω varying from 0 to 1 is taken to be a variable parameter. As the criterion of optimality, the minimum weight of structures is adopted. In conclusion, presented is an algorithm for solving a more complete problem of optimizing the parameters of bendable I-section elements, namely, the web height and the flange width, using the obtained analytical expressions that determine the optimal distribution of the thickness of flanges along the length of the structure.

Keywords: *corrosion, material damage, optimization.*

Introduction

Operation of structures in high temperature conditions and aggressive environments leads to such phenomena as corrosion and material damage. Corrosion leads to a reduction in structural cross-section and, consequently, an increase in stresses. As to material damage, namely, the appearance of micro-cracks and voids resulting from inelastic creep strain, it leads to a deterioration of physical characteristics (for example, the elastic modulus) and a sharp decrease in the stress values at which structural failure occurs. The first kinetic models of changes in material damage were proposed by L. M. Kachanov [1, 2] and Yu. N. Rabotnov [3]. In the first model, a change in material damage is characterized by a continuity parameter, varying from 1 in the initial state to 0 at the moment of structural failure. In the second model, the damage value $\overline{\varpi}$, varying from 0 to 1, is taken to be a variable parameter. Other modifications of Yu. N. Rabotnov's model are considered, for example, in J. Lemaitre's and J. L. Chaboche's works [4, 5]. Using the principle of "separability" and introducing the parameter of normalized time, depending on stress, the above models (in the case of one-dimensional tensile stresses) were modernized in V. P. Golub's work [6]. A new approach to determining structural damage is illustrated by static and cyclic loads. An original damage model was also proposed by L. A. Sosnovsky and S. S. Shcherbakov [7]. A review of studies in this direction was carried out in [8, 9].

The problems of optimizing structures operated in conditions of material damage are the subjects of the works of A. Kostyuk [10], M. Reitman [11], V. Prager [12], Yu. Nemirovsky [13, 14], M. Zhichkovsky [15], etc.

The optimization of structural elements under corrosion conditions is considered in [16–20].

This paper is a continuation of the research in the field of optimal design of structures operating in high temperature conditions, aggressive environment, etc. that contribute to the appearance therein of corrosion and material damage (a similar problem, on the example of the optimization of bendable rectangular-section elements, is presented in [20]).

It discusses the optimization of the lengthwise thickness of flanges of a bendable I-section element, using the same principle of equal damage as in [20]. It is assumed that the flange width and the web height of an I-beam are fixed. Since, during bending, mainly I-beam flanges work (their moment of inertia is 85% of the moment of inertia of the entire cross-section), the beam web is not taken into account in the calculation.

Problem Formulation

As the equation of corrosion, V. M. Dolinsky' model [21] is adopted, taking into account the influence of the stress state on the corrosion wear of a structure (Fig. 1)

$$\frac{ds}{dt} = -2\left(\alpha + \beta\left|\sigma_{\max}\right|\right), \tag{1}$$

where α and β are constant coefficients; s_0 and s are the initial and current thicknesses of I-section element flanges; $\sigma_{\max}$ are the maximum stresses in the current cross-section.

It is assumed that the upper and lower faces of the cross-section are subject to corrosion, and to the same extent (by taking the maximum stress modulo), as evidenced by coefficient 2 in (1).

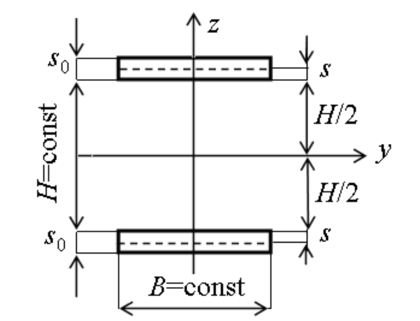

Fig. 1. Cross-section of a bendable I-section element

As the kinetic equation describing the change (from 0 to 1) of the material damage parameter $\overline{\varpi}$, Yu. N. Rabotnov's model [3] is adopted (as in [20])

$$\frac{d\overline{\varpi}}{dt} = a_k \left(\frac{\sigma_{\max}}{1-\overline{\varpi}}\right)^{b_k}, \tag{2}$$

where a_k and b_k are constants.

When determining stresses in an I-beam, both the effect of creep and the factor of high temperatures are taken into account. The law of creep is adopted in the form [3]

$$\dot{\varepsilon} = A_1\sigma^n, \tag{3}$$

where A_1 and n are constant values at a given temperature.

With n=1, we obtain the distribution of stresses in the elastic element, with $n \to \infty$, in the element of perfectly plastic material. In practice, the value of n usually does not exceed 12.

Taking into account the hypothesis of flat cross-sections $\dot{\varepsilon} = \dot{x}z$, from (3) we find

$$\sigma = k\dot{\varepsilon}^{1/n} = k(\dot{x}z)^{1/n}, \qquad (4)$$

where $k = 1/A_1^n$; $\dot{x}$ is the rate of change of the curvature of the neutral layer.

We write the expression for the bending moment in a cross-section x for an arbitrary time moment t

$$M(x) = \int z\sigma dA = k(x)^{1/n} \cdot 2B \int_{H_n}^{H_n+S} z^{1+1/n} dz = \frac{k(\dot{x})^{1/n}}{2+1/n} \cdot 2B\left[(H_n+s)^{2+1/n} - H_n^{2+1/n}\right],$$

where $H_n = H/2$.

This yields

$$k(\dot{x})^{1/n} = \frac{M(2+1/n)}{2B\left[(H_n+s)^{2+1/n} - H_n^{2+1/n}\right]}.$$

Substituting this expression into (4), for $z = H_n + s$, we have

$$\sigma_{\max} = \frac{M(2+1/n)(H_n+s)^{1/n}}{2B\left[(H_n+s)^{2+1/n} - H_n^{2+1/n}\right]} = \frac{Mm}{2B\left[(H_n+s)^2 - \dfrac{H_n^{2+1/n}}{(H_n+s)^{1/n}}\right]}, \qquad (5)$$

where $m = 2+1/n$.

Since at high temperatures the quantity $n \geq 3$, it is correct to make the following simplifications in expression (5):

$$\frac{H_n^{2+1/n}}{(H_n+s)^{1/n}} \approx \frac{H_n^{2+1/n}}{H_n^{1/n}} = H_n^2,$$

$$(H_n+s)^2 - H_n^2 = 2H_n s + s^2 \approx 2H_n s.$$

Finally, we get

$$\sigma_{\max} = \frac{Mm}{4BH_n s}. \qquad (6)$$

In this case, equations (1) and (2) (with account taken of (6)) have the form

$$\frac{ds}{dt} = -2\left(\alpha + \frac{\beta Mm}{4BH_n s}\right), \qquad (7)$$

$$\frac{d\varpi}{dt} = a_k \left(\frac{\dfrac{Mm}{4BH_n s}}{1-\varpi}\right)^{b_k}. \qquad (8)$$

Solving Equations of Corrosion and Material Damage

Due to the independence of the corrosion process, as in [20], we first solve equation (7).

Separate the variables in equation (7) and integrate both sides

$$\int_{S_o}^{S} \frac{ds_1}{1 + \dfrac{\beta Mm}{4BH_n \alpha s_1}} = -2\alpha \int_0^T dt,$$

where T is the lifetime of the structure.

After integration, we have

$$s_o - s + \frac{\beta Mm}{4BH_n\alpha}\ln\frac{s+\dfrac{\beta Mm}{4BH_n\alpha}}{s_o+\dfrac{\beta Mm}{4BH_n\alpha}} = 2\alpha T\ . \qquad (9)$$

To solve equation (8), we separate the variables and, substituting the expression dt from (7), we integrate both sides

$$\int_0^1 (1-\varpi)^{b_k}\, d\varpi = \frac{1}{b_k+1} = -\frac{a_k}{2\alpha}\left(\frac{Mm}{4BH_n}\right)^{b_k}\int_{s_o}^{s}\frac{ds_1}{s_1^{b_k}\left(1+\dfrac{\beta Mm}{4BH_n\alpha s_1}\right)}\ . \qquad (10)$$

Optimization Problem. General case

In the general case, the optimization problem is presented as follows. As an objective function, the volume V of the structure is chosen. The vector of the variable parameters of a bendable I-beam is taken to be $\overline{X} = \{H_0, B_0, s_0\}^{\mathrm{T}}$. The goal of the optimization problem is to find the optimal construction vector $\overline{X}_{op}$ for which

$$V \to \min,\ \overline{\varpi}(x) \le 1\ ,$$

where the structure volume is

$$V = 2\int_0^L H_0(x)\, B_0(x)\, s_0(x)\, dx\ . \qquad (11)$$

This problem of nonlinear mathematical programming can be solved, for example, using the random search algorithm [22]. In short, the algorithm for this solution is as follows. Given arbitrary (within specified limits) values of H_0 and B_0, according to the methodology below, the optimal flange lengthwise-thickness values $s_0(x)$ are found. After that, using formula (11), the structure volume is found and another attempt is made to specify new H_0 and B_0 parameters, in this case, with finding the corresponding (optimal) $s_0(x)$ values. The beam volume is again determined, etc. The procedure is repeated until the optimal parameters H_{op} and B_{op} are found, giving the minimum volume of the structure.

Optimization Problem. Special case

Let us dwell on an optimization sub-problem. It is assumed that the flange width and the web height of an I-section element are fixed (as shown in Fig. 1). The optimization of the lengthwise thickness of flanges of the bendable I-section element is considered using the obtained solutions of equations of corrosion and material damage. By virtue of the assumptions made, two equations (9) and (10) with two unknowns s and s_0 are obtained. Solving them together for each fixed value of x, from the principle of equal damage at the final moment of the structure's life (for $\overline{\varpi} = 1$), the optimal distribution of the lengthwise thickness of flanges of the bendable I-section element $s_0(x)$, which gives the minimum mass of the structure, is determined. The last statement is true due to the dependence

$$V = \int_0^L A(x)dx = 2B\int_0^L s_0(x)dx\ . \qquad (12)$$

Special Solution Cases

The obtained integral expression of metal damage (10), as in [20], can be simplified in two particular cases. So, for b_k=2 we have

$$\frac{8\beta BH_n}{3a_k Mm} = \ln\frac{1+\dfrac{\beta Mm}{4BH_n\alpha s}}{1+\dfrac{\beta Mm}{4BH_n\alpha s_o}}\ .$$

Hence,

$$s=\frac{\dfrac{s_o\beta Mm}{4BH_n\alpha}}{s_o\left(\exp\dfrac{8BH_n}{3a_kMm}-1\right)+\dfrac{\beta Mm}{4BH_n\alpha}\exp\dfrac{8BH_n}{3a_kMm}}. \tag{13}$$

For b_k=3 we get

$$\frac{2\beta^2BH_n}{Mm\alpha a_k}=\frac{\beta Mm}{4BH_n\alpha}\left(\frac{1}{s}-\frac{1}{s_o}\right)-\ln\frac{s+\dfrac{\beta Mm}{4BH_n\alpha}}{s_o+\dfrac{\beta Mm}{4BH_n\alpha}}+\ln\frac{s}{s_o}. \tag{14}$$

As an example, consider the optimization of a cantilever beam with a force P at the end (Fig. 2). In this case, $M=Px$. Proceeding to dimensionless quantities, denoting $T_*=\dfrac{8TBH_n\alpha^2}{\beta PLm}$, $\xi=x/L$, $S=\dfrac{4BH_n\alpha^2 s}{\beta PLm}$, $S_0=\dfrac{4s_oBH_n\alpha}{\beta PLm}$, $C=\dfrac{8\beta BH_n}{3a_kPLm}$, $D=\dfrac{2\beta^2BH_n}{a_kPLm}$, we have the following transformations of equations (9), (13) and (14):

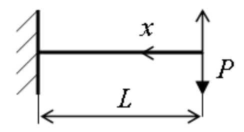

Fig. 2. Design diagram of the beam

$$S_0-S+\xi\ln\frac{s+\xi}{s_0+\xi}=T_*, \tag{15}$$

$$S=\frac{S_0\xi}{S_0\left(\exp\dfrac{c}{\xi}-1\right)+\xi\exp\dfrac{c}{\xi}}, \tag{16}$$

$$D=\xi^2\left(\frac{1}{S}-\frac{1}{S_0}\right)-\xi\ln\frac{S+\xi}{S_0+\xi}+\xi\ln\frac{S}{S_0}. \tag{17}$$

Let us first dwell on the solution to the problem in the case of b_k=2.

Substituting the expression for the value of S (16) into (15), we obtain a transcendental equation with one unknown S_0. Given $0\le\xi\le 1$, its solution is found using the random search algorithm [22].

In the case of b_k=3, combining equations (15) and (17), we have

$$D+T_*=\xi^2\left(\frac{1}{S}-\frac{1}{S_0}\right)+\xi\ln\frac{S}{S_0}+S_0-S. \tag{18}$$

Since $0<S/S_0<1$, we can use the following expansion:

$$\ln\frac{S}{S_0}\approx\left(\frac{S}{S_0}-1\right)-\frac{\left(\dfrac{S}{S_0}-1\right)^2}{2}.$$

Substituting it into (18), after simple transformations, we obtain the third-degree equation:

$$S^3+r_1S^2+r_2S+r_3=0,$$

where $r_1=\dfrac{2S_0^2}{\xi}-4S_0$, $r_2=2D_1S_0^2/\xi+2\xi S_0+3S_0^3-2S_0^3/\xi$, $r_3=-2\xi S_0^2$, $D_1=D+T_*$.

We find its solution by applying the Cardano formula to its reduced form

$$y^3+py+q=0,$$

where $y=S+r_1/3$, $p=\dfrac{3r_2-r_1^2}{3}$, $q=\dfrac{2r_1^3}{27}-\dfrac{r_1r_2}{3}+r_3$.

If $D_s = (p/3)^3 + (q/2)^2 > 0$, then $y = u + v$, where $u = \sqrt[3]{-q/2 + \sqrt{D_s}}$, $v = \sqrt[3]{-q/2 - \sqrt{D}}$.

If $D_s < 0$, then $y = 2\sqrt[3]{r}\cos(\varphi/3)$, where $r = \sqrt{-p^3/27}$, $\cos\varphi = -q/2r$.

After the dependence between S_0 and S is found, given $0 \le \xi \le 1$, we solve equation (18), as in the previous case, using the random search method. As a result, in both cases (for b_k=2 and b_k=3), we have the optimal values of the thickness of flanges of the I-section element of a cantilever beam of minimum weight (with account taken of (12)).

Numerical Results

As a numerical illustration, we consider the following calculation options: for b_k=2, *a*) T^*_a=1, D_a=0.5, and *b*) T^*_b=0.8, D_b=0.4; for b_k=3, *c*) T^*_c=1, D_{1c}=0.375 and *d*) T^*_d=0.8, D_{1d}=0.3. We conditionally assume that options *b* and *d* correspond to an elastic element (n=1, m=6), and options *a* and *c* correspond to an element of perfectly plastic material($n \to \infty$; m=4). Therefore, $T^*_a/T^*_b=D_a/D_b=T^*_c/T^*_d=D_{1c}/D_{1d}$.

The selection of the coefficients D_1 with respect to the coefficients D in the corresponding options can be carried out as follows. Since $D_1/D=3/4*\beta/\alpha$, taking $\beta/\alpha=1$, we have $D_1=0.75D$.

The optimal values of the initial thickness of cantilever I-beam flanges $S_0(\xi)$ as well as at the moment of destruction $S_*(\xi)$ for all options are shown in Fig. 3.

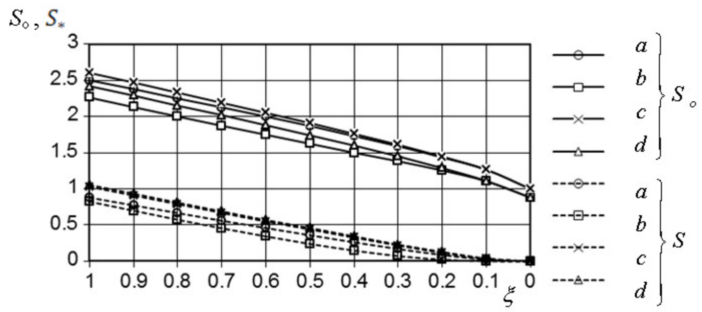

Fig. 3. Optimization results for I-beam element flanges

Conclusions

The problem of optimum design of bendable I-section elements, used in conditions of corrosion and material damage, has been posed and solved.

From the results obtained (Fig. 3), it is seen that in all optimal projects, $S_T(0)$=0 and $S_0(0)=T^*$. The first equality indicates that there is no overspending of the material in the cross-section with a zero stress. In turn, the inequality to zero, $S_0(0)$, is explained by the fact that corrosion also acts in the unstressed section, and its value follows directly from equation (14) with S_T=0 and $\xi \to 0$.

Comparing the optimal projects *a* and *c*, respectively, with projects b and d, we can conclude that taking into account creep gives an increase in the initial thickness of flanges of the cantilever I-beam, in this case, by an average of 12% with an increase in the stress value at which fracture occurs, as evidenced by the parameter $S_T(\xi)$.

In conclusion, it should be noted that the proposed approach to solving the problems of optimal design of structures operating under conditions of corrosion and damage to the material can be used to solve similar problems by applying both analytical solutions and numerical methods.

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Received 10 March 2020

Оптимізація згинних елементів двотаврового перерізу в умовах корозії і пошкодження матеріалу

М. М. Фрідман

Криворізький металургійний інститут Національної металургійної академії України,
50006, Україна, Дніпропетровська обл., м. Кривий Ріг, вул. Степана Тільги, 5

Експлуатація конструкцій в умовах високої температури і агресивного середовища призводить до появи в них таких явищ, як корозія і пошкодженість матеріалу. В результаті корозії зменшується переріз конструкції, і, як наслідок, в ній збільшуються напруження. Що стосується пошкодження матеріалу, а саме, появи в ньому мікротріщин і порожнеч, в результаті непружної деформації (повзучості), то вона призводить до погіршення фізичних характеристик (наприклад, модуля пружності) і різкого зниження величин напружень, за яких відбувається руйнування конструкції. Дана робота є продовженням дослідження в області оптимального проектування конструкцій, що працюють в умовах, які сприяють появі в них корозії і пошкодження матеріалу (висока температура, агресивне середовище і т.д.). Попередня робота в цій області була присвячена оптимізації згинних елементів пря-

мокутного перерізу. У цій статті розглядається оптимізація товщини полиць згинного двотаврового перерізу по його довжині, використовується той же принцип рівнопошкодженості, який був застосований під час оптимізації згинних елементів прямокутного перерізу. Приймається, що ширина полиць і висота стінок елемента двотаврового перерізу фіксовані. Оскільки під час вигину працюють, в основному, полки двотавра (їх момент інерції досягає 85% від моменту інерції всього перерізу), то його стінка в розрахунку не береться до уваги. Як рівняння корозії приймається модель В. М. Долинського, що враховує вплив напружень на корозійний знос конструкцій. Як кінетичне рівняння, що описує зміну пошкодження матеріалу, приймається модель Ю. М. Работнова, де як змінюваний параметр прийнята величина пошкодження, що варіюється від 0 до 1. Критерієм оптимальності приймається мінімум маси конструкції. Наприкінці роботи наведено алгоритм розв'язання більш повної задачі оптимізації параметрів згинних елементів двотаврового перерізу, а саме, висоти стінки і ширини полиці, з використанням отриманих аналітичних виразів, що визначають оптимальний розподіл товщини полиць по довжині конструкції.

***Ключові слова**: корозія, пошкодженість матеріалу, оптимізація.*

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