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BENDING OF PLATES WITH COMPLEX SHAPE MADE FROM MATERIALS THAT DIFFERENTLY RESIST TO TENSION AND COMPRESSION

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A new numerical-analytical method for solving physically nonlinear bending problems of thin plates with complex shape made from materials that differently resist to tension and compression is developed. The uninterrupted parameter continuation method is used to formulate and linearize the problem of physically nonlinear bending. For the linearized problem, a functional in the Lagrange form, given on the kinematically possible displacement rates, is constructed. The main unknown problems (displacements, strains, stresses) were found from the solution of the initial problem, which was solved by the Runge-Kutta-Merson method with automatic step selection, by the parameter related to the load. The initial conditions are found from the solution of the problem of linear elastic deformation. The right-hand sides of the differential equations at fixed values of the load parameter corresponding to the Runge-Kutta-Merson scheme are found from the solution of the variational problem for the functional in the Lagrange form. Variational problems are solved using the Ritz method in combination with the R-function method, which allows to submit an approximate solution in the form of a formula – a solution structure that exactly satisfies the boundary conditions and is invariant with respect to the shape of the domain where the approximate solution is sought. The test problem for the nonlinear elastic bending of a square hinged plate is solved. Satisfactory agreement with the three-dimensional solution is obtained. The bending problem of the plate of complex shape with combined fixation conditions is solved. The influence of the geometric shape and fixation conditions on the stress-strain state is studied. It is shown that failure to take into account the different behavior of the material under tensile and compression can lead to significant errors in the calculations of the stress-strain state parameters.

Keywords: *thin plate, physically nonlinear bending, complex shape, R-function method.*

Introduction

Experimental studies of the mechanical properties of many materials (gray cast iron, light alloys, polymers, composites, etc.) testify to their unequal tensile and compression resistance beyond linear elasticity [1–4]. The deformation diagrams of such materials are nonlinear and contain a small initial linear section where the Young's moduli during tensile and compression are approximately the same. At a higher load, the nonlinear nature of deformation, in which the diagrams of deformation during tensile and compression differ significantly, is manifested. The different resistance of materials to tension and compression is one of the effects of phenomenon the material deformation characteristics dependence on the kind of loading, which was first systematized and analyzed in detail in [4].

Theories and methods of the stress-strain state of plates calculation in a physically nonlinear setting are widely presented in the literature and continue developing. A fairly complete overview of the current state of the problem of solving physically nonlinear problems of the theory of plates and shells made of materials with characteristics that depend on the type of loading is presented in [4, 5]. Most often, in research, plates and shells of canonical geometric shape are considered. For example, in papers [6–9], nonlinear-elastic and elastic-plastic deformation of rectangular plates, thick-walled cylinders, cylindrical and conical shells made from materials that differently resist to tension and compression were studied for the first time. In this case, the discrete orthogonalization methods of S. K. Godunov, uninterrupted parameter continuation, iterative methods and the Runge-Kutta-Merson method for the integration of initial problems were used to solve physically nonlinear problems.

The analysis of the available literature showed that currently there are no papers devoted to the study of physically nonlinear deformation of plates with complex shape made from materials that differently resist to tension and compression.

In the case of plates with a complex geometric shape, there is a need to use universal calculation methods, such as the finite elements method [10], the R-function method [11, 12], the "immersion" method [13], etc.

The purpose of the paper is to develop a numerical-analytical method for solving the problems of physically nonlinear bending of thin plates with complex shape made of materials that differently resist to tension and compression, based on the use of the R-function method.

Formulation of the problem. Solution method

An isotropic thin plate with a thickness h and with an arbitrary shape Ω is considered in a rectangular Cartesian coordinate system $O x_1 x_2 z$. The plate is under the action of a transverse load of $q=q(x_1, x_2)$ intensity.

It is assumed that the strain tensor components ε_{kl} consist of "linear" e_{kl}, which are subject to Hooke's law, and "nonlinear" η_{kl} (elastic or plastic) components, i.e.

$$\varepsilon_{kl} = e_{kl} + \eta_{kl} . \qquad (1)$$

Kirchhoff's hypotheses are assumed to be correct. Then the deformations in the plate are related to the displacements by relations

$$\varepsilon_{11} = u_{1,1} - zw_{,11}, \; \varepsilon_{22} = u_{2,2} - zw_{,22}, \quad \gamma_{12} = 2\varepsilon_{12} = u_{1,2} + u_{2,1} - 2zw_{,12}, \; \varepsilon_{i3} = 0, \left(i = 1,2,3\right), \qquad (2)$$

where $u_1(x_1, x_2)$, $u_2(x_1, x_2)$, $w(x_1, x_2)$ are displacement of the points of the coordinate surface along the axes Ox_1, Ox_2, Oz respectively.

Stress and "linear" deformations e_{kl} are related by Hooke's law

$$\sigma_{11} = \frac{E}{1-\nu^2}\left(e_{11} + \nu e_{22}\right), \; \sigma_{22} = \frac{E}{1-\nu^2}\left(e_{22} + \nu e_{11}\right), \; \sigma_{12} = 2Ge_{12}, \qquad (3)$$

where E, ν are Young's modulus and Poisson's ratio of the plate material; $G = \frac{E}{2(1+\nu)}$ is the shear modulus.

To formulate and linearize the problem of physically nonlinear bending of plates, the method of uninterrupted parameter continuation of the solution [14] will be used. A parameter $t \in \left[t_0, t_*\right]$, associated with an external load, is introduced into consideration. Initial value $t=t_0$ is set in such a way that it ensures the deformation of the plate material within the limits of linear elasticity, and t_* corresponds to the given load level $q(t_*)=q^*$.

The derivative with respect to the parameter t is marked with a dot above the symbol. Further, according to the text of the paper, derivatives by parameter t will be called rates.

For the variational statement of the problem, we will use a functional in the Lagrange form, given on the kinematically possible displacement rates, which for a three-dimensional body has the form [15]

$$L(\dot{\text{v}}_i) = 0.5\iiint\limits_V \dot{\sigma}_{kl}\dot{e}_{kl}dV - \iint\limits_{S_p} \dot{P}_i\dot{v}_i dS . \qquad (4)$$

In our case, the functional (4) will be written by the following formula

$$L(\dot{u}_1, \dot{u}_2, \dot{w}) = 0,5\iint\limits_\Omega \int\limits_{(h)} \left(\dot{\sigma}_{11}\left(\dot{\varepsilon}_{11} - \dot{\eta}_{11}\right) + \dot{\sigma}_{22}\left(\dot{\varepsilon}_{22} - \dot{\eta}_{22}\right) + \dot{\sigma}_{12}\left(\dot{\gamma}_{12} - 2\dot{\eta}_{12}\right)\right)dx_1dx_2dz - \iint\limits_\Omega \dot{q}\dot{w}dx_1dx_2 , \qquad (5)$$

where the rates of the "nonlinear" components $\dot{\eta}_{11}, \dot{\eta}_{22}, \dot{\eta}_{12}$ are considered given for each fixed value of the parameter and do not vary.

After differentiating relations (1)–(3) by parameter t and substituting in (5), we obtain a functional in the Lagrange form for the bending problem of a thin plate

$$L = 0,5\iint\limits_\Omega \Big[A_1\left(\dot{u}_{1,1}^2 + \dot{u}_{2,2}^2\right) + A_2\dot{u}_{1,1}\dot{u}_{2,2} + A_3\left(\dot{u}_{1,2} + \dot{u}_{2,1}\right)^2 - 2B_1\left(\dot{u}_{1,1}\dot{w}_{,11} + \dot{u}_{2,2}\dot{w}_{,22}\right) - 2B_2\left(\dot{u}_{1,1}\dot{w}_{,22} + \dot{u}_{2,2}\dot{w}_{,11}\right) - 2B_3\dot{w}_{,12}\left(\dot{u}_{1,2} + \dot{u}_{2,1}\right)$$

$$+ D_1\left(\dot{w}_{,11}^2 + \dot{w}_{,22}^2\right) + 2D_2\dot{w}_{,11}\,\dot{w}_{,22} + D_3\dot{w}_{,12}^2 \;\Big]dx_1dx_2 - \iint\limits_\Omega \left(\dot{N}_{11}^f\dot{u}_{1,1} + \dot{N}_{22}^f\dot{u}_{2,2} + \dot{N}_{12}^f\left(\dot{u}_{1,2} + \dot{u}_{2,1}\right)\right)dx_1dx_2 +$$

$$+ \iint\limits_\Omega \left(\dot{M}_{11}^f\dot{w}_{,11} + \dot{M}_{22}^f\dot{w}_{,22} + 2\dot{M}_{12}^f\dot{w}_{,12}\right)dx_1dx_2 - \iint\limits_\Omega \dot{q}\dot{w}dx_1dx_2 , \qquad (6)$$

where

$$A_1=\int_{(h)}\frac{E}{1-\nu^2}dz,\ A_2=\nu A_1,\ A_3=\int_{(h)}Gdz,\ B_1=\int_{(h)}\frac{Ez}{1-\nu^2}dz,\ B_2=\nu B_1,\ B_3=2\int_{(h)}Gzdz,$$

$$D_1=\int_{(h)}\frac{Ez^2}{1-\nu^2}dz,\ D_2=\nu D_1,\ D_3=4\int_{(h)}Gz^2dz; \tag{7}$$

$$\dot N_{11}^f=\int_{(h)}\frac{E}{1-\nu^2}(\dot\eta_{11}+\nu\dot\eta_{22})dz,\ \dot N_{22}^f=\int_{(h)}\frac{E}{1-\nu^2}(\dot\eta_{22}+\nu\dot\eta_{11})dz,\ \dot N_{12}^f=2\int_{(h)}G\dot\eta_{12}dz,$$

$$\dot M_{11}^f=\int_{(h)}\frac{Ez}{1-\nu^2}(\dot\eta_{11}+\nu\dot\eta_{22})dz,\ \dot M_{22}^f=\int_{(h)}\frac{Ez}{1-\nu^2}(\dot\eta_{22}+\nu\dot\eta_{11})dz,\ \dot M_{12}^f=2\int_{(h)}G\dot\eta_{12}zdz. \tag{8}$$

The stiffness parameters of the plate and "fictitious" forces caused by nonlinear components are calculated by formulas (7), (8).

Solution of the variational equation $\delta L=0$ gives the distribution of the displacement rates for fixed values of the parameter $t>t_0$ at any point on the plate. The main unknown problems of nonlinear plate bending can be found by integrating the corresponding rates from the solution of the Cauchy problem by the load parameter for a system of differential equations

$$\frac{du_1}{dt}=\dot u_1,\ \frac{du_2}{dt}=\dot u_2,\ \frac{dw}{dt}=\dot w,$$

$$\frac{d\varepsilon_{11}}{dt}=\dot u_{1,1}-z\dot w_{,11},\ \frac{d\varepsilon_{22}}{dt}=\dot u_{2,2}-z\dot w_{,22},\ \frac{d\gamma_{12}}{dt}=\dot u_{1,2}+\dot u_{2,1}-2z\dot w_{,12},$$

$$\frac{d\sigma_{11}}{dt}=\frac{E}{1-\nu^2}\left(\dot u_{1,1}+\nu\dot u_{2,2}-z(\dot w_{,11}+\nu\dot w_{,22})-(\dot\eta_{11}+\nu\dot\eta_{22})\right),$$

$$\frac{d\sigma_{22}}{dt}=\frac{E}{1-\nu^2}\left(\dot u_{2,2}+\nu\dot u_{1,1}-z(\dot w_{,22}+\nu\dot w_{,11})-(\dot\eta_{22}+\nu\dot\eta_{11})\right),$$

$$\frac{d\sigma_{12}}{dt}=G\left(\dot u_{1,2}+\dot u_{2,1}-2z\dot w_{,12}-2\dot\eta_{12}\right),$$

$$\frac{d\eta_{11}}{dt}=\dot\eta_{11},\ \frac{d\eta_{22}}{dt}=\dot\eta_{22},\ \frac{d\eta_{12}}{dt}=\dot\eta_{12}. \tag{9}$$

The nonlinearity of the system (9) is due to the nonlinearity of the constitutive equations for $\dot\eta_{kl}$ $(k,l=1,2)$, which will be specified below. The initial conditions for the required functions are found from the solution of the linear deformation problem at $q(t_0)=q_0$. For this, it is possible to use a functional of the form (6), in which the rates of the functions must be replaced by the functions themselves and it should be assumed that the "fictitious" forces are $\dot N_{kl}^f=0$, $\dot M_{kl}^f=0$ (k, l=1, 2).

We will solve the initial problem for the system of equations (9) using the Runge-Kutta-Merson (RKM) method with automatic step selection [16]. To calculate the right-hand sides of equations (9) at fixed values $t>t_0$, corresponding to the RKM scheme, it is necessary to solve the variational problems for the functional (6) for five times at each step. Variational problems were solved using the Ritz method in combination with the R-function method [11].

Numerical results

As a test example, nonlinear elastic bending of a square ($2a\times2a$) hinged plate made of gray cast iron СЧ 15-32 is considered. The geometric dimensions are as follows: a=0.05 m, thickness h=0.01 m.

For gray cast iron, the equality of the modulus of elasticity under tensile and compression on the initial linear sections of the deformation diagrams was experimentally established. At the same time, beyond the limits of linear elasticity, the diagrams differ significantly and depend on the type of loading [1].

Young's modulus and Poisson's ratio of the material: E=1.07×10[5] MPa, ν=0.22. Tensile strength is $\sigma_в$=150 MPa. It should be noted that under compression the strength limit for cast irons is higher than under tensile by approximately 2.5–4.5 times [17].

The plate is under the influence of a transverse load

$$q(x_1, x_2) = q_0 \cos\frac{\pi x_1}{2a} \cos\frac{\pi x_2}{2a},$$

where q_0=8.0 MPa.

A linear law for the amplitude of the external load is set as

$$q_0(t) = q_{01} + t q_{02}, \tag{10}$$

where $t \in [0, t_*]$.

To describe the nonlinear behavior of the material, tensor-linear constitutive equations that describe the dependence of the material characteristics on the type of loading [18] are used

$$\dot{\eta}_{ij} = \nu(\sigma_e)\dot{\sigma}_e\left(\frac{C\sigma_{ij} + AI_1\delta_{ij}}{\sigma_{e2}} + B\delta_{ij}\right). \tag{11}$$

Here $\sigma_e = \sigma_{e2} + \sigma_{e1}$ is a equivalent stress; $\sigma_{e1} = BI_1$, $\sigma_{e2}^2 = AI_1^2 + CI_2$; $I_1 = \mathrm{tr}(\sigma) = \delta_{ij}\sigma_{ij}$, $I_2 = \mathrm{tr}(\sigma^2) = \sigma_{ij}\sigma_{ij}$ $(i, j$=1, 2, 3) are linear and quadratic invariants of the stress tensor; A, B, C are material parameters determined from experimental data.

The simplest approximation for the function $\nu(\sigma_e)$ in (11) may be a power dependence [18]

$$\nu(\sigma_e) = n\sigma_e^{n-1}.$$

Parameters A, B, C, n for the gray cast iron СЧ 15-32 material were found from the data of basic experiments on tensile, compression and torsion, which are given in [5] (Part 1)

$$A = -1.024 \cdot 10^{-5}\ \text{MPa}^{-\frac{2n}{n+1}},\ B = 1.148 \cdot 10^{-3}\ \text{MPa}^{-\frac{n}{n+1}},\ C = 2.891 \cdot 10^{-5}\ \text{MPa}^{-\frac{2n}{n+1}},\ n=4.4.$$

If the elastic-plastic deformation is studied, then relations (11) should be supplemented with the condition of plasticity [4, 5, 18].

The kinematic boundary conditions for hinge fixation have the form

$$\dot{w} = 0,\ \dot{u}_\tau = 0, \tag{12}$$

where $\dot{u}_\tau = \dot{u}_2 n_1 - \dot{u}_1 n_2$; n_1, n_2 are direction cosines of the external normal n to the contour of the plate $\partial\Omega$.

The structure of the solution satisfying condition (12) can be written as follows:

$$\dot{w} = \omega\Phi_1,\ \dot{u}_1 = \omega_{,1}\Phi_2 + \omega\Phi_3,\ \dot{u}_2 = \omega_{,1}\Phi_2 + \omega\Phi_4,$$

where Φ_i, i=1, …, 4 are undefined components of the solution structure; function $\omega=\omega(x_1, x_2)$ is obtained using the R-function theory and satisfies the conditions [11]: ω=0, $\omega_{,n}$=-1 on the border $\partial\Omega$, ω>0 inside Ω. Requirement of function ω normalization to the first order ($\omega_{,n}$=-1) in some cases is not required.

For a square plate, the function has the form

$$\omega = \omega_1 \wedge_0 \omega_2,$$

where $\omega_1 = \frac{1}{2a}(a^2 - x_2^2)$, $\omega_2 = \frac{1}{2a}(a^2 - x_1^2)$, and symbol $\wedge_0$ denotes the R-conjunction [7]: $f_1 \wedge_0 f_2 = f_1 + f_2 - \sqrt{f_1^2 + f_2^2}$.

In the numerical implementation, the undefined components of the solution structure were given in the form of finite series $\Phi_i(x_1, x_2, t) = \sum_n C_n^{(i)}(t) f_n^{(i)}(x_1, x_2)$, where $C_n^{(i)}(t)$ are undefined coefficients, which are found at each step by the Ritz method; t is the fixed value of the load parameter; $\{f_n^{(i)}\}$ are systems of linearly independent functions. Power polynomials of the form $x_1^k x_2^l$ were used as $\{f_n^{(i)}\}$. Rates of equivalent stresses $\dot{\sigma}_e$ in the constitutive equations, at each step by t, were assumed to be constant and were calculated from the values of stresses and stress rates in the previous step.

Tables 1–3 compare the results of calculation of deflections and normal stresses obtained in a three-dimensional setting (3D) [7] and using the R-function method (RFM). The spatial solution is obtained by expanding the unknown functions into trigonometric Fourier series. Linearization was performed by movement along the load in combination with iterative refinement of the solution. Tables 1, 2 show data for deflections and stresses on the lower surface of the plate (z=h/2) in section x_2=0, and Table 3 shows stress distribution across the thickness in the center of the plate.

Table 1. Deflections w·10^4, m

x_1/a	3D	RFM
0.0	2.60	2.48
0.2	2.47	2.35
0.4	2.09	1.99
0.6	1.51	1.43
0.8	0.79	0.75

Table 2. Normal stresses σ$_{11}$, MPa

x_1/a	3D	RFM
0.0	107.0	92.2
0.2	104.0	91.0
0.4	93.7	86.6
0.6	75.7	71.7
0.8	43.3	43.4

Table 3. Normal stresses σ$_{11}$, MPa in the center of the plate

$2z/h$	3D	RFM
-1.0	-168.0	-164.9
-0.5	-77.8	-76.9
0.0	10.7	11.6
0.5	77.5	78.5
1.0	107.0	92.2

From the Table 3, it can be seen that the influence of different behavior of the material under tension and compression is manifested in a significant difference in the absolute values of stresses in the tensiled and compressed regions. At the same time, membrane stresses appear in the plate, and the neutral surface shifts toward the compressed fibers.

From the above results, it can be concluded that the method proposed in the paper provides a satisfactory match with the results of the three-dimensional solution. The maximum relative error for deflections is 5.3%, and for stresses – 13.8%.

Next, we will consider a square hinged plate and a plate with complex shape made of gray cast iron СЧ 15-32 material under the action of a uniformly distributed load q_0=6.0 MPa (Fig. 1). The rectilinear sections of the plate with complex shape contour are hinged, and the circular corner cutouts are free from fixation. Geometric dimensions: 2a=2b=0.1 m, r=0.02 m, h=0.01 m.

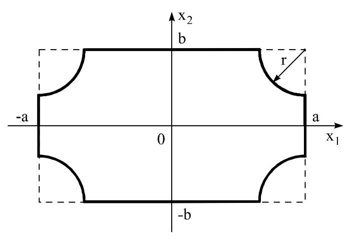

Fig. 1. Plate with circular cutouts

The area border equation shown in Fig. 1 has the form

$$\omega = (\omega_1 \wedge_0 \omega_2) \wedge_0 ((\omega_3 \wedge_0 \omega_4) \wedge_0 (\omega_5 \wedge_0 \omega_6)) = 0\,, \qquad (13)$$

where $\omega_1 = \frac{1}{2b}\left(b^2 - x_2^2\right)$, $\omega_2 = \frac{1}{2a}\left(a^2 - x_1^2\right)$, $\omega_3 = \frac{1}{2r}\left((x_1 - a)^2 + (x_2 - b)^2 - r^2\right)$,

$\omega_4 = \frac{1}{2r}\left((x_1 + a)^2 + (x_2 - b)^2 - r^2\right)$, $\omega_5 = \frac{1}{2r}\left((x_1 + a)^2 + (x_2 + b)^2 - r^2\right)$, $\omega_6 = \frac{1}{2r}\left((x_1 - a)^2 + (x_2 + b)^2 - r^2\right)$.

In this case, the structure of the solution can be written as follows:

$$\dot{w} = \omega_0 \Phi_1\,,\ \ \dot{u}_1 = \omega_1 \Phi_2\,,\ \ \dot{u}_2 = \omega_2 \Phi_3\,, \qquad (14)$$

where $\omega_0 = \omega_1 \wedge_0 \omega_2$.

It should be noted that since the structure of the solution (14) satisfies only the kinematic boundary conditions, the function ω defined by the formula (13) is not included in the structure and is used only for finding the coordinates of the nodes of the integration grid over the area Ω when calculating the Ritz matrix.

Fig. 2 shows graphs of the dependence of the deflections in the center of the plate with complex shape on the intensity of the transverse load q. Solid lines correspond to the nonlinear solution, and dashed lines – to the linear solution. Fig. 3 shows similar graphs for a square plate.

Figs. 4, 5 show the results of calculating the absolute values of stresses in the center of a plate with complex shape and in the center of a square plate. In the figures, the number 1 corresponds to the stresses on the upper surface (z=-h/2), and the number 2 – on the bottom surface (z=h/2). Dashed lines show the linear solution.

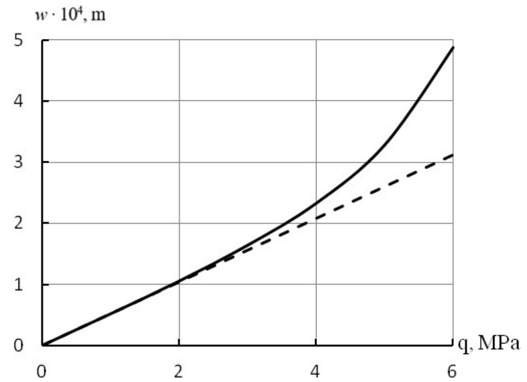

Fig. 2. Deflections in the center of the plate with complex shape

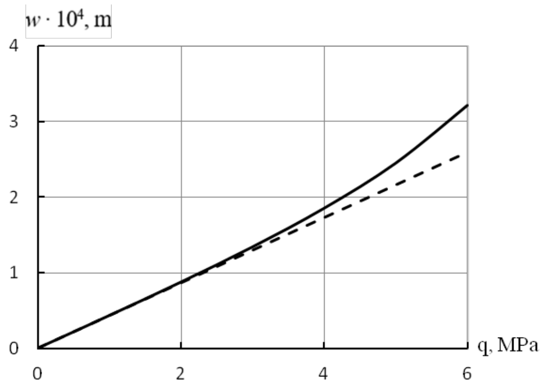

Fig. 3. Deflections in the center of the square plate

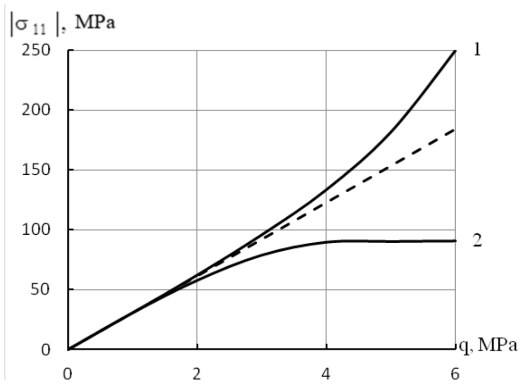

Fig. 4. Stress in the center of the plate with complex shape

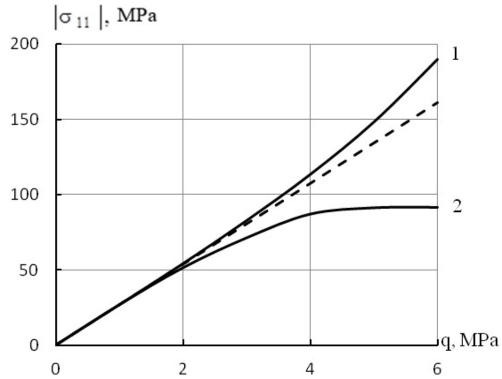

Fig. 5. Stress in the center of the square plate

In the numerical calculations, both in the test one and in this example, the load in formula (10) was taken as: q_{01}=1.0 MPa, q_{02}=10^{-1} MPa, and the initial step and the given calculation error in the RKM method were equal to: $\Delta t_0=10^{-2}$, $\varepsilon=10^{-5}$.

The graphs shown in Figs. 2–5 show that the presence of corner cutouts in the plate that are free from fixation leads to an increase in deflections and stresses, and failure to take into account the different behavior of the material under tensile and compression significantly affects the calculation results.

Conclusions

A new numerical-analytical method for solving the problems of physically nonlinear bending of plates with complex shape made from materials that differently resist to tension and compression has been developed. The method is based on the joint use of R-function method and methods of uninterrupted parameter continuation and Runge-Kutta-Merson.

The test problem was solved, a match with the spatial solution was obtained. The calculation of the nonlinear elastic bending of the plate of complex shape with combined fixation conditions was performed. The influence of the geometric shape and fixation conditions on the stress-strain state was studied. It is shown that failure to take into account the different behavior of the material under tensile and compression leads to significant errors in the results of the calculation of SSS parameters.

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Згин пластин складної форми із матеріалів, що неоднаково опираються розтягу і стиску

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У статті розроблено новий чисельно-аналітичний метод розв'язання фізично нелінійних задач згину тонких пластин складної форми із матеріалів, що неоднаково опираються розтягу і стиску. Для постановки й лінеаризації задачі фізично нелінійного згину використовувався метод неперервного продовження за параметром. Для лінеаризованої задачі побудовано функціонал у формі Лагранжа, заданий на кінематично можливих швидкостях переміщень. Основні невідомі задачі (переміщення, деформації, напруження) знаходилися із розв'язку початкової задачі, яка розв'язувалася методом Рунґе-Кутта-Мерсона з автоматичним вибором кроку, за параметром, пов'язаним із навантаженням. Початкові умови знаходилися із розв'язку задачі лінійно-пружного деформування. Праві частини диференціальних рівнянь при фіксованих значеннях параметра навантаження, що відповідають схемі Рунґе-Кутта-Мерсона, знаходилися із розв'язку варіаційної задачі для функціонала у формі Лагранжа. Варіаційні задачі розв'язувалися методом Рітца в поєднанні з методом R-функцій, який дозволяє подати наближений розв'язок у вигляді формули – структури розв'язку, яка точно задовольняє граничним умовам і є інваріантною стосовно форми області, де відшукується наближений розв'язок. Розв'язано тестову задачу для нелінійно-пружного згину квадратної шарнірно опертої пластини. Отримано задовільний збіг із тривимірним розв'язком. Розв'язано задачу згину пластини складної форми з комбінованими умовами закріплення. Досліджено вплив геометричної форми й умов закріплення на напружено-деформований стан. Показано, що неврахування різної поведінки матеріалу за розтягу і стиску може призвести до суттєвих похибок у розрахунках параметрів напружено-деформованого стану.

***Ключові слова**: тонка пластина, фізично нелінійний згин, складна форма, метод R-функцій.*

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