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STATIC BUCKLING OF A PRE-LOADED COMPLEX NANO-COMPOSITE SHELL

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This paper describes a technique for analyzing the phenomenon of static buckling of a pre-loaded complex nano-composite shell. Most of the works devoted to the analysis of complex structures consider vibration processes, while the phenomenon of buckling can be an important factor that limits the use of new materials in space-rocket hardware. A nano-composite constant-thickness shell consisting of two spherical covers and a cylindrical body is considered. It is acted upon by internal pressure and an axial compressive force. This shell simulates the fuel tank of a launch vehicle. Conditions under which the shell is deformed non-axisymmetrically, buckling statically, are investigated. A technique is proposed that allows the problem to be divided into the analysis of the pre-loaded state of the shell and the analysis of buckling. Further analysis is performed using a technique based on the high-order shear deformation theory and the Ritz method. The problem is discretized by representing the variables that determine the state of the shell in the form of expansions in basis functions with unknown coefficients. Thus, it is the expansion coefficients that become unknown in the problem. The problem of analyzing the pre-stressed state of a structure is reduced to solving a system of linear algebraic equations with respect to the expansion coefficients. The problem of buckling analysis can be reduced to the problem of eigenvalues. The solution to this problem makes it possible to find the minimum value of the compressive load at which the shell buckles, as well as the forms of buckling. The results of applying the developed technique were compared with those of finite element modeling of a structure made of the simplest nano-composite material. The comparison results indicate a high accuracy of the technique described. At the same time, the use of the finite element method for the analysis of large-scale thin-walled structures made of functionally gradient materials is extremely difficult, in contrast to the methodology proposed in the paper. Comparison of various types of nano-reinforcement showed that a rational choice of the type of reinforcement can significantly increase the critical load. In this case, the internal pressure on the shell also significantly affects the critical load.

Keywords: *static buckling, complex shell, nano-composite material, carbon nanotubes, Ritz method.*

Introduction

Modern materials are widely used in the aerospace industry. The use of carbon nano-tubes (CNT) as a reinforcing agent makes it possible to obtain materials with high specific stiffness and strength in comparison with the base material (matrix) [1].

The problem of analyzing the static buckling of nano-composite shells was solved in [2–13]. The buckling of cylindrical shells was studied in [5–8, 10–13]. In these works, the longitudinal compression of the shell is considered in combination with thermal and electromagnetic loading. Also, in [12], the buckling of a pre-loaded shell is considered.

Most of the thin-walled structures used in the aerospace industry are composite. The analysis of such structures in the scientific literature is mainly limited to vibration analysis and dynamic problems. Thus, in [14], equations of motion were constructed for a conical-cylindrical-conical shell. The designs obtained as a result of a combination of spherical and cylindrical shells are considered in [15, 16]. Here, the task is limited to the analysis of free vibrations of structures. An analysis of the vibration dynamics of conical-cylindrical shells is given in [17–19].

Most of the works devoted to the analysis of complex structures consider vibration processes, while the phenomenon of buckling can be an important factor that limits the use of new materials in space-rocket hardware.

This paper considers the static buckling of a complex nano-composite structure under the action of internal pressure and an external axial load. A method for determining the critical axial load on the shell, based on the Ritz method, is proposed.

Formulation of the Problem

We consider the stress-strain state (SSS) of a thin-walled structure of constant thickness *h* (Fig. 1), consisting of two spherical covers and a cylindrical body of length *L*. Such a geometric model is used to represent fuel tanks for launch vehicles. In this paper, to simplify the presentation, a symmetric construction is considered. Nevertheless, no fundamental changes in the described methodology are required for the analysis of an asymmetric structure. The radius of the covers is denoted as R_s, and the radius of the cylinder, as $R_c \le R_s$. Further, the spherical covers and the cylindrical body will be called geometric primitives. At the point of their attachment, the angle between the tangent to the sphere in the plane of the structure axis and the cylinder wall is α.

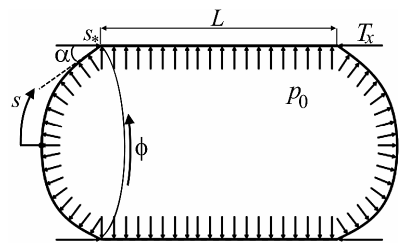

Fig. 1. Sketch of the shell

The position of a point in the shell is described by a bound coordinate system (s, ϕ, z): the coordinate s (Fig. 1) is directed along the shell generatrix from the pole of one of the covers; the angular coordinate ϕ is measured in the plane perpendicular to the shell axis counterclockwise; the z coordinate is directed along the normal to the middle surface of the shell.

The structure is made of a nano-composite material, which is a PmPV polymer reinforced with guided nano-tubes along the shell generatrix.

Two types of loads act on the structure: the internal fuel pressure p_0, uniformly distributed over the shell surface, and the axial compressive load T_x, uniformly applied to the attachment interface between the covers and the body. The task is to find the minimum axial load, which leads to the static buckling of a given structure at a given internal pressure. Thus, the pressure p_0 acts as a preliminary load within the static stability analysis.

Material Properties

A nano-composite material (nano-composite) is a matrix, usually isotropic, reinforced with CNTs. In this paper, we consider a nano-composite material reinforced with the CNTs directed along the generatrix of the shell of revolution. The definition of the characteristics of such a material is described in detail in the scientific literature [20, 21]. A nano-composite is an orthotropic material whose properties can change depending on the z coordinate of the shell. The most common types of nano-composites are: UD, FGX, FGO, FGV, FGΛ. The dependence of the volumetric content of CNTs on the z coordinate in these materials is shown in Fig. 2.

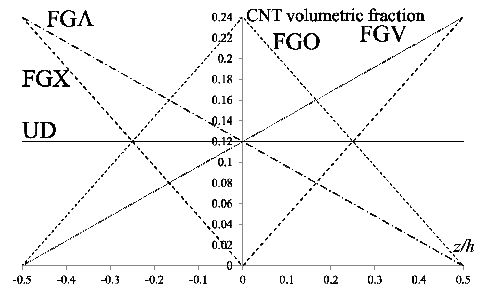

Fig. 2. Dependence of the content of CNTs on the z coordinate in different nano-composites

Solution method

The problem of the static buckling of a composite thin-walled structure is solved in four stages:

– expressions for the potential energy of the geometric primitives of a structure are derived;
– continuity of displacements at the attachment points of geometric primitives is ensured;
– a variational approach to assess the SSS of the structure under the action of internal pressure is used;
– critical longitudinal loads are calculated.

Structure Model

The deformation of the shell is described using the high-order shear deformation theory [22, 23], in which the displacements of an arbitrary point of the shell are described by the following expressions:

$$u_1 = \left(1 + \frac{z}{R_1}\right)u + z\phi_1 + z^2\psi_1 + z^3\gamma_1; \qquad u_2 = \left(1 + \frac{z}{R_2}\right)v + z\phi_2 + z^2\psi_2 + z^3\gamma_2; \qquad u_3 = w,$$

where u_1, u_2, u_3 are the projections of displacements of an arbitrary point of the shell onto the associated coordinate axes; R_1, R_2 are the radii of curvature of the middle surface of an undeformed shell along the corresponding connected axes; z is the distance from an arbitrary point of the shell to the middle surface; ψ_i; γ_i; $i = 1,2$ are the unknown functions to be determined. To define these functions, the following boundary conditions are used [22]:

$$\gamma_{13}\big|_{z=\mp 0.5h}=0\,;\ \gamma_{23}\big|_{z=\mp 0.5h}=0\,,$$

where γ_{13}, γ_{23} are shear stresses. Thus, the SSS of the shell is described by five independent functions: u, ϕ_1, v, ϕ_2, w.

The conditions for the continuity of the displacements of shell points at the attachment interface of geometric primitives $s=s_*$ and $s=s_*+L=L_1$ are as follows:

$$w^{(2)}\Big|_{s=s_*+0}=w^{(1)}\Big|_{s=s_*-0}\cos(\alpha)+u^{(1)}\Big|_{s=s_*-0}\sin(\alpha)\,;\quad u^{(2)}\Big|_{s=s_*+0}=u^{(1)}\Big|_{s=s_*-0}\cos(\alpha)-w^{(1)}\Big|_{s=s_*-0}\sin(\alpha)\,;$$

$$v^{(2)}\Big|_{s=s_*+0}=v^{(1)}\Big|_{s=s_*-0}\,;\ \phi_1^{(2)}\Big|_{s=s_*+0}=\phi_1^{(2)}\Big|_{s=s_*-0}\,;\quad \phi_2^{(2)}\Big|_{s=s_*+0}=\phi_2^{(1)}\Big|_{s=s_*-0}\cos(\alpha)\,; \tag{1}$$

$$w^{(2)}\Big|_{s=L_1-0}=w^{(3)}\Big|_{s=L_1+0}\cos(\alpha)+u^{(3)}\Big|_{s=L_1+0}\sin(\alpha);\quad u^{(2)}\Big|_{s=L_1-0}=w^{(3)}\Big|_{s=L_1+0}\sin(\alpha)-u^{(3)}\Big|_{s=L_1+0}\cos(\alpha);$$

$$v^{(2)}\Big|_{s=L_1-0}=-v^{(3)}\Big|_{s=L_1+0}\,;\ \phi_1^{(2)}\Big|_{s=L_1-0}=-\phi_1^{(3)}\Big|_{s=L_1+0}\,;\ \phi_2^{(2)}\Big|_{s=L_1-0}=-\phi_2^{(3)}\Big|_{s=L_1+0}\cos(\alpha), \tag{2}$$

where functions with superscripts $^{(1)}$ and $^{(3)}$ correspond to the displacements and angles of rotation of the normal on the spherical parts of the structure; superscript $^{(2)}$ corresponds to the cylindrical part of the structure.

Shell SSS Model

The deformation analysis of the structure is performed by the Ritz method. The potential energy of the shell is constructed under the assumption that the displacements of the shell due to static buckling are much smaller than those due to the action of internal pressure. Then these displacements can be considered as independent quantities, similar to the approach used in the method of multiple scales

$$u=u_0+\mu u_1\,;\ v=v_0+\mu v_1\,;\ w=w_0+\mu w_1\,;\ \phi_1=\phi_{1,0}+\mu\phi_{1,1}\,;\ \phi_2=\phi_{2,0}+\mu\phi_{2,1}\,,$$

where μ is a small parameter; u_0, v_0, w_0, $\phi_{1,0}$, $\phi_{2,0}$ are the displacements and angles of rotation of the normal under axisymmetric SSS due to the action of internal pressure; u_1, v_1, w_1, $\phi_{1,1}$, $\phi_{2,}$ are the displacements and angles of rotation of the normal under axisymmetric SSS due to the buckling of the shell. Such an approach allows us to separate the phenomena of the static deformation of the shell and its static instability.

To analyze the SSS of the shell, the shell model is discretized. For this, the displacements of the structure are represented as an expansion in basis functions, and the expansion coefficients are the required parameters that determine the SSS. The basis functions are chosen so as to satisfy the displacement continuity conditions (1), (2).

Axisymmetric SSS Analysis

The analysis of the axisymmetric SSS of the shell due to the action of internal pressure is reduced to minimizing the potential energy depending on the expansion coefficients. Since the potential energy is a quadratic form with respect to the variables u_0, v_0, w_0, $\phi_{1,0}$, $\phi_{2,0}$, its local extremum corresponds to its minimum. Thus, the axisymmetric SSS of the structure can be determined from the system of linear algebraic equations

$$\frac{\partial\Pi^{(0)}}{\partial A_i}=0, i=1,\ldots,N_\Omega\,,$$

where $\Pi^{(0)}$ is the potential energy of the structure under axisymmetric deformation; A_i is the expansion coefficients of the variables u_0, v_0, w_0, $\phi_{1,0}$, $\phi_{2,0}$ in a given basis; N_Ω is the total number of the expansion coefficients. It should be noted that under axisymmetric SSS $v_0\equiv 0$; $\phi_{2,0}\equiv 0$ which is why the unknown coefficients should be sought only for the variables u_0, w_0, $\phi_{1,0}$.

Static Buckling Analysis

Under the action of an axial load, a cylindrical shell can buckle, which leads to non-axisymmetric deformation and a sudden loss of structural rigidity [11]. The analysis of nonaxisymmetric SSS due to the static buckling of the shell under the action of the axial forces T_x on it also begins with discretizing the system. The variables u_1, v_1, w_1, $\phi_{1,1}$, $\phi_{2,1}$ are represented as expansions in the basis functions

$$w_1 = \sum_{i=1}^{L_w} B_i R_i^{(w)}(s)\cos(n\phi);\; w_1 = \sum_{i=1}^{L_u} B_{L_w+i} R_i^{(u)}(s)\cos(n\phi);$$

$$v_1 = \sum_{i=1}^{L_v} B_{L_w+L_u+i} R_i^{(v)}(s)\sin(n\phi);\phi_{1,1} = \sum_{i=1}^{L_X} B_{L_w+L_u+L_v+i} R_i^{(X)}(s)\cos(n\phi);\; \phi_{2,1} = \sum_{i=1}^{L_Y} B_{L_w+L_u+L_v+L_X+i} R_i^{(Y)}(s)\cos(n\phi),$$

where $R_i^{(w)}$, $R_i^{(u)}$, $R_i^{(v)}$, $R_i^{(X)}$, $R_i^{(Y)}$, $i = 1,2,\ldots$ are the basis functions that satisfy conditions (1), (2); (B_1, $B_2,\ldots,B_M$), $M = L_w + L_u + L_v + L_X + L_Y$ is the vector of unknown expansion coefficients.

The potential energy of the system has the form

$$E_\Sigma = \Pi_\Sigma - V_\Sigma; \tag{3}$$

$$V_\Sigma = \frac{T_x}{2}\int_0^L dx \int_0^{2\pi} \left(\left(\frac{\partial w_1^{(2)}}{\partial x} \right)^2 + \left(\frac{\partial u_1^{(2)}}{\partial x} \right)^2 + \left(\frac{\partial v_1^{(2)}}{\partial x} \right)^2 \right) R_c d\phi,$$

where Π_Σ is the potential energy of the shell; V_Σ is the potential energy of the axial load T_x [24]. The equilibrium position of the system is characterized by the minimum potential energy (3), which enables us to formulate a homogeneous system of linear algebraic equations $\frac{\partial E_\Sigma}{\partial B_i} = 0; i = 1,\ldots,M$, which makes it possible to construct the eigenvalue problem in the form

$$\begin{bmatrix} K_{11} & \cdots & K_{15} \\ \vdots & \ddots & \vdots \\ K_{51} & \cdots & K_{55} \end{bmatrix} \begin{bmatrix} B_w \\ B_u \\ B_v \\ B_X \\ B_Y \end{bmatrix} = T_x \begin{bmatrix} V_{11} & V_{12} & V_{13} & 0 & 0 \\ V_{21} & V_{22} & V_{23} & 0 & 0 \\ V_{31} & V_{32} & V_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} B_w \\ B_u \\ B_v \\ B_X \\ B_Y \end{bmatrix}, \tag{4}$$

where $K_{11},\ldots, K_{55}$, $V_{11},\ldots, V_{33}$ are matrices. The vector of the coefficients is presented in matrix form $(B_1,\ldots, B_M) = [B_w\, B_u\, B_v\, B_X\, B_Y]$. Since the matrix on the right-hand side of (4) is a degenerate one, the eigenvalue problem cannot be solved in form (4). This is due to the fact that the parameters B_w, B_u, B_v are linearly dependent on the parameters B_X, B_Y. To search for this dependence, consider the last two lines of equation (4)

$$\begin{bmatrix} K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \end{bmatrix} \begin{bmatrix} B_w \\ B_u \\ B_v \end{bmatrix} = -\begin{bmatrix} K_{44} & K_{45} \\ K_{54} & K_{55} \end{bmatrix} \begin{bmatrix} B_X \\ B_Y \end{bmatrix}.$$

Thus, $\begin{bmatrix} B_X \\ B_Y \end{bmatrix} = -\begin{bmatrix} K_{44} & K_{45} \\ K_{54} & K_{55} \end{bmatrix}^{-1} \begin{bmatrix} K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \end{bmatrix} \begin{bmatrix} B_w \\ B_u \\ B_v \end{bmatrix}.$

Substitution of this relation in the first three lines of (4) allows us to construct the well-posed eigenvalue problem

$$[\Gamma] \begin{bmatrix} B_w \\ B_u \\ B_v \end{bmatrix} = T_x \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \begin{bmatrix} B_w \\ B_u \\ B_v \end{bmatrix},$$

where $[\Gamma] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} - \begin{bmatrix} K_{14} & K_{15} \\ K_{24} & K_{25} \\ K_{34} & K_{35} \end{bmatrix} \begin{bmatrix} K_{44} & K_{45} \\ K_{54} & K_{55} \end{bmatrix}^{-1} \begin{bmatrix} K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \end{bmatrix}$

From this eigenvalue problem, critical axial loads and buckling patterns are determined.

Results

The analysis of the static buckling of the nano-composite shell was carried out (Fig. 3).

The structural parameters are as follows: $R_s = R_c = 0.25$ m; $h = 5$ mm; $L = 1$ m, $\alpha = 0^\circ$. The dependence of the critical pressure on the type of nano-reinforcement and the value of the internal pressure was investigated.

A comparison was made with the results of calculating the critical pressure on the structure that were obtained using the ANSYS package for the UD nano-composite.

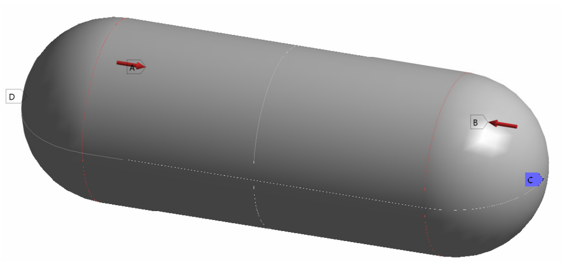

Fig. 3. Shell picture

The calculation results are shown in Fig. 4. The solid line shows the results of calculations by the developed method, the symbols ■, ▲, and ♦ show the results of finite element analysis, the numbers 0.28, 0.17, 0.12 indicate the results corresponding to these volume fractions of nano-tubes in the material.

The results of calculations by the developed method and the finite element method are close (Fig. 4).

The dependence of the critical pressure on the type of a nano-composite and internal pressure for composites with a nano-tube volume fraction of 0.28 is shown in Fig. 5. It is seen that the type of a nano-composite can significantly (up to 1.5 times) affect the critical pressure; in this case, the type of the dependence of the critical pressure on the internal one remains practically unchanged.

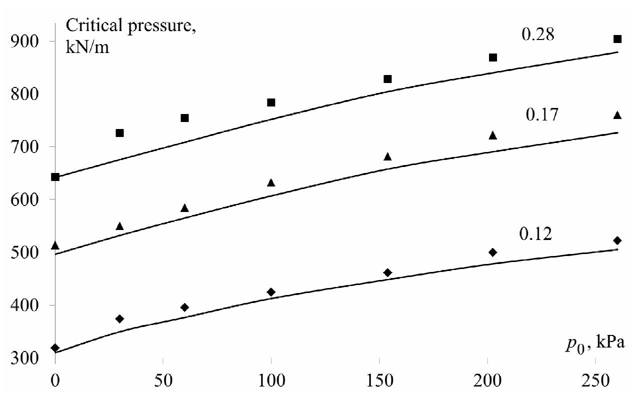

Fig. 4. Comparison of the developed and finite element methods

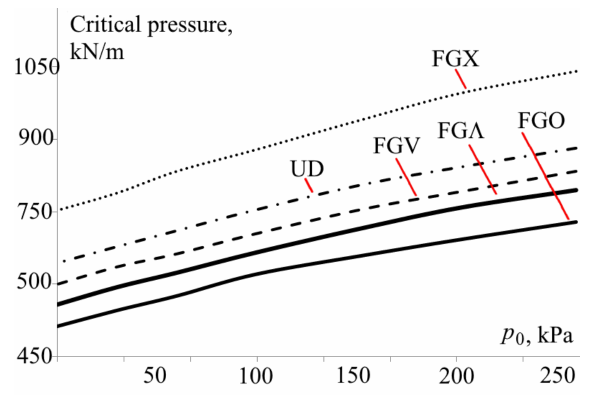

Fig. 5. Dependence of the critical pressure on the type of a nano-composite

Conclusions

A method for analyzing the static buckling of a pre-loaded complex nano-composite shell under the action of a compressive axial load is proposed. The method is based on the high-order shear theory and the Ritz method. The comparison of this method with the finite element analysis shows a high accuracy of results.

The analysis of the dependence of the critical pressure on the type of a nano-composite shows that a rational choice of the type of nano-reinforcement without changing the volume fraction of CNTs in it can increase the critical pressure up to one and a half times.

The proposed method makes it possible to analyze large-scale thin-walled structures, as well as structures made of functional-gradient materials. These objects are a traditional weak point of finite element analysis methods.

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У статті описано методику аналізу явища втрати статичної стійкості попередньо навантаженої нанокомпозитної складеної оболонки. Більшість робіт, які присвячено аналізу складених конструкцій, розглядають вібраційні процеси. Водночас явище втрати стійкості може стати важливим фактором, що обмежує використання нових матеріалів у ракетно-космічній техніці. Розглянуто нанокомпозитну оболонку постійної товщини, яка складається з двох сферичних кришок та циліндричного корпуса. На оболонку діє внутрішній тиск та осьова стискаюча сила. Така оболонка може моделювати паливний бак ракети-носія. Досліджуються умови, за яких оболонка деформується невісесиметрично внаслідок втрати статичної стійкості. Запропоновано методику, яка дозволяє розділити задачу на аналіз попередньо навантаженого стану оболонки та аналіз втрати стійкості. Подальший аналіз здійснюється допомогою методики, яка базується на теорії зсуву високого порядку та методі Рітца. Проводиться дискретизація задачі шляхом подання змінних, що визначають стан оболонки, в формі розкладень за базисними функціями з невідомими коефіцієнтами. Таким чином, невідомими задачі стають коефіцієнти розкладень. Задача аналізу попередньо навантаженого стану конструкції зводиться до розв'язання системи лінійних алгебраїчних рівнянь відносно коефіцієнтів розкладень. Задачу аналізу втрати стійкості може бути зведено до задачі власних значень. Розв'язавши цю задачу, можна знайти мінімальне значення стискаючого навантаження, що призводить до втрати стійкості оболонки, а також форми втрати стійкості. Результати застосування розробленої методики було порівняно з результатами скінченноелементного моделювання на конструкції з найпростішого нанокомпозитного матеріалу. Результати порівняння свідчать про високу точність описаної методики. При цьому використання методу скінченних елементів для аналізу масштабних тонкостінних конструкцій з функціонально градієнтних матеріалів є надзвичайно ускладненим, на відміну від методики, яку запропоновано у статті. Порівняння різних видів наноармування свідчить про те, що раціональний вибір типу армування може суттєво підвищити критичне навантаження. При цьому на критичне навантаження також значно впливає внутрішній тиск на оболонку.

Ключові слова: *статична нестійкість, складена оболонка, нанокомпозитний матеріал, вуглецеві нанотрубки, метод Рітца.*

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