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## AEROELASTIC CHARACTERISTICS OF ROTOR BLADES OF LAST STAGE OF A POWERFUL STEAM TURBINE

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*Blades of powerful steam turbines are subjected to significant unsteady loads, which, in some cases, can lead to the appearance of self-excited oscillations or auto-oscillations. These fluctuations are extremely dangerous and negatively affect the life time of the blading. When developing new or upgrading existing turbine stages, it is necessary to carry out research on the aeroelastic behavior of the rotor blades. As a result of the modernization of a low-pressure cylinder of a 1000 MW steam turbine, the length of the rotor blades of the last stage increased to 1650 mm. In this regard, a numerical analysis of the aeroelastic characteristics of the last-stage rotor blades in the nominal operation mode was carried out. The analysis used the method of solving the coupled problem of unsteady aerodynamics and elastic blade vibrations, which allows the prediction of the amplitude-frequency spectrum of unsteady loads and blade vibrations in a viscous gas flow. The paper presents the results of numerical analysis of aeroelastic characteristics of the last stage rotor blades both for the mode of controlled harmonic oscillations with a given amplitude and inter-blade phase shift, and for the mode of coupled oscillations of the blades under influence of unsteady aerodynamic forces. The results of the simulation of coupled oscillations of blades for the first five natural forms are presented in the form of the time distribution of displacement of the blade peripheral cross-section, as well as the time distribution of forces and moments acting on the peripheral cross-section. The corresponding amplitude-frequency spectra of displacements and loads in the peripheral section are also given. The results of the calculations showed a positive damping of oscillations, the absence of flutter and auto-oscillations for the first five natural forms of oscillations of the blades in the nominal operation mode of the steam turbine.*

**Keywords:** aeroelasticity, flutter, steam turbine, modal method, CFD.

### Introduction

The blade systems of turbomachines are subjected to significant non-stationary aerodynamic loads during operation even in nominal modes. These loads are caused not only by the circular unevenness of the flow in the blade channels but also by the mechanical vibrations of the blades themselves. In some cases, this can lead to self-excited oscillations, even if the fluctuation frequencies of the blade and the flow do not coincide. This phenomenon of aeroelasticity is dangerous and can cause blade damage, especially in the last stages of steam turbines [1–4].

The study of aeroelastic phenomena is also necessary for other blade machines, in particular, wind turbines [5], fans [6–10], compressors [11–14], etc. The method of modeling aeroelastic phenomena in turbomachines consists in solving the problem of the interaction of two physical media (liquid and elastic). One of the low-cost and fast methods is solving the problem in the frequency domain [15–17]. This approach uses the linearization of the equations of unsteady fluid motion and is effective only for small fluctuations in simple flows. To obtain a more complete picture of the interaction between the fluid and the structure, it is necessary to simultaneously solve the Navier-Stokes equations and equations of blade movement with an exchange of data at the contact surfaces [18–20].

As a result of a review of the current state of the aeroelasticity problem in turbomachines and the existing methods of flutter prediction, it was found that the most promising method of studying the aeroelastic behavior of turbomachine blades is the simultaneous modeling of three-dimensional unsteady aerodynamics and modeling of blade motion by the modal method (coupled aeroelastic problem) [21–22]. This method of solving the coupled problem of unsteady aerodynamics and elastic blade oscillations allows to obtain the amplitude-frequency spectrum of blade oscillations in a three-dimensional gas flow, both for controlled oscillations and for self-excited oscillations, as well as to identify the conditions for the occurrence of uncontrolled blade oscillations in order to increase the reliability of blade rows of turbomachines.

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The paper uses a mathematical model of a viscous gas flow described by a system of unsteady Reynolds averaged Navier-Stokes equations, supplemented by a Baldwin-Lomax turbulence model [23]. The unsteady motion of the blades is described by a system of differential equations, in which the modal approach is used.

**Problem formulation**

Using the developed numerical method, an analysis of the aeroelastic behavior of the rotor blade row of the low-pressure last stage of a 1000 MW steam turbine was carried out. The rotor blades were improved using the modern method [24], resulting in increasing the blade length to 1650 mm.

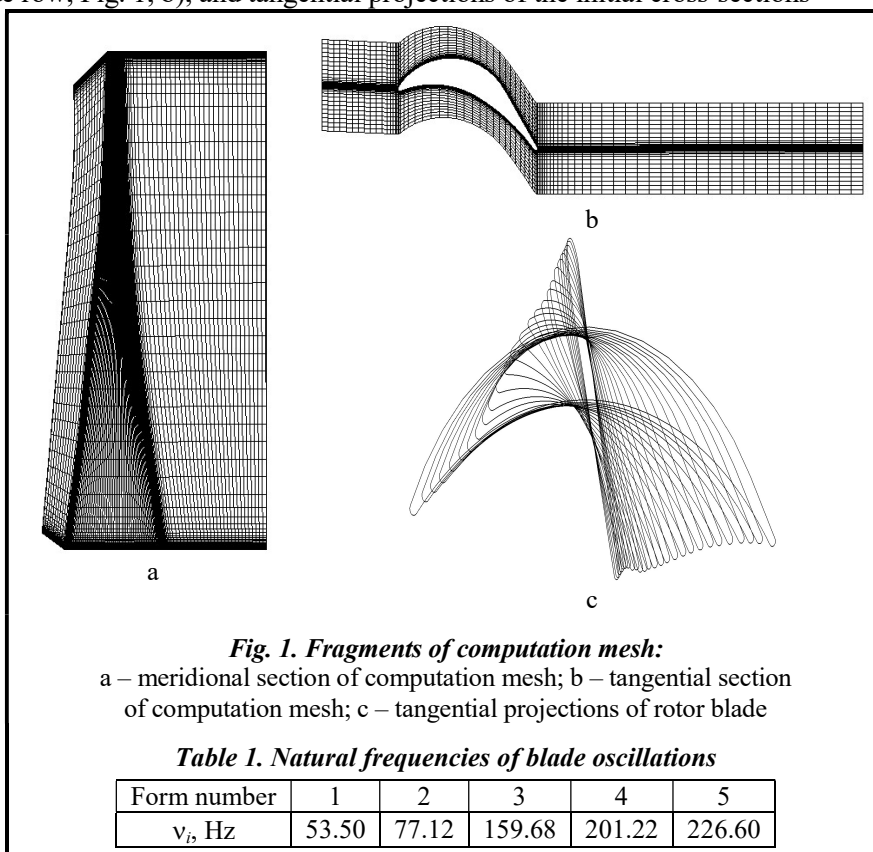
The paper uses a numerical method [25] that performs, sequentially at each iteration, the integration of viscous gas flow equations (Reynolds averaged Navier-Stokes equations) and blade vibration equations under the influence of instantaneous unsteady loads (modal approach). Calculation studies were carried out for the mode of operation of the turbine blade row with a rotation frequency of  $n=1500$  rpm. The analysis of aeroelastic characteristics of the blade row in the spatial steam flow at given harmonic and coupled blades oscillations, taking into account their natural forms and different angles of phase shift of the blade oscillations, is used as a criterion for the flutter stability of the blades.

Fig. 1 presents fragments of the computation mesh in the meridional plane (Fig. 1, a), in the tangential plane (root cross-section of the blade row, Fig. 1, b), and tangential projections of the initial cross-sections forming the blade (Fig. 1, c). Each of the segments of the computational domain is discretized using a hybrid deformed H-O mesh.

The operation mode of the turbine blade row is characterized by the following distribution of gas-dynamic parameters:

- total pressure and temperature at the inlet to the row, which varies by radius  $P_0=18843-20280$  Pa;  $T_0=331-333$  K;
- flow angles in circular ( $\alpha$ ) and radial ( $\gamma$ ) directions;
- variable by radius static pressure behind the row  $P_2=3824-3826$  Pa.

Oscillations of the rotor blades were determined by taking into account the first five natural forms of oscillations. The values of the natural frequencies of blade oscillations are given in Table 1.



**Results of numerical analysis**

On the first step, aeroelastic calculations of the turbine blade row were performed with the given law of blade oscillation. The blades carry out harmonic oscillations in each of their own forms according to the same law with a constant inter-blade phase angle (IBPA)  $\delta=0^\circ; 180^\circ; \pm 90^\circ$ , taking into account the interaction of the first five natural forms.

Fig. 2 presents the distribution by blade height of the dimensionless aerodamping coefficient  $D$  for different IBPAs. With harmonic oscillations of the blades according to a given law, taking into account the interaction of five natural forms of oscillations, "positive aerodamping" takes place, i.e., energy is withdrawn from the oscillating blade into the main flow.

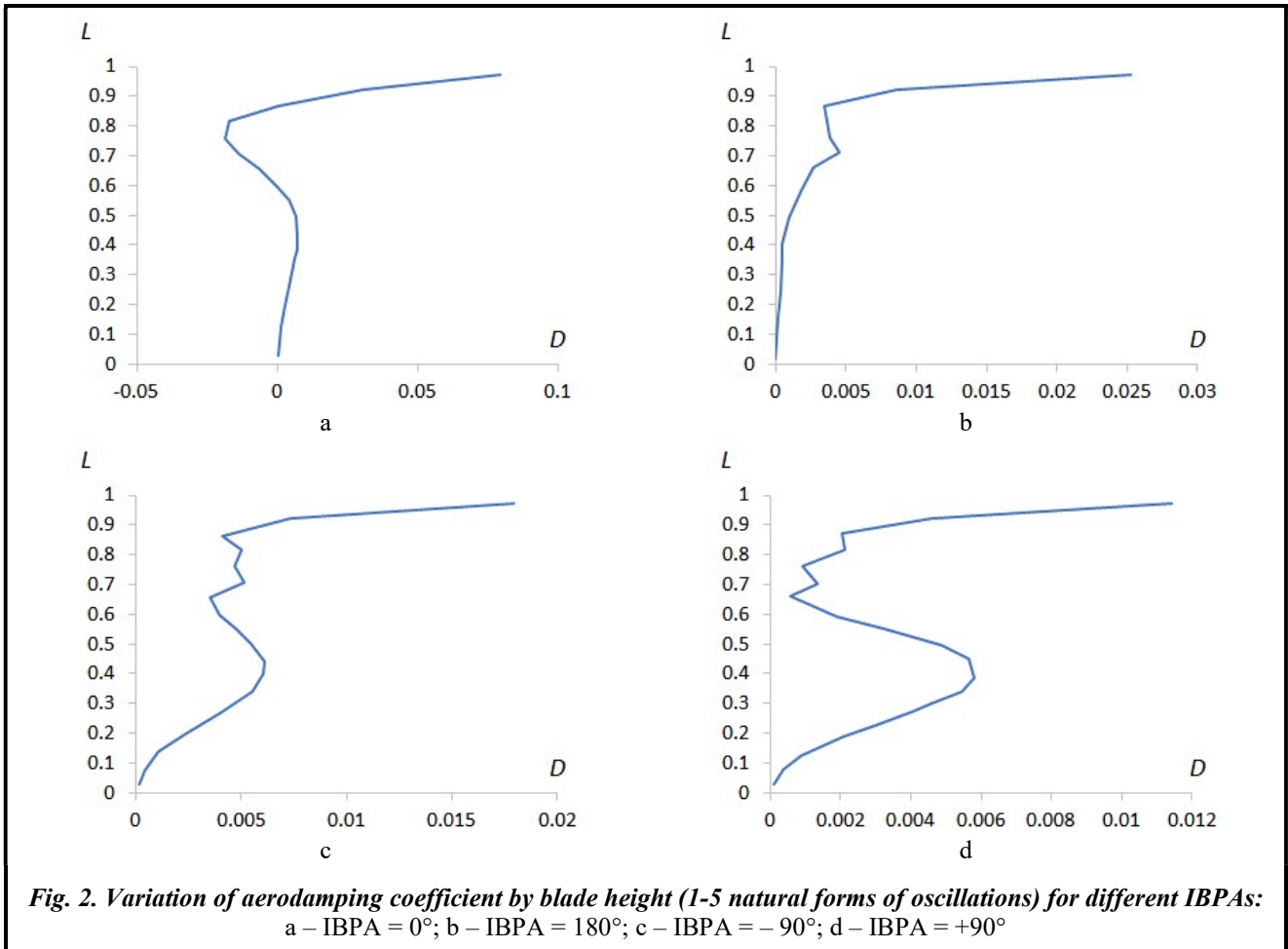
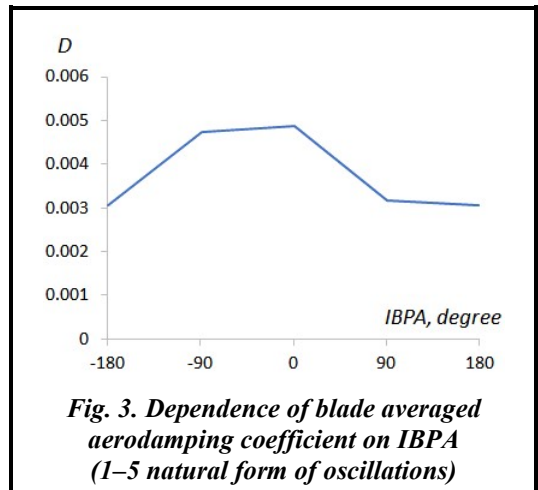


Fig. 3 shows the dependence of the aerodamping coefficient  $D$  on the IBPA. The maximum values of the aerodamping coefficient correspond to  $IBPA=0^\circ$  and  $-90^\circ$ , and the minimum values (the greatest excitation) correspond to  $IBPA=180^\circ$  and  $90^\circ$ .

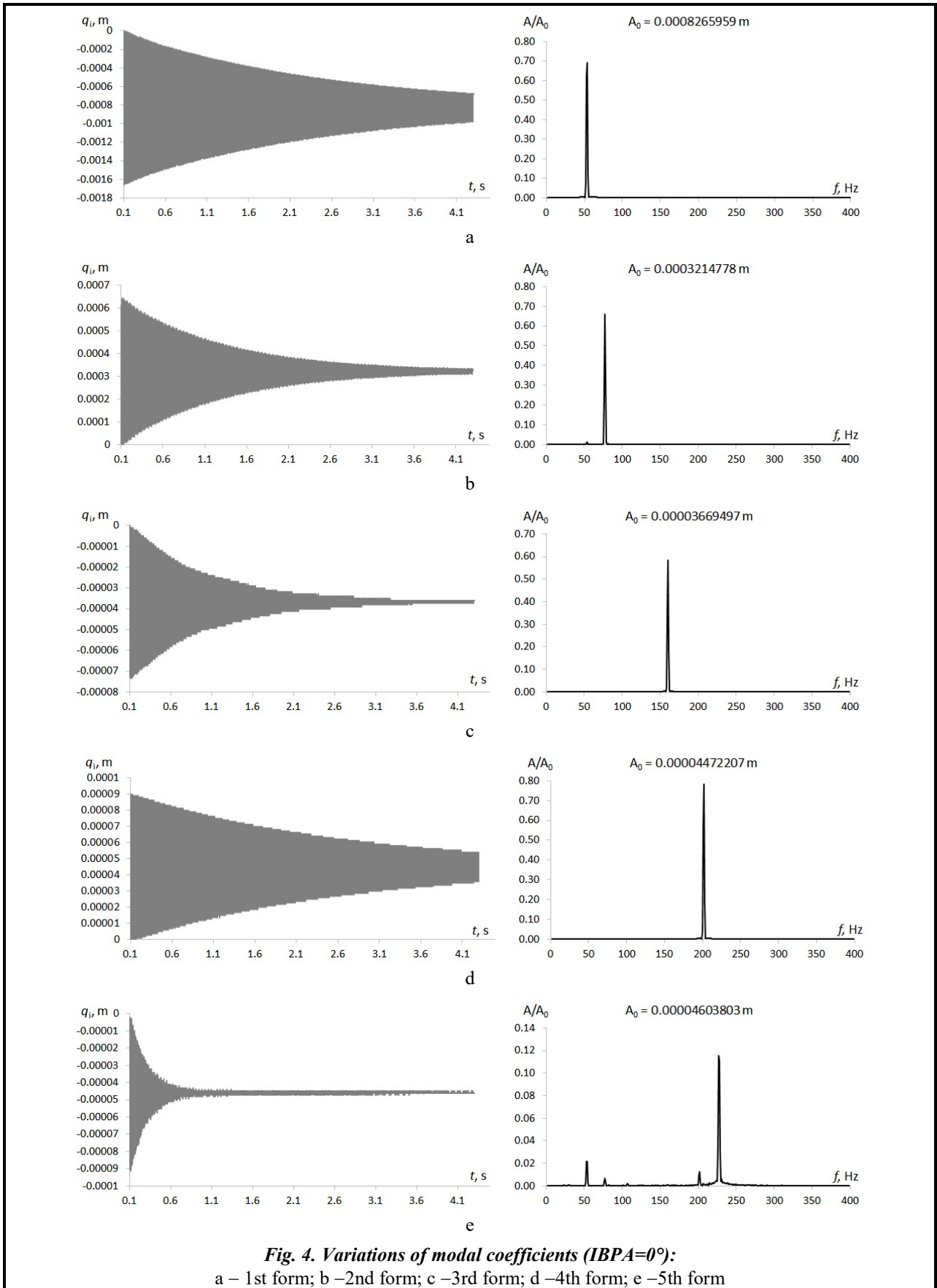
On the second step, modeling of coupled oscillations of the rotor blades was performed, which takes into account the influence of aerodynamic forces on the characteristics of oscillations and vice versa. The simulation was carried out for a period of 4 s with the amplitudes and inter-blade phase angles set at the start for the first five natural forms.

Fig. 4 shows the variations in the modal coefficients characterizing the oscillations of the blades according to five natural forms and their amplitude-frequency spectra.



Oscillations of the blades according to the five natural forms are decaying with frequencies close to the natural frequencies of the oscillations. The intensity of damping of oscillations increases with the increase in the number of the natural form of oscillations. There are no high-frequency harmonics in the vibration spectra of the first five natural forms.

In Fig. 5 are shown the graphs of the peripheral cross-section oscillations of the blade in the circular, axial direction and rotation relative to the center of gravity for  $IBPA=0^\circ$ , taking into account the interaction of five natural forms of oscillations.



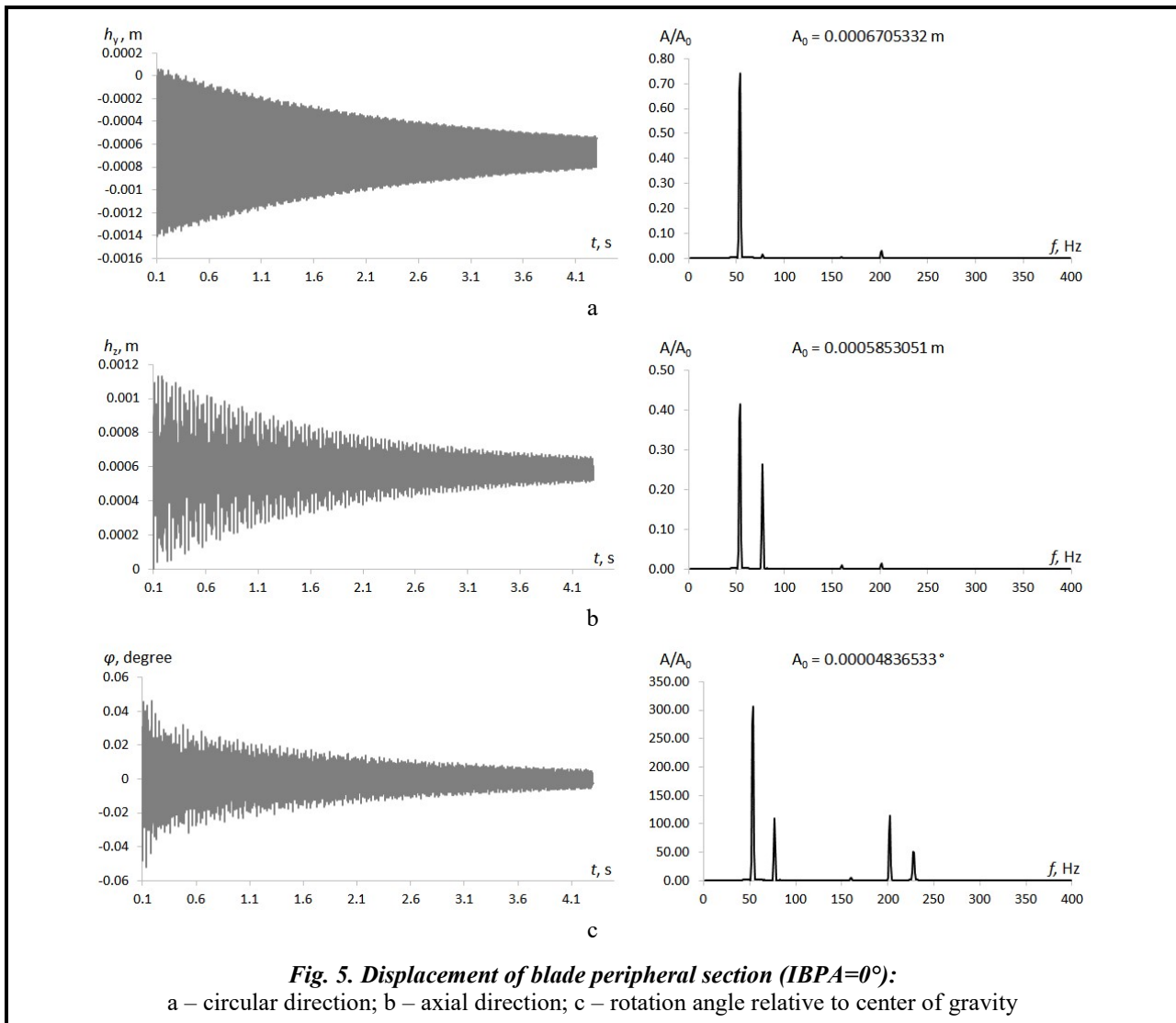
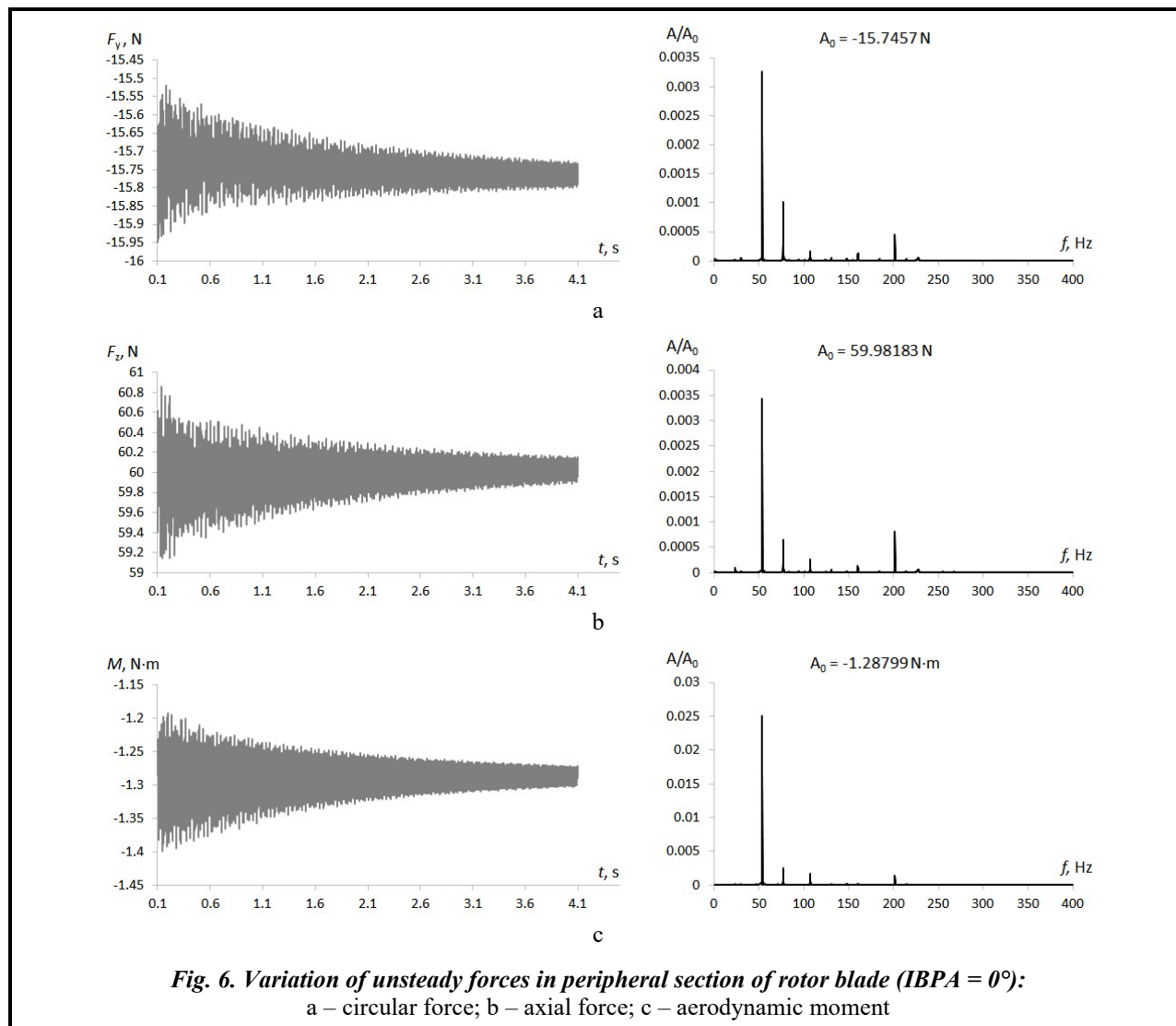


Fig. 6 shows the graphs of unsteady aerodynamic loads (circular force, axial force, and aerodynamic moment) acting in the peripheral section, as well as their amplitude-frequency spectra for  $IBPA = 0^\circ$  (the angle corresponds to the largest damping).

The presented graphs of the dependence of loads on time demonstrate a decrease in the amplitude of load oscillations during coupled simulation, simultaneously with a decrease in the amplitude of oscillations of the blade. The load frequencies are close to the natural frequencies and correspond to the vibration frequencies of the blades, the first natural form has the largest contribution to the oscillations.

The largest contribution to the unsteady components of oscillations in the circular direction is made by a frequency close to the frequency of the 1st natural form (54 Hz), in the axial direction – frequencies close to the frequencies of the 1st and 2nd natural forms (54 and 77 Hz), in torsion oscillations – frequencies close to the frequencies of the 4th and 5th natural forms (201 and 227 Hz). Movements of the blade in all directions are damped during coupled oscillations.



## Conclusions

A numerical analysis of the aeroelastic characteristics of the rotor blade row of the last stage of a powerful 1000 W steam turbine with increased blade length was carried out. The need for analysis arose after the modernization of the stage rotor, as a result of which the length of the blades increased to 1650 m. In the analysis, a numerical method is used, which allows the prediction of the aerodynamic and amplitude-frequency spectra of aerodynamic loads and blade vibrations in a viscous steam flow, including controlled vibrations, self-excited vibrations, or auto-oscillations. Numerical modeling was performed for controlled and coupled oscillations of blades in a flow of viscous steam. The results of the calculations confirmed the damping of blade oscillations for the first five natural forms, which ensures a stable mode of operation of the rotor blades of the last stage of the turbine at the nominal mode.

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## Аеропружні властивості робочих лопаток останнього ступеня потужної парової турбіни

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*Лопатки потужних парових турбін зазнають значних нестационарних навантажень, які, у деяких випадках, можуть призвести до появи самозбудних коливань або автоколивань. Ці коливання вкрай небезпечні та негативно впливають на ресурс лопаткового апарату. При розробці нових чи модернізації існуючих ступенів турбіни необхідно виконувати дослідження аеропружної поведінки робочих лопаток. В результаті модернізації циліндру низького тиску парової турбіни 1000 МВт довжина робочих лопаток останнього ступеня збільшилась до 1650 мм. У зв'язку з цим було проведено чисельний аналіз аеропружних характеристик робочих лопаток останнього ступеня у номінальному режимі роботи. При аналізі використовувався метод розв'язання зв'язаної задачі нестационарної аеродинаміки та пружних коливань лопаток, який дозволяє прогнозувати амплітудно-частотний спектр нестационарних навантажень і коливань лопаток в потоці в'язкого газу. У роботі представлено результати чисельного аналізу аеропружних характеристик лопаткового вінця ротора останнього ступеня як для режиму вимушених гармонійних коливань з заданою амплітудою та міжлопатковим зсувом фаз, так і для режиму зв'язаних коливань лопаток під дією нестационарних аеродинамічних сил. Результати моделювання зв'язаних коливань лопаток для п'яти перших власних форм представлено у формі розподілу за часом переміщення периферійного перетину лопатки, а також сил та моментів, що діють на периферійній перетині. Наведено також відповідні амплітудно-частотні спектри переміщень та навантажень у периферійному перетині. Результати розрахунків показали позитивне демпфування коливань, відсутність флатеру та автоколивань на перших п'яти власних формах коливань лопатки у номінальному режимі роботи парової турбіни.*

**Ключові слова:** аеропружність, флатер, парова турбіна, модальний метод, чисельна аеродинаміка.

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