DOI: https://doi.org/10.15407/pmach2024.02.060

UDC 621.9

TOOLPATH GENERATION ON 3-AXES MILLING USING CONSTANT VOLUME METHOD

Van-Quy Hoang
hoangquyctm@gmail.com
ORCID: 0000-0003-4164-5975

Industrial University
of Ho Chi Minh City,
12 Nguyen Van Bao, Ward 4,
Go Vap District, Ho Chi Minh City,
Vietnam

A new simple method to generate toolpaths when machining based on the fixed base of the residual metal portion left after each longitudinal (forward step) and transverse cutting step (stepover) is presented in this paper. After each longitudinal and transverse cutting step, there will be an unprocessed metal portion, which has the form of a cone with a quadrilateral base, and the lateral edges are curves created by the intersection of the sphere with the diameter of the toolpath. The height of this cone, S_c*, is called the scallop height, and its projection on a cutting plane is called the cusp. However, the entire volume of this unprocessed metal portion was examined in the paper. From there, it was proposed to adjust the toolpath in such a way that the volume of this portion remains constant in each step of the tool, resulting in ensured machining quality on the entire surface. Unlike the previous studies where toolpath is generated using iso-scallop, iso-parametric, or iso-planar methods, a new method based on calculating the volume of uncut metal after each step of the toolpath horizontally and vertically is offered in this paper. This method is called constant volume. Compared to existing methods, this approach is superior because it makes possible to calculate the volume of remaining metal, thereby ensuring more uniform surface quality and more efficient toolpath. To ensure the correctness of the proposed method, a script used to generate a toolpath with a simple surface was implemented by Matlab2010a. The toolpath generated by the proposed method was compared with toolpath generated by the traditional method already available in CAD/CAM software. The results showed that the proposed method had good accuracy and fast toolpath generation time. This method can be extended to complex surfaces and is an option for application in CAD/CAM software as well as providing another toolpath generation solution for mechanical machining in general.*

Keywords*: toolpath, toolpath generation, toolpath optimization, 3-axis CNC machine, constant volume.*

Introduction

Generally, the methods for generating toolpaths mainly rely on basic methods, such as iso-parametric, iso-scallop ones. These methods are often used to calculate toolpaths by fixing the distance between two successive horizontal stepovers in the toolpath. The methods for toolpaths generating have been extensively studied by many researchers. The results of these studies have been applied in commercial CAD/CAM software or presented in influential scientific publications. Subhajit Sarkar and Partha P. Dey [1, 2] introduced a new method based on iso-parametric toolpath planning for machining form surfaces called the Boundary Interpola-

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tion method. The authors have developed a modern algorithm based on the proposed method. Comparative results with the traditional finite element method demonstrate significant improvements. Chen Zhi-Tong et al [3] presented an optimization method for toolpath selection in surface machining. This method estimates the average strip width to generate the next toolpath. While this method has many advantages in surface machining quality, toolpath guidance may be fragmented in regions with varying surface characteristics, leading to increased machining time. Goran M. Mladenovic et al [4] investigated a method to maintain constant cutting force during surface machining. The results obtained thanks to the method demonstrate its effectiveness in rough surface machining, as the excess material during rough machining undergoes continuous variation, leading to significant changes in values. In contrast, during finish machining, the excess material is relatively small, resulting in insignificant changes in cutting force. In a study on toolpath generation for freeform surfaces, Guillermo H. Kumazawa et al [5] focused on optimizing the width of the machining strips. The authors analyzed the Preferred Feed Direction (PFD) field of the surface and divided it into separate regions by identifying inflection points and forming separating curves. PFD toolpath lines, similar to iso-scallop height, were then created for each region to reduce the overlapping of machining paths. The results of the study showed that the toolpath lines were shorter than the existing method. Young-Joon Kim et al [6], introduced a new approach to generating toolpaths that result in high-quality machining, the research group analyzed hyper-osculating circles (HOCs), particularly the curvature difference between tools and surfaces. The process of detecting and handling HOCs needs to be continuously performed throughout the machining operation. Taejung Kim and Sanjay E. Sarma [7], were presented to generate toolpaths based on a vector field modeled as a set of straight lines. To solve this problem, the authors proposed applying the Greedy Algorithm to find the best path for the cutting tool. The results showed that the efficiency of the toolpath was better than in previous studies. D. Walhof [8] has studied a new solution for toolpaths generating when machining freeform surfaces on a 3-axis CNC milling machine using ball-end milling cutters. This method aims to fix the scallop height during the machining process. Unlike previous studies on constant scallop height, this method does not approximate in 2D but instead on a 3D geometric object. As a result, it is possible to get a more accurate machined surface and a shorter toolpath length. N. Shokrollahi and E. Shojaei [9] compared three algorithms used in CAD/CAM software for surface machining: iso-parametric, iso-planar, and iso-scallop ones. The research group conducted experiments to evaluate the algorithms. The results showed that the iso-parametric strategy was not optimal in terms of time and surface quality; the iso-planar strategy was optimal in terms of time, but not in terms of surface quality; the iso-scallop height strategy had good surface quality but was not optimal in terms of machining time. This suggests that improving the time aspect based on the iso-scallop height method is a feasible research direction and can be a good solution for machining details on a 3-axis CNC milling machine. The vector field linearization approach is used as the basis for selecting toolpaths based on the iso-scallop height method. The surface is parameterized, then the initial toolpath is selected, and subsequent toolpaths are computed based on the fit to the initial toolpath. Subsequent toolpaths are determined by the distance from the farthest baseline path while ensuring that the scallop height remains within the limits. Aman Kukreja and S.S. Pande [8] proposed two approaches for machining free-form surfaces on a 3-axis CNC milling machine. The first approach involves directly simulating the CNC program on CAM software to generate a CAD model with the surface created by the scallop profile. The second approach involves generating the CNC program directly from CL points generated by the scallop profile. The authors' method is accurate, fast, and robust, with the scallop height being directly created from the CNC program. Van-Quy Hoang et al [10] presented an evaluation of the influence of process parameters on surface accuracy when machining on a 3-axis CNC milling machine. The results showed that the influence of stepover on machining accuracy is the most significant. V. Q. Hoang and D. T. Nguyen [11] proposed a simple solution for toolpaths generating when machining a conical surface based on the scallop height not exceeding a predetermined limit.

Materials and methods

In this study, the author proposes a new method called "Constant Metal Volume Leftover after Machining" or simply "Constant Volume" to determine the parameters of cutting toolpaths. This method is more comprehensive than other methods that only consider scallop height or cusp, because it also considers the volume of uncut metal, which, in addition to evaluating the degree of influence of the details on the surface quality, also evaluates the level of energy use, machine wear, tool wear, etc. When calculating the volume of metal left after machining, there are three constraints including the amount of vertical tool feed (F_r), the amount of horizontal tool feed stepover (S_z), and the scallop height. When one of these factors is changed, the other two will change accordingly, so that the volume remains constant across the entire machined surface. Surface accuracy will be calculated to ensure compliance with technical requirements, and parameters will be calculated to achieve the optimal values, where the total machining time or toolpath distance is smaller than other methods.

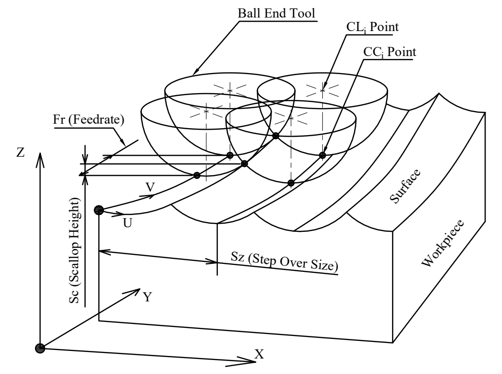


Fig. 1. Basic parameter of the toolpath

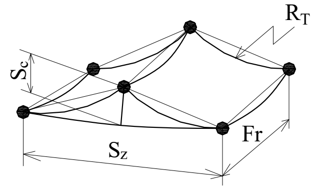


Fig. 2. Volume 1 (V_1)

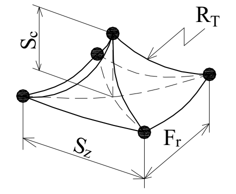


Fig. 3. Volume 2 (V_2)

To determine the toolpath, the parameters of the toolpath must be identified. In Fig. 1, the basic parameters for forming the toolpath and detail surface include: cutting tool radius R_T; stepover size (S_z); forward step size (F_r); and scallop height (S_c). After each stepover and forward step, the large amount of metal is removed, but still, the amount of metal left on the surface forms a block structure as shown in Fig. 2, which contains four parameters: S_c; R_T; S_z and F_r. This is the volume of metal that is not machined after one stepover and forward step. When the tool curvature is not considered, the volume of this block is calculated according to Eq (1)

$$V_1 = \frac{1}{2} S_c \cdot S_z \cdot F_r \,, \tag{1}$$

where V_1 is the volume of the block; S_c is scallop height; S_z is stepover size; F_r is forward step size.

However, due to the constant R_T curvature of the ball-end tool on the 3-axis CNC milling machine, the volume of this part can be accurately calculated using Eq (2)

$$V_1^* = V_1 - V_{R_T} = \frac{1}{2}\left\{ S_c \cdot S_z - \frac{R_T^2}{2}\left(\pi \cdot \frac{\theta}{180} - \sin\theta\right)\right\} \cdot F_r \,. \tag{2}$$

V_{R_T} is the volume created by the curvature of the cutting tool Eq (3), as shown in Fig. 2

$$V_{R_T} = \frac{R_T^2}{2}\left(\pi \cdot \frac{\theta}{180} - \sin\theta\right), \tag{3}$$

where θ (Fig. 4) is the angle formed by the arc CC_i, and point X is the intersection of the representative circle of the tool at two consecutive steps.

In addition, there is another approach to calculating the amount of metal left after each combination of horizontal and vertical steps. This volume is formed when two consecutive horizontal and vertical steps are combined, as shown in Fig. 3, and they are different from the volume calculated in Eq (1), which is only calculated in a single step

$$V_2 = \frac{1}{3} S_c \cdot 2S_\Delta = \frac{1}{3} S_c \cdot 2\frac{1}{2} S_z \cdot F_r = \frac{1}{3} S_c \cdot S_z \cdot F_r \,. \tag{4}$$

After comparing these two volumes (V_1^* and V_2), it becomes evident that the calculation method for V_2 is simpler, and factors such as S_c, S_t, and F_r are more explicit.When V_2 is constant, one of the three factors changes, and the remaining two factors change so that this product is always a constant. However, using this approach, some volumes will not be calculated outside the boundary, resulting in lower accuracy than the V_1^* solution, which is a constant. Therefore, to achieve higher accuracy, we use the metal volume calculated in Eq (2) as a basis for determining the values of the path parameters in this paper. Typically, during tool processing, the cutting tool is selected in advance, meaning R_T is usually fixed. Therefore, changing the remaining parameters will change the V_1^* volume. In Fig. 4, it is also easy to see that θ depends on R_T and S_z according to Eq (5)

$$\sin\theta = \frac{S_z}{2R_T}. \tag{5}$$

In Fig. 4, the relation between S_z and S_c is written in the form of Eq (6)

$$S_z = 2\sqrt{R_T^2 - (R_T - S_c)^2}\,. \tag{6}$$

Therefore, V_1^* only depends on two parameters: S_c and F_r. This means that from an initial *CC* point, three consecutive *CC* points can be determined to create a group of four points for each machining step. In practical machining, the surface accuracy is usually selected, so S_c does not exceed a predetermined value, usually the scallop height $[S_c]$ is limited. Thus, depending on the curvature of the surface at different positions, S_c can be calculated through S_z and R_T as in Eq (6). This means that at this point, V_1^* can only be a constant if the F_r value changes accordingly. Therefore, it is necessary to adjust the cutting tool along the direction of F_r value change depending on the position of the surface. The F_r value can be derived from Eq (3) as shown in Eq (7)

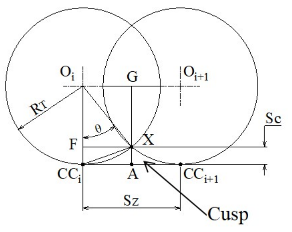


Fig. 4. The relation between S_z and S_c

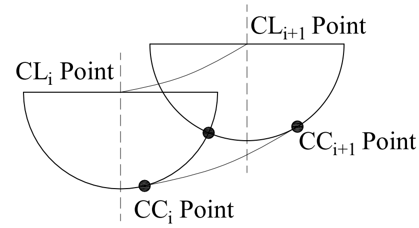


Fig. 5. Calculation of CL point from CC point

$$F_r = 2V_1^*\left\{\frac{1}{S_c \cdot S_z - \frac{R_T^2}{2}\cdot\left(\pi\cdot\frac{\theta}{180} - \sin\theta\right)}\right\}. \tag{7}$$

To create a toolpath, it is necessary to determine the CC_i point, which is positioned between consecutive points. In other words, a horizontal tool shift of S_z will perform a combination of vertical movements with a vertical displacement step of F_r. After moving vertically over the tool surface, the cutting tool will shift horizontally by one step S_z and repeat the process until the entire surface has been machined. To create the toolpath after determining the CC_i point, it is necessary to determine the CL_i point.

In Fig. 5, the positions of CC_{i+1} point, and CL_{i+1} point are generated from CC_i point and CL_i point. Therefore, to generate the toolpath, it is necessary to determine all CL_i points.

$$CL_i = CC_i + nR_T = P(u,v) + \frac{P_u x P_v}{|P_u x P_v|}R_T\,. \tag{8}$$

The coordinates of the CL_i point are X_{CLi}, Y_{CLi}, and Z_{CLi}, while the coordinates of the CC_i point are X_{CCi}, Y_{CCi}, and Z_{CCi}. The coordinate equation of the CL_i point is expressed in Eq (8) [11]. γ is the angle formed by the unit normal vector of the surface and the tool axis

$$\begin{bmatrix} X_{CLi} \\ Y_{CLi} \\ Z_{CLi} \\ 1 \end{bmatrix} = \left\{\begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right\}\begin{bmatrix} X_{CLi} \\ Y_{CLi} \\ Z_{CLi} \\ 1 \end{bmatrix}. \tag{9}$$

The algorithm for toolpath generation is shown in Table 1.

Results and discussion

To verify the accuracy of the constant volume method, the evaluation was conducted by generating a zigzag toolpath for machining the surface of a workpiece with dimensions of length×width×height=100×50×10 mm. The aim was to achieve a machining accuracy of 0.05 mm, with a horizontal tool step of 1mm, a maximum cutting depth of 2 mm, and using Ball end mill with a diameter of 5 mm. Machining simulation was performed using commercial CAD/CAM software. It was done to obtain the toolpath and machining details as shown in Fig. 6.

After testing the workpiece model with varied accuracy requirements while maintaining a constant tool diameter, in a table of statistics presented in Table 2 was obtained. The accuracy values and volume of metal are predetermined, and the algorithm will calculate the parameters of the toolpath.

In Table 2, (S_c, V) and R_T are established with predetermined accuracy and volume of the remaining metal after machining, and the parameters of the toolpath S_z and F_r are calculated. The value of θ will also be calculated depending on S_z and R_T. Table 2 is established with varying R_T parameters, and V variations, thereby providing a clearer understanding of the influence of fixed parameters S_c, R_T, and V on the toolpath parameters S_z, F_r, and θ.

Table 1. Toolpath generation using the Constant Volume method

```
Begin
Input: x, y, $R_T$, V, Width, Long
Calculate $S_c$, $S_z$, $F_r$
theta = asin($S_z$/(2*$R_T$));
$S_c$ = $R_T$*(1 - cos(theta));
$S_z$ = 2*sqrt($R_T$^2-($R_T$-$S_c$)^2);
n = Width/$S_z$ + $S_z$/2;
m = Long/$F_r$
temp = [x, y];
for i = 1:n/2
    x = x + $F_r$*m;
    temp = [temp; x y];
    y = y + $S_z$;
    temp = [temp; x y];
    x = x – $F_r$*m;
    temp = [temp; x y];
    y = y + $S_z$;
    temp = [temp; x y];
end
plot (temp(:, 1), temp(:, 2);
export list of coordinates (x, y)
export G-code
End
```

Table 2. Table of the toolpath parameters

S_c, mm	R_T, mm	Stepover S_z, mm	Forward step F_r, mm	Angles θ, º	Volume V, mm^3
0.05	2.0	0.88882	2.07882	0.22408	0.5
0.05	**2.5**	**0.99499**	**3.02704**	**0.20034**	**1.0**
0.05	3.0	1.09087	3.49508	0.18283	1.5
0.05	3.5	1.17898	3.72974	0.16923	2.0
0.03	2.0	0.69022	2.77963	0.17342	0.5
0.03	2.5	0.77227	4.02116	0.15507	1.0
0.03	3.0	0.84640	4.62188	0.14154	1.5
0.03	3.5	0.91455	4.91617	0.13102	2.0

The set of parameters in row 2 was inputted into the algorithm, and a Matlab script was written to generate the toolpath using the Constant Volume method (Table 1). The resulting toolpath is shown in a zigzag pattern in Fig. 7.

The total length of the toolpath includes both rapid tool movement and machining routes. The statistics were conducted when generating the toolpath using the traditional method in commercial CAD/CAM software and the proposed algorithm described in Table 2.

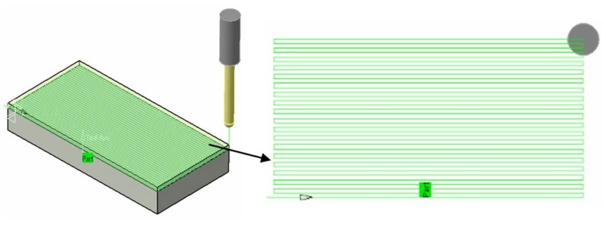


Fig. 6. Toolpath generating using commerce CAD/CAM

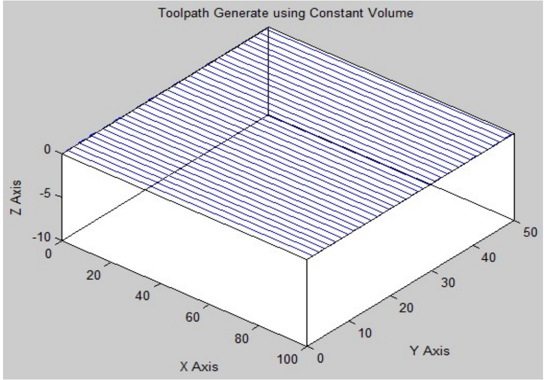
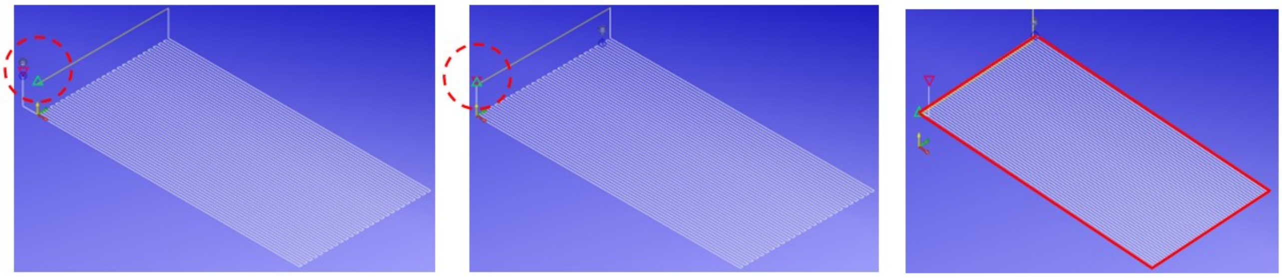


Fig. 7. Toolpath generating using the Constant Volume Method

1. *In terms of toolpath:* The noticeable difference here lies in the tool advance and retract positions after machining. For the traditional method (Fig. 8, a), there's a slight deviation of the cutting tool movement from the surface to be machined. This path may either leave some material uncut or overcut, often opting for overcutting to ensure a flawless machining process, or select boundary cutting to ensure the removal of all excess material for a perfect surface (Fig. 8, c). This process is more complex compared to the proposed method (Fig. 8, b).



a b c

Fig. 8. Compared toolpath:
a – the toolpath generating by commercial CAD/CAM; b – the toolpath generating using Constant Volume Method;
c – the boundary toolpath

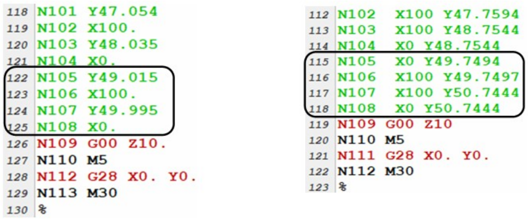
To further evaluate the proposed method, some statistics data was compiled in Table 3, including the length of the cutting toolpath during processing and the total length of the cutting toolpath for two similar cases, except for a slight deviation in the toolpath generated by commercial CAD/CAM software when moving the cutting tool from position (-5.55, 0, 10) to the workpiece origin (0, 0, 0) during processing, while for the Constant Volume method, the cutting was moved straight down to the machining area. The tool moved from position (0, 0, 10) to (0, 0, 0) for processing (as circled in Fig. 8). This can be easily adjusted during the programming process, so it is not a significant issue.

Table 3. Comparison of the cutting toolpaths generated by the traditional method and the Constant Volume method

Traditional Method		Constant Volume Method	
Machining length, mm	Toolpath length, mm	Machining length, mm	Toolpath length, mm
5275,540	5325,535	5270,744	5321,488

Machining length is the length of the toolpath where the tool is engaged in cutting. Total length includes both the length of the toolpath when the tool is engaged in cutting and rapid motion.

2. *Machining Process:* In traditional method testing, an issue arose where the calculation to create a horizontal step may be slightly incomplete if the final step does not have enough horizontal tool displacement equal to the amount of vertical tool displacement S_z, resulting in incorrect tool displacement for machining the final contour. In Fig. 9, a, instead of N107 Y50, it should be N107 Y49.995, which leads to the surface accuracy at the final step being often not as precise as the surface accuracy in the previous steps. In Fig. 9, b, the author calculated the toolpath to overcome the error left at the final machining step. In case of stopping at the command N106 X100 Y49.7497, the surface will still be machined, but like in Fig. 9, a, the final toolpath will leave a different error than the surface errors left in the previous steps. The command N107 X100, Y50.744 will solve this issue, and will be noted in the Constant Volume algorithm.



```
N101 Y47.054
N102 X100.
N103 Y48.035
N104 X0.
N105 Y49.015
N106 X100.
N107 Y49.995
N108 X0.
N109 G00 Z10.
N110 M5
N112 G28 X0. Y0.
N113 M30
%
```

```
N102  X100 Y47.7594
N103  X100 Y48.7544
N104  X0 Y48.7544
N105  X0 Y49.7494
N106  X100 Y49.7497
N107  X100 Y50.7444
N108  X0 Y50.7444
N109 G00 Z10
N110 M5
N111 G28 X0. Y0.
N112 M30
%
```

a b

Fig. 9. The G-code to end the program:
a – generated by a commercial CAD/CAM system;
b – generated by the Constant Volume algorithm, which processes the final toolpath

Conclusion

In this study, a new method to determine the volume of remaining metal residue after machining, thereby proposing a toolpath generation method called Constant Volume, has been developed. Based on the required accuracy of the machined part, the lateral tool compensation and longitudinal tool stepover are calculated based on the volume of this metal residue to determine the parameters of the toolpath. The results indicate that the proposed method yields a shorter toolpath compared to the traditional method. However, this method also demonstrates the advantage of determining the F_r vertical stepover parameter, which is extremely important when creating tool paths for complex surfaces. In addition, the Constant Volume method also has an advantage in the corresponding edge regions with the final toolpath using traditional method in commercial CAD/CAM software, which is often not accurately calculated and can cause errors in these regions. In the proposed Constant Volume method, this issue can be addressed. Although the toolpath may be added with an additional offset, the accuracy remains the same for the entire surface and the total length is less than one in the traditional method. The author proposes several future research directions, including the toolpath optimizing using the Constant Volume method by employing genetic algorithms to find optimal stepover parameters, aiming to enhance productivity and reduce machining time.

References

1. Sarkar, S. & Dey, P. P. (2014). A new iso-parametric machining algorithm for free-form surface. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, vol. 228, iss. 3, pp. 197–209. https://doi.org/10.1177/0954408913495191.
2. Sarkar, S. & Dey, P. P. (2015). Tool path planning for machining free-form surfaces. *Transactions of FAMENA*, vol. 39, iss. 1, pp. 65–78.
3. Zhu, Y., Chen, Z.-T., Ning, T., & Xu, R.-F. (2016). Tool orientation optimization for 3+2-axis CNC machining of sculptured surface. *Computer Aided Design*, vol. 77, pp. 60–72. https://doi.org/10.1016/j.cad.2016.02.007.
4. Mladenović, G. M., Tanovic, L. M., & Ehmann, K. F. (2015). Tool path generation for milling of free form surfaces with feed rate scheduling. *FME Transactions*, vol. 43, iss. 1, pp. 9–15. https://doi.org/10.5937/fmet1501009M.
5. Kumazawa, G. H., Feng, H. Y., & Barakchi Fard, M. J. (2015). Preferred feed direction field: A new tool path generation method for efficient sculptured surface machining. *Computer-Aided Design*, vol. 67–68, pp. 1–12. https://doi.org/10.1016/j.cad.2015.04.011.
6. Kim, Y.-J., Elber, G., Bartoň, M., & Pottmann, H. (2015). Precise gouging-free tool orientations for 5-axis CNC machining. *Computer-Aided Design*, vol. 58, pp. 220–229. https://doi.org/10.1016/j.cad.2014.08.010.
7. Kim, T. & Sarma, S. E. (2002). Toolpath generation along directions of maximum kinematic performance; A first cut at machine-optimal paths. *Computer-Aided Design*, vol. 34, iss. 6, pp. 453–468. https://doi.org/10.1016/S0010-4485(01)00116-6.
8. Walhof, D. (2016). The democratic theory of Hans-Georg Gadamer. Palgrave Macmillan Cham, 145 p. https://doi.org/10.1007/978-3-319-46864-8.
9. Chandran, S. & Abraham Mathews, J. (2016). Simulation and optimization of CNC controlled grinding processes: Analysis and simulation of automated robot finishing process. LAP LAMBERT Academic Publishing, 196 p.
10. Van, Q. H. & Ngoc, T. B. (2019). Effect of feed rate, tool-path and step over on geometric accuracy of freeform surfaces when 3 axis CNC milling. *Applied Mechanics and Materials*, vol. 889, pp. 107–114. https://doi.org/10.4028/www.scientific.net/amm.889.107.
11. Hoang, V. Q. & Nguyen, D. T. (2022). A simple method to tool path generation for cone surface on CNC 3 axis milling. In: Long, B. T., Kim, H. S., Ishizaki, K., Toan, N. D., Parinov, I. A., Kim, Y. H. (eds) *Proceedings of the International Conference on Advanced Mechanical Engineering, Automation, and Sustainable Development 2021 (AMAS 2021)*. Lecture Notes in Mechanical Engineering. Cham: Springer, vol. 2, pp. 481–492. https://doi.org/10.1007/978-3-030-99666-6_70.

Received 31 March 2024

Побудова траєкторії інструменту на тривісній фрезі за допомогою методу постійного об'єму

Van-Quy Hoang

Хошимінський індустріальний університет,
12 Nguyen Van Bao, Ward 4, Go Vap District, Ho Chi Minh City, В'єтнам

У цій статті представлено новий простий метод побудови траєкторій інструменту під час механічної обробки на базі фіксованої основи залишкової металевої частини, що залишилася після кожного поздовжнього (передній крок) і поперечного кроку різання (перехід). Після кожного поздовжнього і поперечного кроку різання залишиться необроблена частина металу, яка має форму конуса з чотирикутною основою, а її бічні грані є кривими, створеними перетином сфери з діаметром траєкторії інструменту. Висота цього конуса, S_c, *називається висотою гребінця, а його проекція на січну площину – вершиною. Однак весь об'єм цієї* *необробленої частини металу було проаналізовано у даній статті. Виходячи з цього, було запропоновано коригування траєкторії інструменту таким чином, щоб об'єм цієї частини залишався постійним на кожному кроці роботи інструменту, в результаті чого забезпечується якість обробки всієї поверхні. На відміну від попередніх досліджень, у яких траєкторію інструменту генерували за допомогою ізоскалопного, ізопараметричного або ізопланарного методів, в цьому дослідженні запропоновано новий метод, заснований на обчисленні об'єму нерозрізаного металу після кожного кроку траєкторії інструменту по горизонталі та вертикалі. Цей метод називається методом постійного об'єму. Порівняно з існуючими методами, цей підхід є кращим, оскільки завдяки ньому можна обчислити об'єм металу, що залишився, таким чином забезпечуючи більш однорідну якість поверхні та більш ефективну траєкторію інструменту. Щоб переконатися в коректності запропонованого методу, у Matlab2010a був реалізований сценарій, який використовувався для створення траєкторії інструменту з простою поверхнею. Траєкторію інструменту, створену запропонованим методом, порівнювали з тією, що створена традиційним методом, який уже доступний у програмному забезпеченні CAD/CAM. Результати показали, що запропонований метод мав хорошу точність і швидкий час формування траєкторії. Цей метод можна розширити до складних поверхонь і він є одним з варіантів для застосування в програмному забезпеченні CAD/CAM, а також надає інше рішення для створення траєкторії інструменту для механічної обробки в цілому.*

Ключові слова*: траєкторія, побудова траєкторії, оптимізація траєкторії, тривісний верстат з ЧПК, постійний об'єм.*

Література

1. Sarkar S., Dey P. P. A new iso-parametric machining algorithm for free-form surface. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*. 2014. Vol. 228. Iss. 3. P. 197–209. https://doi.org/10.1177/0954408913495191.
2. Sarkar S., Dey P. P. Tool path planning for machining free-form surfaces. *Transactions of FAMENA*. 2015. Vol. 39. Iss. 1. P. 65–78.
3. Zhu Y., Chen Z.-T., Ning T., Xu R.-F. Tool orientation optimization for 3+2-axis CNC machining of sculptured surface. *Computer Aided Design*. 2016. Vol. 77. P. 60–72. https://doi.org/10.1016/j.cad.2016.02.007.
4. Mladenović G. M., Tanovic L. M., Ehmann K. F. Tool path generation for milling of free form surfaces with feed rate scheduling. *FME Transactions*. 2015. Vol. 43. Iss. 1. P. 9–15. https://doi.org/10.5937/fmet1501009M.
5. Kumazawa G. H., Feng H. Y., Barakchi Fard M. J. Preferred feed direction field: A new tool path generation method for efficient sculptured surface machining. *Computer-Aided Design*. 2015. Vol. 67–68. P. 1–12. https://doi.org/10.1016/j.cad.2015.04.011.
6. Kim Y.-J., Elber G., Bartoň M., Pottmann H. Precise gouging-free tool orientations for 5-axis CNC machining. *Computer-Aided Design*. 2015. Vol. 58. P. 220–229. https://doi.org/10.1016/j.cad.2014.08.010.
7. Kim T., Sarma S. E. Toolpath generation along directions of maximum kinematic performance; A first cut at machine-optimal paths. *Computer-Aided Design*. 2002. Vol. 34. Iss. 6. P. 453–468. https://doi.org/10.1016/S0010-4485(01)00116-6.
8. Walhof D. The democratic theory of Hans-Georg Gadamer. Palgrave Macmillan Cham, 2016. 145 p. https://doi.org/10.1007/978-3-319-46864-8.
9. Chandran S., Abraham Mathews J. Simulation and optimization of CNC controlled grinding processes: Analysis and simulation of automated robot finishing process. LAP LAMBERT Academic Publishing, 2016. 196 p.
10. Van Q. H., Ngoc T. B. Effect of feed rate, tool-path and step over on geometric accuracy of freeform surfaces when 3 axis CNC milling. *Applied Mechanics and Materials*. 2019. Vol. 889. P. 107–114. https://doi.org/10.4028/www.scientific.net/amm.889.107.
11. Hoang V. Q., Nguyen D. T. A simple method to tool path generation for cone surface on CNC 3 axis milling. In: Long B. T., Kim H. S., Ishizaki K., Toan N. D., Parinov I. A., Kim Y. H. (eds) *Proceedings of the International Conference on Advanced Mechanical Engineering, Automation, and Sustainable Development 2021 (AMAS 2021)*. Lecture Notes in Mechanical Engineering. Cham: Springer, 2022. Vol. 2. P. 481–492. https://doi.org/10.1007/978-3-030-99666-6_70.