DOI: https://doi.org/10.15407/pmach2021.03.070

UDC 517.95+518.517

MATHEMATICAL AND COMPUTER SIMULATION OF HEX HEAD CREWS FOR IMPLEMENTATION ON A 3D PRINTER

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In this paper, based on the R-functions theory, methods have been developed and equations have been constructed for the 3D printing of hex-head screws with Bristol, Pentalobe, Polydrive and other types of screw slots. Such screws are used both in personal computers and other high-end equipment. The Bristol slot has four or six radial grooved beams. The advantage of the design of this slot is the correct perpendicular, rather than tangential, vector of force application when the slot is rotated by a tool, which minimizes the risk of stripping out the slot. For this reason, the Bristol slot is used in soft metal screws. Compared to the internal hex, the Bristol slot allows a noticeably higher torque, only slightly higher than that of the Torx slot. This type of slot is used in aviation, high-end telecommunications equipment, cameras, air brakes, agricultural equipment, astronomical equipment, and foreign military equipment. Variations with a pin in the center are found in game consoles to prevent the use of a flat-blade screwdriver as an improvised key. The Pentalobe slot is a five-point slot designed by Apple and used in its products to limit unauthorized disassembly. It was first used in mid 2009 to mount MacBook Pro batteries. Its miniature version was used in the iPhone 4 and later models, in the MacBook Air (available since late 2010 models), and the MacBook Pro with Retina screens. The Polydrive slot is a starlike slot with rounded star points, used in the automotive industry for applications requiring high tightening torque. The Torq-set slot is a cross slot for fasteners requiring high tightening torque. The grooves are slightly offset, not intersecting at one point. Fasteners with this type of slot are used in military aviation, for example, in E-3, P-3, F-16, Airbus, Embraer, and Bombardier Inc. The Phillips Screw Company owns the trademark and manufactures fasteners with this type of slot. The slot design standards are National Aerospace Standard NASM 3781 and NASM 4191 for the ribbed version. The resulting equations for the surfaces of screws were checked during the modeling of the screws before 3D printing. The 3D printing technology allows us to reduce the cost and labor intensity of manufacturing products, including complex slot screws. The analytical recording of designed objects makes it possible to use alphabetic geometric parameters, complex superposition of functions, which, in turn, allows us to quickly change their design elements. The positivity property of the constructed functions at the internal points of an object is very convenient for the implementation of 3D printing.

Keywords: *R-functions, mathematical model, screw, slot, 3D printing.*

Introduction

One of the new technologies that has been gaining popularity in recent years is 3D printing. It allows us to create volumetric models of any objects using special equipment – a 3D printer. Advantages of using modern 3D printers are reducing the cost of manufacturing products and the timing of their emergence on the market, modeling objects of any shape and complexity, speed and high precision of manufacturing, the ability to use various materials. However, there is a problem with specifying the printing information. In the process of preparing products for 3D printing, it is necessary to create a 3D computer model of the desired object. In studies on the computer modeling of solid bodies, carried out in the works of A. G. Requicha [1–3], eight main presentation schemes have been identified: 1) engineering drawings; 2) wire frame representation; 3) representation with primitives; 4) voxel presentation; 5) discrete models; 6) Constructive Solid Geometry; 7) schematic sweep representation; 8) boundary representation (Brep). The analysis of the above classical representations showed that their practical application is limited, or requires significant efforts in constructing models of complex-shaped objects. From the point of view of universality, one of the most promising is the functional representation, which is based on the use of the language of implicit mathematical functions with the constructive capabilities of the R-functions theory developed by academician V. L. Rvachev [4].

The aim of this paper is to develop techniques, based on the R-functions theory, and construct equations for the surfaces of various types of slotted screws to be subsequently 3D printed.

Fastener slots are recesses in the heads of threaded fasteners for transferring torque from the tool to the slots. The slots can be straight, cross, square, multi- square, inner hexagonal, five-pointed, star-shaped, combined, etc. In addition to the common, widely used types, there are less common ones used as anti-vandal or designed to prevent unauthorized access. Slots are anti-vandal if their task is both to complicate unauthorized access in public places and complicate the self-opening and repair of electronic devices. Anti-vandalism is often based on the fact that screwdrivers for the required slot are not available in standard tool sets, and it is rather difficult to find them on the market. In some cases, the manufacturer can replace a rare but standardized type of slot with its own, proprietary, patent-protected, which enables not only to make unauthorized untwisting as difficult as possible, but also prosecute any independent screwdriver manufacturer that does not have a manufacturing license from the patent holder. Examples of proprietary slots include T-Groove, Slot-Lok, Pentagon, Tork-Nut, T-Slope.

Main Part

When constructing mathematical models by the R-functions method, both the simplest R-operations $\begin{cases} x \wedge_0 y \equiv x + y - \sqrt{x^2 + y^2} \\ x \vee_0 y \equiv x + y + \sqrt{x^2 + y^2} \\ \bar{x} \equiv -x \end{cases}$ and R-operations $\begin{cases} x \wedge_\rho y = x + y - \sqrt{x^2 + y^2 + \dfrac{SR}{8\rho^2}\left(SR + \left|SR\right|\right)} \\ x \vee_\rho y = x + y + \sqrt{x^2 + y^2 + \dfrac{SR}{8\rho^2}\left(SR + \left|SR\right|\right)} \\ \bar{x} = -x \qquad SR = \rho^2 - x^2 - y^2 \end{cases}$ will be used to smooth sharp edges and corners, where ρ is the radius of rounded corners.

When constructing equations corresponding to geometric objects with a cyclic point symmetry, the results of the following theorem [5] will be used to reduce the number of R-operations.

Theorem. Let the translation domain $\Sigma_0 = \left[\sigma_0(x, y) \geq 0\right]$ be symmetrical about the abscissa axis, and let it be possible for the domain $\Sigma_1 = \left[\sigma_0(x - r_0, y) \geq 0\right]$ to be placed inside the sector $-\alpha \leq \theta \leq \alpha,\ 0 < \alpha < \dfrac{\pi}{n}$. The domains $\Sigma_k = \left[\sigma_0\left(r\cos(\theta - 2\pi k/n) - r_0, r\sin(\theta - 2\pi k/n)\right) \geq 0\right]$ $k = 0,1,\ldots,n-1$ have been obtained as a result of rotating the domain $\Sigma_1 = \left[\sigma_0(x - r_0, y) \geq 0\right]$ around the origin by angles $2\pi k/n$ ($k = 0,1,\ldots,n-1$). Then the equation of the boundary $\partial\Omega$ of the domain $\Omega = \bigcup_{k=0}^{n-1}\Sigma_k$ has the form $\omega(x, y) \equiv \sigma_0\left(r\cos\mu(\theta, n) - r_0, r\sin\mu(\theta, n)\right) = 0$, $\left(r = \sqrt{x^2 + y^2},\ \theta = arctg\,\dfrac{y}{x}\right)$, where $\mu(n\theta) = \dfrac{8}{n\pi}\sum_k (-1)^{k+1} \dfrac{\sin\left[(2k-1)\dfrac{n\theta}{2}\right]}{(2k-1)^2}$.

For chamfering, we use the equation of a conical surface, for which the guide is described by the equation $\omega(x, y) = 0$, and the vertex is at a point $A(x_0, y_0, z_0)$. As a result, the equation $\omega(x, y) = 0$ of the conical surface is obtained by changing the variables $\begin{cases} x \Leftarrow x_0 - z_0\dfrac{x - x_0}{z - z_0} \\ y \Leftarrow y_0 - z_0\dfrac{y - y_0}{z - z_0} \end{cases}$ in the equation $\omega(x, y) = 0$, namely $\omega\left(x_0 - z_0\dfrac{x - x_0}{z - z_0}, y_0 - z_0\dfrac{y - y_0}{z - z_0}\right) = 0$ [5].

Let us build a mathematical model of a 6-32 (UTS) cross-slotted hex head screw used in personal computers

$$\rho=\sqrt{x^2+y^2}\ ;\ \theta=arctg\frac{y}{x}\ ;\ \mu=\frac{8}{no\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{no\theta}{2}\right]}{(2k-1)^2}\ ;$$

$$f1=3-\rho\cos\mu\ ;\ fp1=3.7-\rho\cos\mu\ ;\ x0=0\ ;\ y0=0\ ;\ z0=4\ ;\ no=6;\ rr=1;\ r1=1.8;$$

$$xk=x0-z0\frac{x-x0}{z-z0}\ ;\ yk=y0-z0\frac{y-y0}{z-z0}\ ;\ \omega k=16-xk^2-yk^2\wedge_0 z+1\geq 0\ ;$$

$$f2=\left(f1\wedge_0\left(1-z^2\right)/2\right)\wedge_0\omega k\geq 0\ ;\ f4=\left(2.6^2-x^2-y^2\right)/5.2\wedge_0 z-0.6\geq 0\ ;$$

$$f5=\left(\left(16-x^2-y^2\right)/8\wedge_0(-1-z)(1.3+z)\right)\wedge_0 fp1\geq 0\ ;\ h=1;\ ri=1.6;$$

$$\mu1=\frac{4h}{\pi^2}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)(\frac{\pi z}{h}-\frac{\theta}{2})\right]}{(2k-1)^2}\ ;\ ff3=0.4^2-(\rho-1.6)^2-\mu1^2\geq 0;\ fi=ri^2-\rho^2\geq 0\ ;$$

$$kk=\left(fi\wedge_0\overline{ff3}\right)\wedge_0(-1-z)(6+z)\geq 0\ ;$$

$$\begin{cases} t1=\left(0.25^2-x^2\right)/0.5;\ t2=\left(0.25^2-y^2\right)/0.5; \\ f31=t1\vee_\rho t2\geq 0;\ \rho=1;\ f3=f31\wedge_0\left(4-x^2-y^2\right)/4\geq 0;\ fs1=f2\wedge_0-f3\geq 0; \end{cases}\qquad(1)$$

$$fs2=fs1\wedge_0-f4\geq 0\ ;\ fs=fs2\vee_0 f5\geq 0\ ;$$

$$res=(((fi\wedge_0-ff3)\wedge_0(-1-z)(6+z)))\vee_0 fs\geq 0\ .$$

Fig. 1 shows a hex washer head screw constructed by the formula $\left((f2\vee_0 f5)\wedge_0\overline{f4}\right)\vee_0 kk\geq 0$. A rod with an external thread is attached to the screw head according to the formula $\left((f2\vee_0 f5)\wedge_0\overline{f4}\right)\vee_0 kk\geq 0$ (Fig. 2), the equation of which can be constructed according to the results of work [6]. Substituting the formula for the cross slot (1), we finally get the screw model 6-32 (UTS) (Fig. 3).

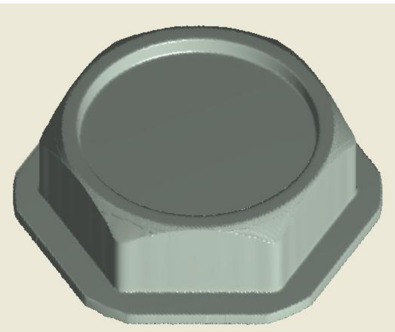

Fig. 1. Hex washer head screw

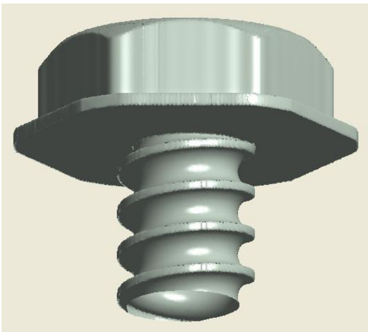

Fig. 2. Screw with an external threaded rod

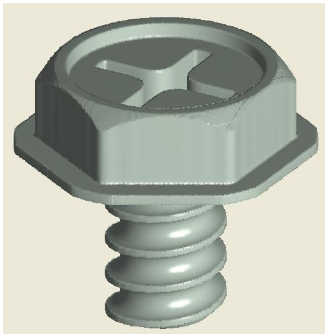

Fig. 3. Screw model 6-32 (UTS)

Replacing (1) with

$$t1=1-\frac{\rho^2\sin^2\mu}{0.25^2}-\frac{\rho^2\cos^2\mu}{2^2}\geq 0\ ;\ t2=0.8^2-x^2-y^2\geq 0\ ;\ f31=t1\vee_\rho t2\geq 0\ ;\ fs1=f2\wedge_0-f31\geq 0\ ,$$

we get a Torx hex head screw (Fig. 4).

Replacing (1) with

$$no1=5\,;\ \mu 1=\frac{8}{no1\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{no1\theta}{2}\right]}{(2k-1)^2}\,;$$

$$f31=0.8^2-(ro\cos\mu 1-1.3)^2-(ro\sin\mu 1)^2\,;\ f3=f31\vee_0 1.3^2-x^2-y^2\geq 0\,,$$

we get a Pentalobe hex head screw (Fig. 5), which is used by Apple and Meizu in personal computers.

Replacing (1) with $f31=-0.6^2+(ro\cos\mu-2)^2+(ro\sin\mu)^2\geq 0$; $f3=f31\wedge_0 2^2-x^2-y^2\geq 0$, we get a Polydrive hex head screw (Fig. 6).

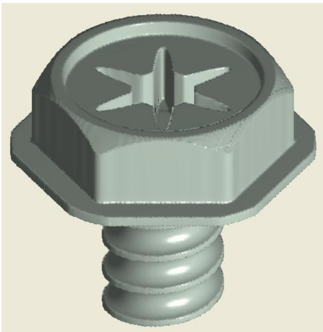

Fig. 4. Torx screw

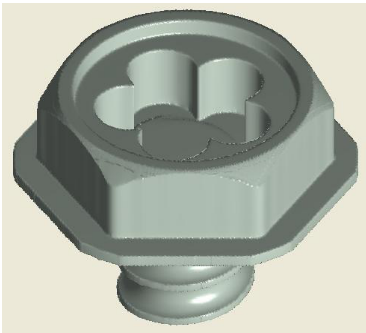

Fig. 5. Pentalobe hex head screw

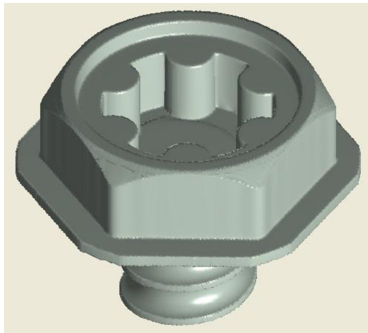

Fig. 6. Polydrive hex head screw

A Bristol hex head screw (Fig. 7) can be obtained by replacing (1) with

$$f31=(0.5^2-(ro\sin\mu)^2)\wedge_0(2^2-(ro\cos\mu)^2-(ro\sin\mu)^2)((ro\cos\mu)^2+(ro\sin\mu)^2-1.5^2)\geq 0\,;$$

$$f3=-f31\wedge_0 2^2-x^2-y^2\geq 0\,,$$

and a Torq-set hex head screw (Fig. 8), with

$$d1=(3-x)(x+0.7)\wedge_0(0.7-y)y\geq 0\,;\ d2=(x+0.7)(-x)\wedge_0(3-y)y\geq 0\,;$$

$$d3=(x+3)(0.7-x)\wedge_0(0.7+y)(-y)\geq 0\,;\ d4=(0.7-x)x\wedge_0(0.7-y)(3+y)\geq 0\,;$$

$$f31=((d1\vee_0 d2)\vee_0 d3)\vee_0 d4\geq 0\,;\ f3=f31\wedge_0 2^2-x^2-y^2\geq 0\,.$$

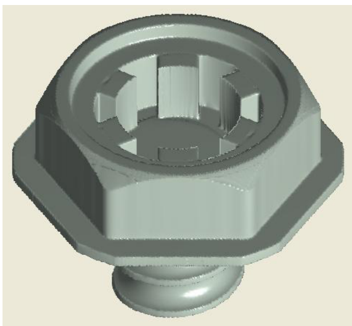

Fig. 7. Bristol hex head screw

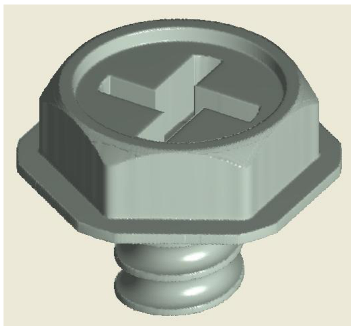

Fig. 8. Torq-set hex head screw

Thus, having built a model of the screw base in the form of a hex washer head with external threaded rod, and then, by including in the formula of the main screw block only the logical formula of the required slot, we obtain the desired result. At the same time, construction and manufacturing costs are minimized.

Fig. 9 shows the results of the 3D printing of some of the constructed screw models.

Fig. 9. Results of the 3D-printing of screw models

Conclusions

Building a mathematical model is a central stage in studying or designing any system. All the subsequent analysis of the object depends on the quality of the model. The model must be sufficiently accurate, adequate, and easy to use.

Summing up, it should be said that in this work, for the first time, thanks to the R-functions theory, methods have been developed and equations have been constructed for various hex head slotted screws, which are used both in personal computers and in other high-class equipment, for the implementation on a 3D printer. The analytical recording of the designed objects makes it possible to use alphabetic geometric parameters, complex superposition of functions, which, in turn, allows us to quickly change their structural elements.

The proposed method for specifying the shape of products, using a limited number of parameters, can significantly reduce the complexity of work in cases where we need to view a large number of design options in search of an optimal solution. With parametric assignment, the change in the computational domains is carried out almost instantly.

With the help of R-functions, an algorithm for the step-by-step construction of the equations of screws has been developed and implemented, which allows us to check and make adjustments to the model at each stage of its construction.

The reliability of the results obtained, their adequacy to the designed objects is confirmed by visualization both in the operating conditions of the RFPreview program and by implementation on a 3D printer.

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Received 26 July 2021

Математичне та комп'ютерне моделювання гвинтів з шестигранною голівкою для реалізації на 3D-принтері

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У даній статті на основі теорії R-функцій розроблено методики і побудовано рівняння для моделювання гвинтів із шестигранною головкою та шліцами типу Bristol, Pentalobe, Polydrive та ін., які застосовуються як в персональних комп'ютерах, так і в іншому обладнанні високого класу, для їхнього подальшого друку на 3D-принтері. Шліц типу Bristol має чотири або шість радіальних променів-заглиблень. Перевагою конструкції даного шліца є правильний перпендикулярний, а не дотичний вектор прикладання сили при обертанні шліца інструментом, що мінімізує небезпеку його зриву. Через це шліц Bristol використовується в гвинтах з м'яких металів. Порівняно з внутрішнім шестигранником шліц Bristol допускає помітно більший крутний момент, лише трохи більше такого, ніж у шліцах Torx. Цей тип шліців використовується в авіації, телекомунікаційному обладнанні високого класу, камерах, повітряних гальмах, сільгосптехніці, астрономічному обладнанні та зарубіжній військовій техніці. Різновиди зі штифтом в центрі зустрічаються в ігрових приставках, для запобігання використанню плоскої шліцьової викрутки як імпровізованого ключа. Шліц Pentalobe – п'ятипроменевий шліц, розроблений компанією Apple і використовуваний нею в своїх продуктах для обмеження несанкціонованого розбирання. Вперше використаний в середині 2009 року для кріплення акумулятора MacBook Pro. Мініатюрна версія використовувалася в iPhone 4 і подальших моделях, в MacBook Air (в моделях з кінця 2010 р.), в MacBook Pro з екранами Retina. Шліц Polydrive – зіркоподібний шліц з заокругленими вершинами зірки. Застосовується в автомобільній промисловості для задач, що вимагають високого моменту затягування. Шліц Torq-set – хрестоподібний шліц для кріплення з високим моментом затяжки. Пази трохи зміщені і не перетинаються в одній точці. Кріплення з даним видом шліца використовується у військовій авіації, наприклад в E-3, P-3, F-16, Airbus, Embraer і Bombardier Inc. Компанія Phillips Screw Company володіє торговою маркою і виробляє кріплення з даним видом шліца. Стандартами, що описують конструкцію шліца, є National Aerospace Standard NASM 33781 і NASM 14191 для ребристої версії. Отримані рівняння для поверхонь гвинтів було перевірено в ході моделювання останніх перед друком на 3D-принтері. Технологія 3D-друку дозволяє знизити собівартість і трудомісткість виготовлення продукції, в тому числі гвинтів зі складними шліцами. Аналітичний запис проектованих об'єктів дає можливість використовувати буквені геометричні параметри, складні суперпозиції функцій, що, в свою чергу, дозволяє оперативно змінювати їх конструктивні елементи. Властивість позитивності побудованих функцій у внутрішніх точках об'єкта є дуже зручною для реалізації 3D-друку.

Ключові слова: *R-функції, математична модель, гвинт, шліц, 3D-друк.*

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