DOI: https://doi.org/10.15407/pmach2021.02.016

UDC 539. 3

ANALYTICAL CALCULATION OF THE MECHANICAL PROPERTIES OF HONEYCOMBS PRINTED USING THE FDM ADDITIVE MANUFACTURING TECHNOLOGY

[1] **Kostiantyn V. Avramov**
kvavramov@gmail.com
ORCID: 0000-0002-8740-693X

[1] **Borys V. Uspenskyi**
Uspensky.kubes@gmail.com
ORCID: 0000-0001-6360-7430

[2] **Ihor I. Derevianko**
dereviankoii2406@gmail.com
ORCID: 0000-0002-1477-3173

[1] A. Pidhornyi Institute of Mechanical Engineering Problems of NASU
2/10, Pozharskyi St., Kharkiv, 61046, Ukraine

[2] Yuzhnoye State Design Office,
3, Krivorizka Str, Dnipro, 49008, Ukraine

FDM 3D printed honeycombs are investigated. A honeycomb is composed of regular hexagonal cells. A honeycomb is 3D printed so that the fused filament runs along the walls of its cells. We emphasize that the thickness of these walls is one or two times the thickness of the fused filament. When calculating the mechanical properties of a honeycomb, its walls are considered as a Euler-Bernoulli beam bending in one plane. To describe honeycombs, a homogenization procedure is used, which reduces a honeycomb to a homogeneous orthotropic medium. An adequate analytical calculation of the mechanical properties of this medium is the subject of our research. Analytical formulae for calculating the mechanical properties of honeycombs are presented. To assess the adequacy of the calculation results, the analytical data are compared with the results of simulation in the commercial ANSYS package. For this, the mechanical properties of the honeycombs made of the ULTEM 9085 material are determined numerically. To assess these properties, from a large number of analytical formulae are selected those that predict them adequately. As a result of calculations, an analytical prediction of all mechanical properties is obtained, with the exception of the in-plane shear modulus of a honeycomb. This is due to the fact that to simulate such a shear modulus one has to use a three-dimensional theory that does not have an adequate analytical description. A thin aluminum honeycomb was considered. In the future, three-layer structures with such a honeycomb will be investigated. Analytical results for ULTEM 9085 and aluminum honeycombs are similar.

Keywords: *honeycomb, mechanical properties, orthotropic material, additive technology.*

Introduction

Calculation of the mechanical properties of honeycombs is an extremely important fundamental problem, since they are widely used in aerospace engineering as elements of sandwich structures. Without their calculation, it is impossible to design thin-walled sandwich structures. The main purpose of composite sandwich faceplates is to withstand flexural and axial loads. The main purpose of honeycombs is to maintain an adequate distance between the sandwich faceplates, which allows a sandwich to have high stiffness with low weight. Honeycombs are an integral part of thin-walled sandwich structures. To construct their deformation models, the mechanical properties of honeycombs are used. To determine these properties, three groups of approaches are used: analytical, numerical, and experimental. This article only deals with analytical calculation methods.

Honeycombs are such that their shears contribute most to the potential energy of a structure. The first analytical formulae obtained for determining shear moduli are described in [1]. In this work, an analytical estimation of the shear moduli is compared with experimental data. In [2, 3], an approach is proposed for the analytical analysis of the in-plane mechanical properties of a honeycomb. Simple analytical formulae are obtained for calculating the indicated properties. In [4], relationships are derived for the analytical calculation of the out-of-plane shear moduli of a honeycomb. In [5], analytical relationships are obtained for the stiffnesses of honeycombs, on the assumption that the main load in a honeycombs is carried by membrane forces, and bending can be neglected. In [6], approaches are given for calculating shear moduli, with the help of periodic medium homogenization, by constructing asymptotic expansions. Mechanical properties of periodic structures with a negative Poisson's ratio are investigated in [7]. Refined formulae for calculating the nine constants of honeycombs are presented in [8]. In [9, 10], a review of the most reliable analytical relationships for calculating the mechanical properties of a periodic honeycomb-based structure is carried out. A review of the relationships describing the analytical elastic properties of periodic honeycombs is presented in [11].

In this paper, the mechanical properties of FDM 3D printed honeycombs are calculated using simple analytical relationships. The results of numerical simulation in the ANSYS software package are compared with analytical results. Analytical formulae are selected, which give calculation results that coincide with the results of numerical simulation. Static deformation is analyzed; dynamic processes [12–14] are not considered here.

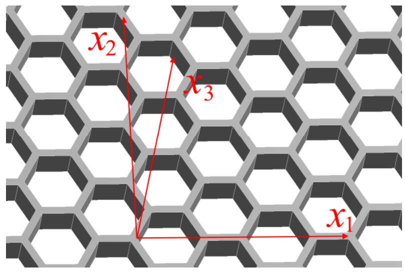

Fig. 1. Sketch of a honeycomb

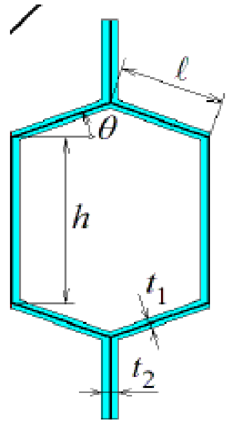

Fig. 2. Sketch of a honeycomb cell

Approach to Analytical Analysis

A honeycomb is 3D printed so that the fused filament runs along the walls of its cells. We emphasize that the thickness of these walls is one or two times the thickness of the fused filament. The elastic properties of the walls in the longitudinal direction are described by the elastic modulus $\tilde{E}_X$ of the polymer material obtained using the FDM technology. When calculating the mechanical properties of a honeycomb, its walls are considered as beams bending in one plane.

In the future, analytical formulae will be presented, which are used to calculate the mechanical properties of honeycombs. A sketch of a honeycomb is shown in Fig. 1. The geometric parameters of one honeycomb cell are shown in figure 2. Consider a particular case of constant honeycomb-cell wall thickness $t=t_1=t_2$. The elastic modulus of a honeycomb in the direction x is determined as follows [15]:

$$E_1 = k_1 \tilde{E}_X \left(\frac{t}{l_B}\right)^3 \frac{\cos(\theta)}{\left(\frac{h}{l}+\sin(\theta)\right)}, \tag{1}$$

$$k_1 = \frac{1}{\left(1+3\frac{S_1}{l_B t}\right)\sin^2(\theta) + 2\kappa\left(1+\tilde{\nu}_{xy}\right)\left(\frac{t}{l_B}\right)^2\left(1+\frac{S_1}{l_B t}\right)\sin^2(\theta) + \left(\frac{t}{l_B}\right)^2\left(1+\frac{S_1}{l_B t}\right)\cos^2(\theta)},$$

where $\tilde{\nu}_{xy}$ is Poisson's ratio of the FDM-made polymer material; parameters t_1, t_2, l, h, θ are shown in Fig. 2; κ is shift coefficient (κ=1,2); l_B is the length of the inclined wall of the honeycomb cell; $l_B = l - t/(2\cos(\theta))$; $S_1 = \frac{A_{cell}}{4} - \left(l_B t + \frac{h_b t}{2}\right) - \frac{A_{void}}{4}$; $A_{void} = (2h + 2l\sin(\theta) - 2t/\cos(\theta) + t\tan(\theta))(2l\cos(\theta) - t)$; h_B is the length of the vertical wall of the honeycomb cell; $h_B = h - t(1-\sin(\theta))/\cos(\theta)$; $A_{cell} = 4l\cos(\theta)(h + l\sin(\theta))$.

To calculate the rest of the in-plane mechanical properties of honeycombs, we will use the following analytical relationships, which are presented in [15]:

$$E_2 = k_2 \tilde{E}_X \left(\frac{t}{l_B}\right)^3 \frac{\left(\frac{h}{l}+\sin(\theta)\right)}{\cos^3(\theta)}; \tag{2}$$

$$\nu_{12} = c_{12} \frac{\cos^2(\theta)\sin(\theta)}{\left(\frac{h}{l}+\sin(\theta)\right)}; \tag{3}$$

$$G_{12}=\tilde{E}_X\left(\frac{t}{l_B}\right)^3\frac{\left(\frac{h}{l}+\sin(\theta)\right)}{\left(\frac{h}{l}\right)^2\cos(\theta)\,\tilde{c}_{12}}, \tag{4}$$

where

$$\frac{1}{k_2}=1+\left[2\kappa\left(1+\tilde{\nu}_{XY}\right)+\tan^2(\theta)+\frac{2h_B}{l_B\cos^2(\theta)}\right]\left(\frac{t}{l_B}\right)^2+\frac{S_1}{l_Bt}\left\{3+\left(\frac{t}{l_B}\right)^2\left[2\kappa\left(1+\nu_{XY}\right)+\tan^2(\theta)\right]\right\}; \tag{5}$$

$$c_{12}=\frac{\left(1+\frac{3S_1}{tl_B}\right)+\left(\frac{t}{l_B}\right)^2\left(1+\frac{S_1}{tl_B}\right)\left[2\kappa\left(1+\tilde{\nu}_{XY}\right)-1\right]}{\left(1+\frac{3S_1}{tl_B}\right)\sin^2(\theta)+\left(\frac{t}{l_B}\right)^2\left(1+\frac{S_1}{tl_B}\right)\left[2\kappa\left(1+\tilde{\nu}_{XY}\right)\sin^2(\theta)+\cos^2(\theta)\right]}; \tag{6}$$

$$\tilde{c}_{12}=1+2\left(\frac{h_B}{l_B}\right)^3\left(\frac{l}{h}\right)^2+\frac{3S_1}{l_Bt}+2\kappa\left(1+\tilde{\nu}_{XY}\right)\left(\frac{t}{l_B}\right)^2\left[1+2\frac{h_B}{l_B}\left(\frac{l}{h}\right)^2\right]+\frac{\left(1+\frac{h}{l}\sin(\theta)\right)^2}{\cos^2(\theta)}\left(\frac{l}{h}\right)^2\left(\frac{t}{l_B}\right)^2. \tag{7}$$

When deriving the relationships for the mechanical properties of honeycombs (1–4), the bending, shear, and tension of their cell walls are taken into account. The simplest formulae for calculating these properties are published in [2]. In these formulae, only the classical bending of honeycombs is taken into account. The formulae can be obtained from (5–7) by discarding the terms that are unessential in such an analysis. Formulae (1–4) are valid in this particular case, and the correcting factors $k_1, k_2, c_{12}, \tilde{c}_{12}$ have the form

$$k_1=c_{12}=\frac{1}{\sin^2(\theta)};\quad k_1=\tilde{c}_{12}=1. \tag{8}$$

The mechanical properties were calculated based on relationships (1–4), and the approach proposed in [10]. Let us consider the main results of this approach. Let us take into account the influence of Poisson's ratios on the deformation of honeycomb cell walls. Then the elastic constants are calculated as follows:

$$E_1=\frac{E_1|_p}{1+k_{11}E_1|_p}; \tag{9}$$

$$E_2=\frac{E_2|_p}{1+k_{22}E_2|_p}; \tag{10}$$

$$G_{12}=\frac{G_{12}|_p}{1+k_{12}G_{12}|_p}; \tag{11}$$

where quantities $E_1|_p$, $E_2|_p$, $G_{12}|_p$ are derived from equations (1–7) by substituting, into these equations, $\frac{\tilde{E}_X}{1-\tilde{\nu}_{XY}^2}$ instead of $\tilde{E}_X$ and $\frac{\tilde{\nu}_{XY}}{1-\tilde{\nu}_{XY}}$ instead of $\tilde{\nu}_{XY}$. The coefficients k_{11}, k_{22}, k_{12} are determined as follows:

$$k_{11}=\frac{\tilde{\nu}_{XY}^2\left(\sin(\theta)+\frac{h}{l}\right)}{\tilde{E}_X\left(t/l\right)}\left\{\cos(\theta)+\frac{6\left(1-\tilde{\nu}_{XY}^2\right)\sin^2(\theta)}{\pi^3\left(t/l\right)^2\left(b/l\right)\cos(\theta)}\sum_{m=1}^{\infty}\frac{H_{m1}}{m^3}\right\}; \tag{12}$$

$$k_{22} = \frac{\tilde{\nu}_{XY}^2 \cos(\theta)}{\tilde{E}_X\left(\sin(\theta)+\dfrac{h}{l}\right)}\left\{\frac{2}{(t/h)}+\frac{\sin^2(\theta)}{(t/l)}+\frac{6\left(1-\tilde{\nu}_{XY}^2\right)\cos^2(\theta)}{\pi^3(t/l)^3(b/l)}\sum_{m=1}^{\infty}\frac{H_{m1}}{m^3}\right\}; \quad (13)$$

$$k_{12} = \frac{\tilde{\nu}_{XY}^2}{\tilde{E}_X\left(\sin(\theta)+\dfrac{h}{l}\right)\cos(\theta)}\left\{\frac{l}{t}\left(\sin(\theta)+\frac{h}{l}\right)^2+\frac{6\left(1-\tilde{\nu}_{XY}^2\right)\cos^2(\theta)}{\pi^3}\left(\frac{h}{b}\right)\left[\left(\frac{l}{t}\right)^2\left(\frac{h}{t}\right)\sum_{m=1}^{\infty}\frac{H_{m1}}{m^3}+2\left(\frac{h}{t}\right)^3\sum_{m=1}^{\infty}\frac{H_{m2}}{m^3}\right]\right\}; \quad (14)$$

$$H_{m1} = \frac{2\sinh^2(\alpha_{m1})}{\left(1-\tilde{\nu}_{XY}\right)\left[\left(3+\tilde{\nu}_{XY}\right)\sinh(\alpha_{m1})\cosh(\alpha_{m1})-\left(1-\nu_{XY}\right)\alpha_{m1}\right]}; \quad \alpha_{m1} = \frac{m\pi b}{l};$$

$$H_{m2} = \frac{2\sinh^2(\alpha_{m2})}{\left(1-\tilde{\nu}_{XY}\right)\left[\left(3+\tilde{\nu}_{XY}\right)\sinh(\alpha_{m2})\cosh(\alpha_{m2})-\left(1+\tilde{\nu}_{XY}\right)\alpha_{m2}\right]}; \quad \alpha_{m2} = \frac{m\pi b}{l}.$$

In [9], to determine the shear moduli G_{13}, G_{23}, the following relationships are used:

$$G_{13} = \tilde{G}_{XZ}\left(\frac{t}{l}\right)\frac{\cos(\theta)}{\left(\dfrac{h}{l}+\sin(\theta)\right)}\frac{1}{C_{13}^*}; \qquad G_{23} = \tilde{G}_{XZ}\left(\frac{t}{l_B}\right)\frac{\left(\dfrac{h}{l}+\sin(\theta)\right)}{\cos(\theta)}\frac{1}{C_{23}^*};$$

$$C_{13}^* = \frac{\kappa l_B}{l}+\frac{S_1}{tl}+\frac{1}{2\left(1+\tilde{\nu}_{XY}\right)}\left(\frac{l_B}{b}\right)^2\left(\frac{l_B}{l}+\frac{3S_1}{tl}\right); \quad C_{23}^* = \kappa+2\left(\frac{h_B}{l_B}\right)+\frac{4S_1}{l_B t}+\frac{1}{2\left(1+\tilde{\nu}_{XY}\right)}\left(\frac{l_B}{b}\right)^2\left(1+\frac{3S_1}{4tl_B}\right).$$

In [8], to determine the shear moduli G_{12}, G_{13}, G_{23}, the following formulae were obtained

$$G_{12} = \tilde{E}_X\left(\frac{t}{l}\right)^3\frac{1+\sin(\theta)}{\cos(\theta)}\frac{1}{C}; \quad (15)$$

$$G_{13} = \tilde{G}_{XZ}\frac{(t/l)}{\left(\dfrac{h}{l}+\sin(\theta)\right)\cos(\theta)}\left[\left(\frac{l_B}{l}\right)\cos^2(\theta)+\frac{3}{4}\left(\frac{t}{l}\right)\tan(\theta)-\frac{\cos(\theta)}{2}\left(\frac{t}{l}\right)\left(2\sin(\theta)-1\right)\right]; \quad (16)$$

$$G_{23} = \tilde{G}_{XZ}\frac{(t/l)}{\left(\dfrac{h}{l}+\sin(\theta)\right)\cos(\theta)}\left[\left(\frac{l_B}{l}\right)\sin^2(\theta)+\frac{h_B}{2l}+\frac{3}{4}\left(\frac{t}{l}\right)\tan(\theta)-\frac{\sin^2(\theta)}{2\cos(\theta)}\left(\frac{t}{l}\right)\left(2\sin(\theta)-1\right)\right], \quad (17)$$

where

$$C = 3+\left(\frac{t}{l}\right)^2\left[\left(2{,}4+1{,}5\tilde{\nu}_{XY}\right)\left(3+\sin(\theta)\right)+\left(1+\sin(\theta)\right)\{\left(1+\sin(\theta)\right)\right]\tan^2(\theta)+\}\sin(\theta).$$

In [6], asymptotic procedures were used to homogenize the mechanical properties of honeycombs. As a result, analytical relationships were obtained for all mechanical properties. We used the following analytical relationships:

$$G_{13} = G_{23} = \frac{\tilde{G}_{XZ}t}{l\sqrt{3}}; \quad (18)$$

$$G_{12} = \frac{2}{3\sqrt{3}l\beta_{XY}}\left(1+\cos(\alpha)+\sin^2(\alpha)\right)\left(1+\cos(\alpha)-\frac{\cos(\alpha)}{r_{XY}}-\frac{2\cos^2(\alpha)}{r_{XY}}\right), \quad (19)$$

where $r_{XY} = 1+\cos(\alpha)$; $\beta_{XY} = \dfrac{3l^2}{2\tilde{E}_X t^3 r_{XY}}+\dfrac{3}{2t\tilde{G}_{XZ}r_{XY}}+\dfrac{r_{XY}}{2\tilde{E}_X t\left(1-\tilde{\nu}_{XY}^2\right)\sin^2(\alpha)}$; $\alpha = \dfrac{\pi}{2}-\theta$.

Analysis of Calculation Results

In [16], the mechanical properties of ULTEM 9085 honeycombs were numerically determined. A general view of such a honeycomb is shown in Fig. 2. Its geometrical parameters are as follows:

$$h=l=6.34 \text{ mm}; \; t_1=t_2=0.4 \text{ mm}; \; b=10 \text{ mm}; \; \theta=30°. \quad (20)$$

The mechanical properties of the ULTEM 9085 material used for printing such a honeycomb were determined experimentally. This procedure is described in [17]. As follows from this work, ULTEM 9085 is an orthotropic polymer with the following mechanical properties relative to the material axes:

$$E_X=2.25 \text{ GPa}; \; E_Y=2.96 \text{ GPa}; \; E_Z=2.41 \text{ GPa}; \quad \nu_{XY}=0.31; \; \nu_{YZ}=0.26; \; \nu_{XZ}=0.33;$$

$$G_{XY}=0.667 \text{ GPa}; \; G_{YZ}=0.889 \text{ GPa}; \; G_{XZ}=0.829 \text{ GPa}.$$

As is well known [5, 6, 11], a honeycomb is homogenized, turning into an homogeneous orthotropic medium. We calculated the mechanical properties, using numerical procedures, which are discussed in detail in [16]. The calculation results are presented in Table 1 (first row). The numerical results were compared with the analytical calculation results. Calculation results 0 (Table 1) were obtained from relationships (1–4), (8). The upper subrow shows the values of the calculated mechanical properties, the lower one – the relative error of the analytical calculation results in comparison with the numerical ones. As one can see from the table, the results given in the upper subrow are in poor agreement with the numerical ones for the mechanical properties. Therefore, we will refine the calculated analytical relationships.

Now let us consider calculation results 1. These results were obtained from relationships (1–7), and are shown in the upper subrow. The lower subrow shows the relative differences between the calculations obtained analytically and numerically. One can see that the calculation results given in this subrow are closer to the numerical simulation results than calculation results 0.

Now let us consider calculation results 2. They are obtained using relationships (9–14). The upper subrow shows the values of the mechanical properties, and the lower one, the relative difference between the results obtained numerically and analytically. The elastic moduli E_1, E_2, E_3 obtained analytically are quite accurate. However, the parameters G_{12}, G_{13}, G_{23} were obtained inaccurately. Therefore, we will look for other analytical approaches to calculate these parameters.

In the row with calculation 3 (Table 1) are shown the results of calculating the shear moduli of honeycombs, based on relationships (15–17). As can be seen from the table, the results of calculating the elastic moduli G_{13}, G_{23}, obtained from the analytical formulae, are close to the numerical simulation results. The results of calculating the shear modulus G_{12} are far from the numerical simulation ones.

To calculate all the shear moduli, analytical formulae (18, 19) are used. The results of these calculations are given in calculation 4 (Table 1). The calculation results of G_{13} and G_{23}, obtained numerically and analytically, are close. The numerical and analytical results of calculating G_{12} differ significantly.

So, all the mechanical properties of an ULTEM 9085 honeycomb (Table 1), obtained numerically and analytically, are close. An exception is the shear modulus G_{12}. To determine it, the honeycomb is deformed so that it experiences a three-dimensional stress state. So far, it has not been possible to obtain a sufficiently accurate analytical description of such a stress state.

Table 1. Mechanical properties of ULTEM honeycombs

Calculation	Mechanical properties								
	E_1, MPa	E_2, MPa	G_{12}, MPa	G_{13}, MPa	G_{23}, MPa	ν_{12}	E_3, MPa	ν_{13}	ν_{23}
Numerical simulation	1.500	1.50	0.576	31.000	31.000	0.98	227	0.003	0.002
0	1.330	1.33	0.332	–	–	1.00	–	–	–
	0.130	0.13	0.420	–	–	0.02	–	–	–
1	0.410	1.41	0.336	–	–	0.98	–	–	–
	0.060	0.06	0.416	–	–	0	–	–	–
2	1.540	1.53	0.366	25.800	30.500	–	227	0.002	0.002
	0.027	0.02	0.360	0.160	0.160	–	0	0.330	0
3	–	–	0.409	32.200	32.200	–	–	–	–
	–	–	0.290	0.070	0.070	–	–	–	–
4	–	–	0.327	33.400	33.400	–	–	–	–
	–	–	0.430	0.077	0.077	–	–	–	–

Table 2. Mechanical properties of aluminum honeycombs

Calculation	Mechanical properties								
	E_1, MPa	E_2, MPa	G_{12}, MPa	G_{13}, MPa	G_{23}, MPa	ν_{12}	E_3, MPa	ν_{13}	ν_{23}
Numerical simulation	0.963	0.928	0.319	259.000	259.000	0.98	1810.00	$1.92 \cdot 10^{-4}$	$1.92 \cdot 10^{-4}$
0	0.840	0.840	0.210	–	–	1.00	–	–	–
	0.120	0.090	0.340	–	–	0.02	–	–	–
1	0.858	0.858	0.211	–	–	0.99	–	–	–
	0.100	0.070	0.330	–	–	0.01	–	–	–
2	0.958	0.958	0.236	249.000	260.000	–	1750.00	$1.80 \cdot 10^{-4}$	$1.80 \cdot 10^{-4}$
	0.005	0.030	0.260	0.038	0.004	–	0.03	0.06	0.06
3	–	–	0.262	263.000	263.000	–	–	–	–
	–	–	0.170	0.015	0.015	–	–	–	–
4	–	–	0.209	266.000	266.000	–	–	–	–
	–	–	0.340	0.025	0.025	–	–	–	–

Let us consider a thin aluminum honeycomb. In the future, we will investigate three-layer structures with such a honeycomb. Its geometric properties (Fig. 2) are as follows:

$$l=h=3.66 \text{ mm}; \quad b=20 \text{ mm}; \quad \delta_1=\delta_2=0.0635 \text{ mm}; \quad \theta=30°.$$

The mechanical properties of the honeycomb material (aluminum) are as follows:

$$E=70 \text{ GPa}; \quad \nu=0.33; \quad G=26.31 \text{ GPa}.$$

The results of the numerical and analytical calculations of the mechanical properties of honeycombs are given in Table 2. The description format for the results in Table 2 is similar to that in Table 1. The analytical and numerical calculation results for all the mechanical properties given in Table 2 are close, except for the shear modulus G_{12}, which could not be predicted analytically with sufficient accuracy.

Conclusions

To calculate the mechanical properties of FDM 3D printed honeycombs, analytical formulae can be used. The results of such calculations are close to the numerical simulation results. The exception is the shear modulus G_{12}, which can only be predicted numerically. It is recommended to use relationships (9–14) to calculate the mechanical properties E_1, E_2. It is proposed to use formulae (16–18) to calculate the shear moduli G_{13} and G_{23}.

Financing

This study was supported by the National Research Foundation of Ukraine (project no. 128 / 02.2020) and the National Academy of Sciences of Ukraine (grant no. 0120U101241).

References

1. Kelsey, S., Gallatly, R. A., & Clark, B. W. (1958). The shear modulus of foil honeycomb cores. *Aircraft Engineering*, vol. 30, iss. 10, pp. 294–302. https://doi.org/10.1108/eb033026.
2. Gibson, L. J., Ashby, M. F., Schajer, G. S. & Robertson, C. I. (1982). The mechanics of two-dimensional cellular materials. *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, vol. 382, iss. 1782, pp. 25–42. https://doi.org/10.1098/rspa.1982.0087.
3. Abd El-Sayed, F. K., Jones, R., & Burgess, I. W. (1979). A theoretical approach to the deformation of honeycomb based composite material. *Composites*, vol. 10, iss. 4, pp. 209–214. https://doi.org/10.1016/0010-4361(79)90021-1.
4. Meraghni, F., Desrumaux, F. & Benzeggagh, M. L. (1999). Mechanical behaviour of cellular core for structural sandwich panels. *Composites Part A: Applied Science and Manufacturing*, vol. 30, iss. 6, pp. 767–779. https://doi.org/10.1016/S1359-835X(98)00182-1.
5. Becker, W. (1998). The in-plane stiffnesses of a honeycomb core including the thickness effect. *Archive of Applied Mechanics*, vol. 68, pp. 334–341. https://doi.org/10.1007/s004190050169.
6. Shi, G. & Tong, P. (1995). The derivation of equivalent constitutive equations of honeycomb structure by two scale method. *Computational Mechanics*, vol. 15, pp. 395–407. https://doi.org/10.1007/BF00350354.
7. Masters, I. G. & Evans, K. E. (1996). Models for the elastic deformation of honeycomb. *Composite Structures*, vol. 35, iss. 4, pp. 403–422. https://doi.org/10.1016/S0263-8223(96)00054-2.
8. Malek, S. & Gibson, L. (2015). Effective elastic properties of periodic hexagonal honeycombs. *Mechanics of Materials*, vol. 91, pp. 226–240. 10.1016/j.mechmat.2015.07.008.

9. Sorohan, S., Constantinescu, D. M., Sandu, M., & Sandu, A. G. (2018). On the homogenization of hexagonal honeycombs under axial and shear loading. Part I: Analytical formulation for free skin effect. *Mechanics of Materials*, vol. 119, pp. 74–91. https://doi.org/10.1016/j.mechmat.2017.09.003.
10. Chen, D.-H., Horii, H., & Ozaki, O. (2009). Analysis of in-plane elastic modulus for a hexagonal honeycomb core: Analysis of Young's modulus and shear modulus. *Journal of Computational Science and Technology*, vol. 3, iss. 1, pp. 1–12. https://doi.org/10.1299/jcst.3.1.
11. Hohe, A. J. & Becker, W. (2002). Effective stress-strain relations for two-dimensional cellular sandwich cores: Homogenization, material models, and properties. *Applied Mechanics Reviews*, vol. 55, iss. 1, pp. 61–87. https://doi.org/10.1115/1.1425394.
12. Avramov, K. V. & Pellicano, F. (2006). Dynamical instability of cylindrical shell with big mass at the end. *Reports of the National Academy of Science of Ukraine*, iss. 5, pp. 41–46.
13. Avramov, K. (2003). Bifurcations of parametric oscillations of beams with three equilibria. *Acta Mechanica*, vol. 164, pp. 115–138. https://doi.org/10.1007/s00707-003-0022-9.
14. Avramov, K. V. (2002). Nonlinear beam oscillations excited by lateral force at combination resonance. *Journal of Sound and Vibration*, vol. 257, iss. 2, pp. 337–359. https://doi.org/10.1006/jsvi.2002.5043.
15. Gibson, L. J. & Ashby, M. F. (1988). Cellular Solids: Structure and properties. Cambridge, United Kingdom: Cambridge University Press, 357 p. https://doi.org/10.1002/adv.1989.060090207.
16. Derevianko, I., Avramov, K., Uspenskyi, B., & Salenko, A. (2021). *Eksperymentalnyi analiz mekhanichnykh kharakterystyk detalei raket-nosiiv, vyhotovlenykh za dopomohoiu FDM adytyvnykh tekhnolohii* [Experimental analysis of mechanical characteristics of parts of launch vehicles manufactured using FDM additive technologies]. *Tekhnichna mekhanika – Technical Mechanics*, iss. 1, pp. 92–100 (in Ukrainian). https://doi.org/10.15407/itm2021.01.092.
17. Uspenskiy, B., Avramov, K., Derevyanko, I., Biblik, I. (2021). *K raschetu mekhanicheskikh kharakteristik sotovykh zapolniteley, izgotovlennykh additivnymi tekhnologiyami FDM* [On calculations of mechanical properties of honeycomb produced by FDM manufacturing]. *Aviatsionno-kosmicheskaya tekhnika i tekhnologiya* – Aerospace Technic and Technology, no. 1, pp. 14–20 (in Russian). https://doi.org/10.32620/aktt.2021.1.02.

Received 19 April 2021

Аналітичний розрахунок механічних характеристик стільникових заповнювачів, які надруковано за допомогою адитивної технології FDM

[1] К. В. Аврамов, [1] Б. В. Успенський, [2] І. І. Деревянко

[1] Інститут проблем машинобудування ім. А. М. Підгорного НАН України,
61046, Україна, м. Харків, вул. Пожарського, 2/10

[2] Державне підприємство «Конструкторське бюро «Південне» ім. М.К. Янгеля»,
49008, Україна, м. Дніпро, вул. Криворізька, 3

Досліджено стільникові заповнювачі, надруковані за допомогою адитивних технологій FDM. Комірка стільникового заповнювача є правильним шестикутником. Стільники друкуються на 3D принтері так, що нитка друку йде уздовж стінки комірки стільника. Підкреслимо, що товщина стінок стільників складає одну–дві товщини нитки. Під час розрахунку механічних характеристик стінки стільникового заповнювача розглядаються як балка Ейлера-Бернуллі, що згинається в одній площині. Для опису стільникових заповнювачів використовується процедура гомогенізації, яка зводить стільниковий заповнювач до однорідного ортотропного середовища. Адекватний аналітичний розрахунок механічних характеристик такого середовища є предметом цих досліджень. Наведено аналітичні формули, за якими здійснюються розрахунки механічних характеристик стільникових заповнювачів. Для оцінки адекватності результатів аналітичні дані порівнюються з результатами моделювання в комерційному пакеті ANSYS. Для цього чисельно визначаються механічні характеристики стільникових заповнювачів з ULTEM 9085. Для оцінки механічних характеристик з великої кількості аналітичних формул вибираються ті, які адекватно описують механічні характеристики стільникових заповнювачів. В результаті розрахунків отримано аналітичний опис всіх механічних характеристик за винятком модуля зсуву в площині стільникового заповнювача. Це пояснюється тим, що для моделювання такого модуля зсуву доводиться використовувати тривимірну теорію, яка не має адекватного аналітичного опису. Розглянуто тонкий стільниковий заповнювач, виготовлений з алюмінію. Надалі будуть досліджуватися тришарові конструкції з таким стільниковим заповнювачем. Результати аналітичного аналізу стільникових заповнювачів з ULTEM і алюмінію є близькими.

Ключові слова: *стільниковий заповнювач, механічні властивості, ортотропний матеріал, адитивні технології.*

Література

1. Kelsey S., Gallatly R. A., Clark B. W. Theshearmodulus of foil honeycomb cores. *Aircraft Eng*. 1958. Vol. 30. Iss. 10. P. 294–302. https://doi.org/10.1108/eb033026.
2. Gibson L. J., Ashby M. F., Schajer G. S., Robertson C. I. The mechanics of two-dimensional cellular materials. *Proc. The Royal Society of London. Ser. A. Math. and Phys. Sci*. 1982. Vol. 382. Iss. 1782. P. 25–42. https://doi.org/10.1098/rspa.1982.0087.
3. Abd El-Sayed F. K., Jones R., Burgess I. W. A theoretical approach to the deformation of honeycomb based composite material. *Composites*. 1979. Vol. 10. Iss. 4. P. 209–214. https://doi.org/10.1016/0010-4361(79)90021-1.
4. Meraghni F., Desrumaux F., Benzeggagh M. L. Mechanical behaviour of cellular core for structural sandwich panels. *Composites Part A: Appl. Sci. and Manufacturing*. 1999. Vol. 30. Iss. 6. P. 767–779. https://doi.org/10.1016/S1359-835X(98)00182-1.
5. Becker W. The in-plane stiffnesses of a honeycomb core includingthe thickness effect. *Archive Appl. Mech*. 1998. Vol. 68. P. 334–341. https://doi.org/10.1007/s004190050169.
6. Shi G., Tong P. The derivation of equivalent constitutive equations of honeycomb structure by two scale method. *Comp. Mech.* 1995. Vol. 15. P.395–407. https://doi.org/10.1007/BF00350354.
7. Masters I. G., Evans K. E. Models for the elastic deformation of honeycomb. *Composite Structures*. 1996. Vol. 35. Iss. 4. P. 403–422. https://doi.org/10.1016/S0263-8223(96)00054-2.
8. Malek S., Gibson L. Effective elastic properties of periodic hexagonal honeycombs. *Mech. Materials*. 2015. Vol. 91. P. 226–240. https://doi.org/10.1016/j.mechmat.2015.07.008.
9. Sorohan S., Constantinescu D. M., Sandu M., Sandu A. G. On the homogenization of hexagonal honeycombs under axial and shear loading. Part I: Analytical formulation for free skin effect. *Mechanics Materials*. 2018. Vol. 119. P. 74–91. https://doi.org/10.1016/j.mechmat.2017.09.003.
10. Chen D.-H., Horii H., Ozaki O. Analysis of in-plane elastic modulus for a hexagonal honeycomb core: Analysis of Young's modulus and shear modulus. *J. Comp. Sci. and Techn.* 2009. Vol. 3. Iss. 1. P. 1–12. https://doi.org/10.1299/jcst.3.1.
11. Hohe A. J., Becker W. Effective stress-strain relations for two-dimensional cellular sandwich cores: Homogenization, material models, and properties. *Appl. Mech. Reviews*. 2002. Vol. 55. Iss. 1. P. 61–87. https://doi.org/10.1115/1.1425394.
12. Avramov K. V., Pellicano F. Dynamical instability of cylindrical shell with big mass at the end. *Reports National Academy Sci. Ukraine*. 2006. Iss. 5. P. 41–46.
13. Avramov K. Bifurcations of parametric oscillations of beams with three equilibria. *Acta Mech.* 2003. Vol. 164. P. 115–138. https://doi.org/10.1007/s00707-003-0022-9.
14. Avramov K. V. Nonlinear beam oscillations excited by lateral force at combination resonance. *J. Sound and Vibration.* 2002. Vol. 257. Iss. 2. P. 337–359. https://doi.org/10.1006/jsvi.2002.5043.
15. Gibson L. J., Ashby M. F. Cellular solids: structure and properties. Cambridge, United Kingdom: Cambridge University Press, 1988. 357 p. https://doi.org/10.1002/adv.1989.060090207.
16. Деревянко І., Аврамов К., Успенський Б., Саленко А. Експериментальний аналіз механічних характеристик деталей ракет-носіїв, виготовлених за допомогою FDM адитивних технологій. *Техн. механіка.* 2021. Вип. 1. С. 92–100. https://doi.org/10.15407/itm2021.01.092.
17. Успенский Б., Аврамов К., Деревянко И., Библик И. К расчету механических характеристик сотовых заполнителей, изготовленных аддитивными технологиями FDM. *Авиац.-косм. техника и технология*. 2021. № 1. С. 14–20. https://doi.org/10.32620/aktt.2021.1.02.