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MATHEMATICAL AND COMPUTER MODELING OF THE FORMS OF MULTI-ZONE FUEL ELEMENTS WITH PLATES

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Seeing the significant increase in the number of nuclear power plants, as well as models and modifications of nuclear reactors, it becomes important to find out/establish the advantages of certain plants. At the same time, designers face a number of questions for which optimal solutions have not yet been found. At nuclear plants, there is the largest turnover of financial funds and the smallest gain in economy brings huge profits, but one should not forget about reliability and costs during the plant construction. This is a complex problem that is solved at the design stage. Calculations of the reactor at the design stage make it possible to determine the main parameters of the active zone, temperature values, etc. Thermohydraulic calculation of the active zone of the reactor is one of the cornerstones in justifying the safe operation of the nuclear power plant. Calculations of coolant parameters and temperatures of fuel elements are carried out at all stages of designing and proving the safety of nuclear power plants. Twisted pipes and finned heat transfer surfaces are widely used in engineering to increase the effective heat transfer coefficient. In particular, longitudinal, transverse, and spiral edges are used for finning the shells of fuel elements of nuclear reactors and the outer surfaces of steam generator pipes. Finning not only increases the heat transfer surface on the side where the heat transfer coefficient has a low value, but also significantly affects the hydrodynamics of the flow, and thus affects this coefficient. It is obvious that the better the medium is mixed in the main flow and in the intercoral zone, the higher the heat transfer coefficient is. The most profitable forms of fuel elements shells finning are chevron and polyzonal finning, which are performed in the form of a multiturn spiral with a large step. The R-function theory turned out to be quite convenient for building mathematical models of finned shells of fuel elements with straight and helical plates, as well as for building the corresponding objects on a 3D printer. From a practical point of view, the relevance of the problem is also determined by the significant spread of twisted cylindrical bodies, twisted channels, coils in the energy, chemical, oil, gas, metallurgical industries and in heat engineering equipment. The flows that arise at this time make it possible to intensify the processes of heat and mass exchange and achieve savings in energy resources

***Keywords**: shells of fuel elements, fuel elements with chevron and polyzonal finning, R-function theory, 3D printer.*

Introduction

Currently, it is very difficult to give an accurate forecast of the ways of further development of the energy base of Ukraine, however, the issue of mathematical modeling of both physical processes in the fuel cartridges of fuel elements and the fuel elements of non-standard form themselves still does not lose its relevance. Fuel elements are classified according to various features: reactor type, coolant parameters, reactor purpose, design, etc. Judging by the classification of fuel elements, their diversity is impressive [1–3]. If only the geometric features are considered, then there are block, rod, ring, tubular, plate, and spherical fuel elements. Block and rod ones are cylinders, the former have a relative length *l*/*d* measured in tens, the latter – in hundreds. Ring fuel elements flow around on both sides, and tubular ones - only from the inside. Some fuel elements are made twisted, thanks to which their self-distancing is ensured. Fuel elements with longitudinal finning (including helical) and straight and helical plates are designed. All of the above creates difficulties in the production of fuel elements of non-standard shape. One of the new technologies that has recently gained popularity is 3D printing, which makes it possible to create three-dimensional models of any objects with the help of special equipment - a 3D printer. 3D printing technology is based on the technology of layer-by-layer

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"growing" of solid objects from different materials. Three-dimensional models are printed from plastic, concrete, hydrogel, wood, metal, and even from living cells. The process of printing on a 3D printer consists of several stages. The first stage is preparatory, during which a computer 3D model of the object to be printed is created. It is possible to create such a model in a 3D modeling program. Once the virtual model is created, it needs to be converted into a set of instructions for the printer, i.e. generate G-code and then start printing. 3D modeling allows to create a three-dimensional model of any object on a computer and, with the help of a 3D printer, obtain a full-fledged physical object that meets the specified parameters.

An important issue in the implementation of 3D printing is the construction of mathematical models of objects [4–8], which is considered the central stage of research or design of any system. All further analysis of the object depends on the quality of the model. The model should be sufficiently accurate, adequate, and also convenient to use. In the course of the analysis of mathematical modeling methods, it can be concluded that the R-function method, developed by academician V. Rvachev [4], is a universal method of creating three-dimensional models, and that is why it is used in the paper. Previously, the thermohydraulic calculation of fuel elements cartridges with violation of the rods packing symmetry was considered [8]. Constructive means and algorithms of the R-function method for mathematical and computer modeling of the combined problem of convective heat transfer in fuel elements grids were improved, and the influence of the packaging type and the cartridge shape on the velocity and temperature fields was studied [8]. The R-function method turned out to be quite convenient for constructing equations corresponding to geometric objects with translation symmetry along a straight line, line segment, with various options of cyclic symmetry, etc. Analytical notation and, therefore, the possibility of introduction of literal parameters into the logical formula allow to quickly and sometimes significantly change the shape of the projected geometric object. The positive property of the constructed function at the interior points of the object is useful in 3D printing.

The aim of this paper is the application of new constructive means of the R-function method for the development of mathematical and computer modeling techniques of multi-zone fuel elements with longitudinal finning and helical plates for their further implementation on a 3D printer.

Main part

R-operations of $\{R_0\}$ systems and superpositions with periodic functions $\mu(x,h)=\frac{4h}{\pi^2}\sum_{i=1}^{\infty}\frac{(-1)^{i+1}}{(2i-1)^2}\sin\frac{(2i-1)x\pi}{h}$ are used in the paper to construct equations that correspond to geometric objects with translation symmetry along a line, and $\mu(n\theta)=\frac{8}{n\pi}\sum_{k}(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n\theta}{2}\right]}{(2k-1)^2}$ – for constructing equations corresponding to geometric objects with point symmetry of the cyclic type [6].

Multi-zone fuel element; longitudinal finning with helical plates

$$\rho=\sqrt{x^2+y^2}\ ;\ \theta=arctg\frac{y}{x};\ ff=\theta*no_1/2;\ \mu_1=\frac{8}{\pi*no_1}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)ff]}{(2k-1)^2};\ \varphi=\frac{2\pi z}{no_3};$$

$$sx_1=\rho\cos\mu_1;\ sy_1=\rho\sin\mu_1;\ \omega1=2^2-x^2-y^2\geq 0;\ \omega22=\omega1\vee_0 1-\frac{(sx_1-2)_1^2}{0.56^2}-\frac{sy_1^2}{0.2^2}\geq 0;$$

$$xs=x\cos(\varphi/3)+y\sin(\varphi/3);\ ys=y\cos(\varphi/3)-x\sin(\varphi/3);\ \rho s=\sqrt{xs^2+ys^2}\ ;\ \theta s=arctg\frac{ys}{xs};$$

$$fff=\theta s*no_2/2;\ \mu=\frac{8}{\pi*no_2}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)fff]}{(2k-1)^2};\ \begin{cases}sx=r\cos\mu\\ sy=r\sin\mu\end{cases};$$

$$\omega lop=\left(0.1^2-sy^2\right)\wedge_0 sx(4-sx)\geq 0;$$

$$\omega_2=\omega_{22}\vee_0\omega lop\geq 0;\ \omega c=x^2+y^2-0.5^2\geq 0;\ \omega t=\left((\omega_2\wedge_0(7.5^2-z^2)\right)\wedge_0\omega c\geq 0\ \text{(Fig. 1).}$$

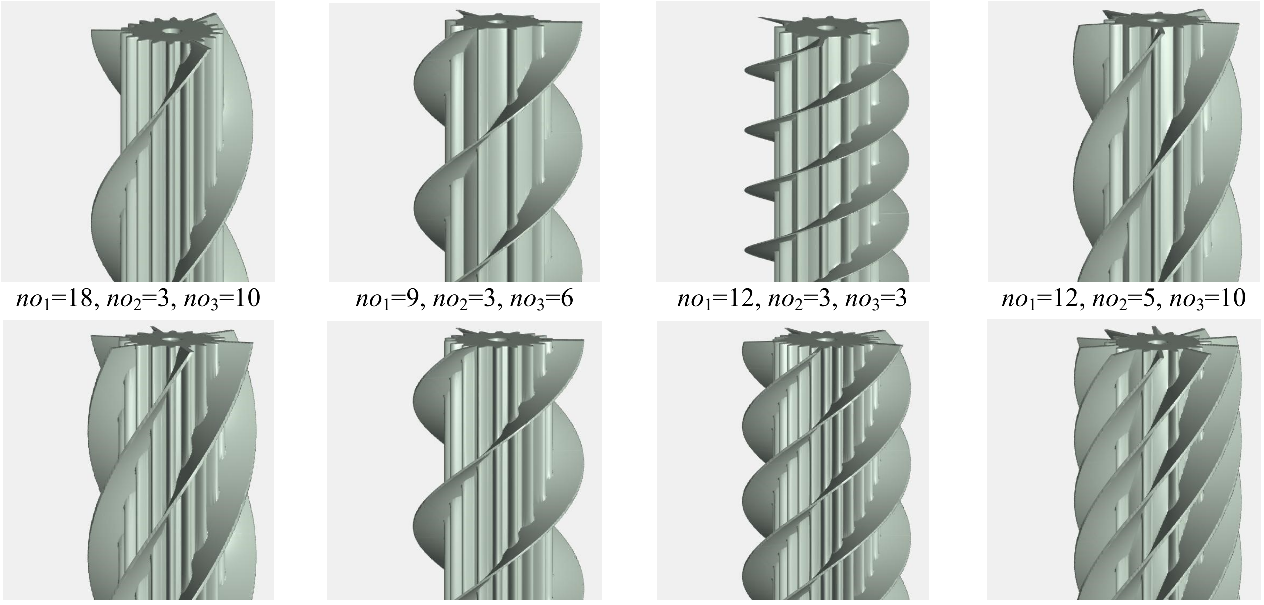

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Fig. 1. Fuel elements with longitudinal shells finning and helical plates

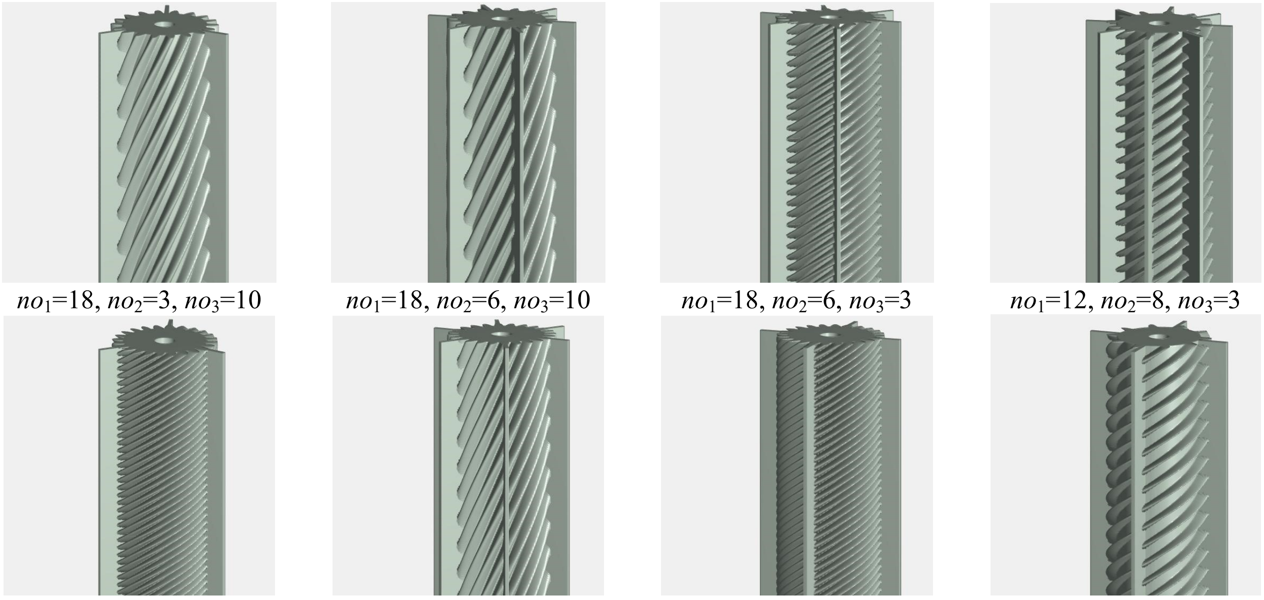

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Fig. 2. Fuel elements with helical shells finning and straight plates

Multi-zone fuel element; helical finning with straight plates

$$\varphi = \frac{2\pi z}{no_3};\ xs = x\cos(\varphi/3) + y\sin(\varphi/3);\ ys = y\cos(\varphi/3) - x\sin(\varphi/3);$$

$$\rho s = \sqrt{xs^2 + ys^2};\ \theta s = arctg\frac{ys}{xs};\ fff = \theta s * no_1/2;\ \mu = \frac{8}{\pi * no_1}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)fff]}{(2k-1)^2};\ \begin{cases} sx = \rho s\cos\mu \\ sy = \rho s\sin\mu \end{cases};$$

$$\omega_1 = 4 - x^2 - y^2 \ge 0;$$

$$\omega_{22} = \omega_1 \vee_0 1 - \frac{(sx-2)^2}{0.56^2} - \frac{sy^2}{0.2^2} \ge 0;$$

$$\rho o=\sqrt{x^2+y^2}\ ;\ \theta=arctg\frac{y}{x};\ ff=\theta*no_2/2;\ \mu_1=\frac{8}{\pi*no_2}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)ff]}{(2k-1)^2};$$

$$sx_1=\rho o\cos\mu_1;\ sy_1=\rho o\sin\mu_1;\ \omega_{lop}=0.1^2-sy_1^2\wedge_0 sx_1(3.5-sx_1)\geq 0;\ \omega_2=\omega_{22}\vee_0\omega_{lop}\geq 0;$$

$$\omega c=x^2+y^2-0.5^2\geq 0;\ \omega t=\left((\omega_2\wedge_0\left(7.5^2-z^2\right)\right)\wedge_0\omega c\geq 0\ \text{(Fig. 2).}$$

Multi-zone fuel elements with chevron finning and straight plates

$$\varphi=\frac{2\pi z}{no_2}\ ;\ \rho=\sqrt{x^2+y^2}\ ;\theta=arctg\frac{y}{x};\ ff_1=\frac{\theta*no_2}{2};\ \mu\mu=\frac{8}{\pi*no_2}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)ff_1]}{(2k-1)^2};$$

$$\begin{cases}xx=\rho\cos\mu\mu\\ yy=\rho\sin\mu\mu\end{cases};\ xs=xx\cos(\varphi/1)+yy\sin(\varphi/1);\ ys=y\cos(\varphi/3)-x\sin(\varphi/3);\ \rho s=\sqrt{xs^2+ys^2}\ ;$$

$$\theta s=arctg\frac{ys}{xs};\ fff=\frac{\theta s*no_1}{2};\ \mu=\frac{8}{\pi*no_1}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)fff]}{(2k-1)^2};\ \begin{cases}sx=\rho s\cos\mu\\ sy=\rho s\sin\mu\end{cases};$$

$$\omega_1=4-x^2-y^2\geq 0;\ ff=\frac{\theta*no_3}{2};\ \mu 1=\frac{8}{\pi*no_3}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)fff]}{(2k-1)^2};\ \begin{cases}sx_1=\rho\cos\mu 1\\ sy\ \ =\rho\sin\mu 1\end{cases};$$

$$\omega_{22}=\omega_1\vee_0 1-\frac{(sx-2)^2}{0.56^2}-\frac{sy^2}{0.2^2}\geq 0;\ \omega_{lop}=0.1^2-sy_1^2\wedge_0 sx_1(3.5-sx_1)\geq 0;\ \omega_2=\omega_{22}\vee_0\omega_{lop}\geq 0;$$

$$\omega c=x^2+y^2-0.5^2\geq 0;\ \omega t=\left((\omega_2\wedge_0\left(7.5^2-z^2\right)\right)\wedge_0\omega c\geq 0\ \text{(Fig. 3).}$$

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Fig. 3. Fuel elements with chevron shells finning and straight plates

Stacked rectilinear fuel elements with helical plates

$$\rho=\sqrt{x^2+y^2}\ ;\ \theta=arctg\frac{y}{x};\ ff=\theta*no_1/2;\ \mu_1=\frac{8}{\pi*no_1}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)ff]}{(2k-1)^2};$$

$$sx_1=\rho\cos\mu_1;\ sy_1=\rho\sin\mu_1;\ \omega 22=sx_1\wedge_0 1-\frac{sx_1^2}{9}-\frac{sy_1^2}{1}\geq 0;$$

$$\varphi = 2\pi z / no_3;\ xs = x\cos(\varphi/3) + y\sin(\varphi/3);\ ys = y\cos(\varphi/3) - x\sin(\varphi/3);$$

$$\rho s = \sqrt{xs^2 + ys^2};\ \theta s = arctg\frac{ys}{xs};\ fff = \theta s * no_2/2;\ \mu = \frac{8}{\pi * no_2}\sum_{k=0}(-1)^{k+1}\frac{\sin[(2k-1)fff]}{(2k-1)^2};$$

$$\begin{cases} sx = r\cos\mu \\ sy = r\sin\mu \end{cases};\ \omega lop = \left(0.1^2 - sy^2\right)\wedge_0 sx(3.5 - sx) \ge 0;$$

$$\omega_2 = \omega_{22} \vee_0 \omega lop \ge 0;\ \omega c = x^2 + y^2 - 0.5^2 \ge 0;\ \omega t = \left(\omega_2 \wedge_0 (10 + z)(5.8 - z)\right) \wedge_0 \omega c \ge 0 \text{ (Fig. 4).}$$

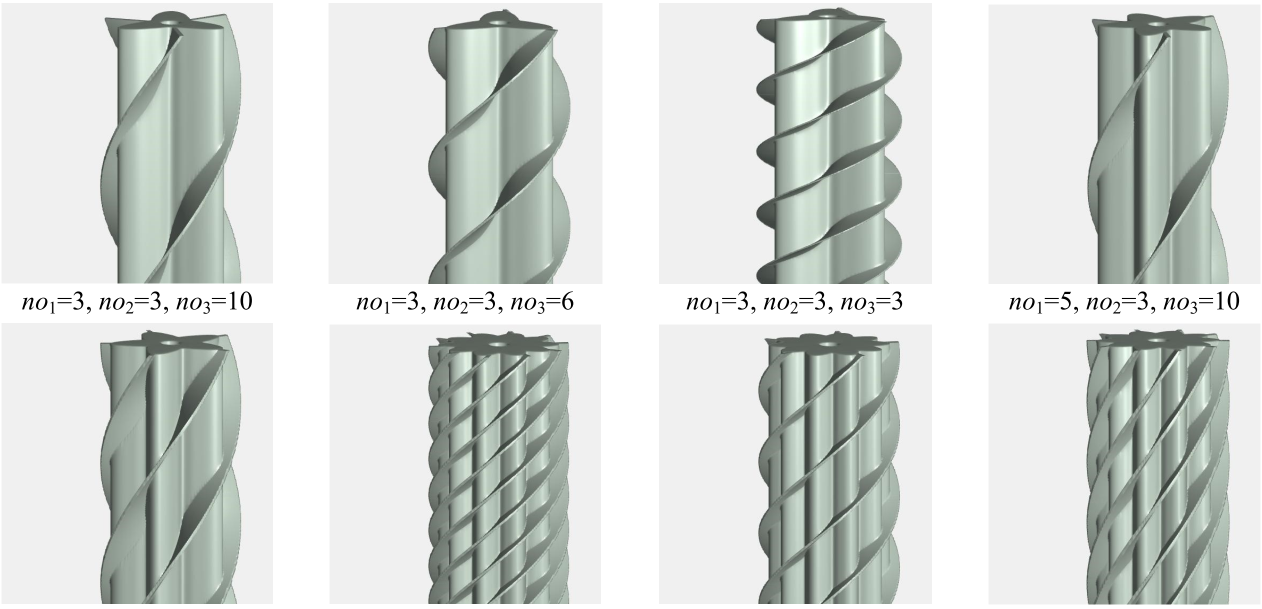

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Fig. 4. Stacked rectilinear fuel elements with helical plates

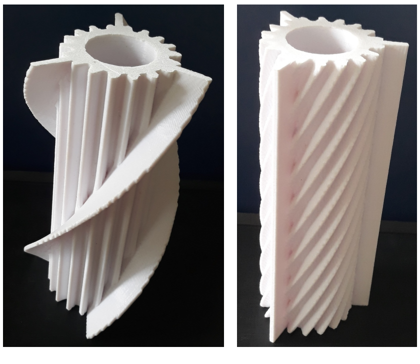

Fig. 5. Fuel elements models made on a 3D printer

Figures 1–4 show the visualization of constructed mathematical models of fuel elements with finning and plates in the RFPreview system [8], and Fig. 5 shows fuel elements models made on a 3D printer.

Conclusions

The creation of mathematical models for the implementation of 3D printing of fuel elements of non-standard shape is of considerable interest, which is connected with the active implementation of 3D printing in various industries. The advantages of using 3D printing are obvious: production of non-standard models, reduction of time for creation of new prototypes, simplicity and low cost of production, use of modern ultra-strong materials. In this paper, the R-function theory is used for mathematical and computer modeling of non-standard fuel elements in the implementation of 3D printing technology. The sequential construction of the equations of the longitudinal, polyzonal and chevron finning of the fuel elements shells allows to track the process of algorithm complication due to the corresponding superpositions in the main equation, the form of which does not change. This is one of the examples of the advantage of analytical identification of geometric objects implemented using R-functions. The reliability of the obtained results, their adequacy to the designed objects are confirmed by visualization both under the operating conditions of the RFPreview program and by implementation on a 3D printer. Analytical notation of designed objects allows to use literal geometric parameters, complex superpositions of functions, which, in turn, allows to quickly change their structural elements.

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Математичне й комп’ютерне моделювання форм багатозонних ТВЕЛів із пластинами

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У зв’язку з суттєвим зростанням кількості атомних станцій, а також моделей і модифікацій ядерних реакторів, важливості набуває з’ясування/встановлення переваг тих або інших установок. У той же час перед конструкторами постає низка питань, для яких оптимальні рішення все ще не знайдено. На атомних станціях іде найбільший оберт фінансових коштів і найменший виграш в економічності приносить величезні прибутки, однак не можна забувати про надійність і витрати при будівництві установки. Це складна комплексна задача, яка вирішується на стадії проєктування. Розрахунки реактора на стадії проєктування дозволяють визначити основні параметри активної зони, значення температури та ін. Теплогідравлічний розрахунок активної зони реактора є одним з наріжних каменів в обґрунтуванні безпечної експлуатації АЕС. Розрахунки параметрів теплоносія й температур тепловиділяючих елементів проводяться на всіх стадіях проєктування й доведення безпеки ядерних енергетичних установок. Для збільшення ефективного коефіцієнта теплопередачі в техніці широко використовуються скручені труби й оребрені теплопередаючі поверхні. Зокрема, для оребрення оболонок ТВЕЛів ядерних реакторів і зовнішніх поверхонь труб парогенераторів застосовують поздовжні, поперечні, спіральні ребра. Оребрення не тільки збільшує поверхню теплообміну з того боку, де коефіцієнт тепловіддачі має низьке значення, а й помітно впливає на гідродинаміку потоку, а тим самим і на цей коефіцієнт. Очевидно, що чим краще перемішується середовище в основному потоці й у міжреберних зазорах, тим вищий коефіцієнт тепловіддачі. Найвигіднішими формами оребрення оболонок ТВЕЛів є шевронне й полізональне оребрення, які виконуються у вигляді багатозаходної спіралі з великим кроком. Теорія R-функцій виявилася достатньо зручною для побудови математичних моделей оребрених оболонок ТВЕЛів із прямими й гвинтовими пластинами, а також для побудови *на 3D-принтері відповідних об'єктів. Із практичної точки зору актуальність задачі також визначається суттєвим поширенням скручених циліндричних тіл, скручених каналів, змійовиків в енергетиці, хімічній, нафтовій, газовій, металургійній галузях промисловості й у теплотехнічному устаткуванні. Потоки, які виникають при цьому, дають можливість інтенсифікувати процеси тепломасообміну й досягти економії енергетичних ресурсів.*

***Ключові слова**: оболонки ТВЕЛів, ТВЕЛи із шевронним і полізональним оребренням, теорія R-функцій, 3D-принтер.*

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