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COMPUTER MODELING OF THE PROCESS FOR MANUFACTURING SPHERICAL VESSELS FROM SHEET STEEL BY HYDROFORMING

Pavlo P. Hontarovskyi
gontarpp@gmail.com
ORCID: 0000-0002-8503-0959

Natalia V. Smetankina
nsmetankina@ukr.net,
ORCID: 0000-0001-9528-3741

Nataliia H. Garmash
garm.nataly@gmail.com
ORCID: 0000-0002-4890-8152

Iryna I. Melezhyk
melezhyk81@gmail.com
ORCID: 0000-0002-8968-5581

Tetiana V. Protasova
tatyprotasova@gmail.com
ORCID: 0000-0003-1489-2081

Anatolii Pidhornyi Institute of Power Machines and Systems of NAS of Ukraine,
2/10, Komunalnykiv str., Kharkiv, 61046, Ukraine

Reliable and safe operation of vessels filled with gas or liquid under high pressure requires compliance with certain requirements for their strength. It is also important to reduce weight and material consumption. Numerous industries, including automotive, chemical engineering, and the rocket and space industry, which supply products in bulk, effectively use hydroforming technology for the manufacture of components. Hydroforming is a metalworking process in which complex-shaped structures are created using fluid pressure and displacement constraints instead of traditional mechanical loads (or in combination with them). The successful implementation of this technology is possible due to the advantages that hydroforming has compared to traditional methods, such as the assembly of blankings by welding. A simulation of the manufacture of spherical vessels from sheet steel by hydroforming is proposed in this paper. The software developed on the basis of the finite element method is used, which allows solving elastoplastic problems of thermomechanics by time or load steps in combination with an iterative process on each of them, during which the geometry of the deformed part is refined. To describe the stress-strain state, a logarithmic measure of deformations is used, which allows reflecting real processes occurring in the workpiece. Plastic deformations are taken into account using deformation theory. Thanks to computer modeling of hydroforming technology, spherical models that have the lowest metal content at high pressure were obtained. The obtained vessel models deform elastically under repeated loading due to an increase in the yield strength of the material, therefore they will not be damaged by low-cycle fatigue. They can be used in aerospace engineering as fuel tanks for liquefied oxygen or fluorine and hydrogen. Computer modeling of the hydroforming process allows to quickly and cheaply set the parameters of vessels of various sizes and from different materials, and to obtain an acceptable result without resorting to multiple experimental attempts.

Keywords: *hydroforming, elastoplastic problem, finite element method, logarithmic strain rate.*

Introduction

Steel cylinders filled with gas or liquid, operating under high pressure, must meet the requirements for their strength and safety of operation. With a small volume, they often are heavy. One of the most important and expensive components of rocket and space technology, for which weight reduction is a significant factor, are the fuel tanks of launch vehicles, the study of the strength of which is a crucial and urgent task [1–3]. Spherical vessels have a minimum metal content and a uniform distribution of stresses in the walls at a given pressure. In addition, they are characterized by the maximum ratio of volume to surface area, which is very important when storing liquids or liquefied gases at low temperatures, protecting them from heating caused by environment.

Computer modeling of the technology of manufacturing vessels of a shape close to spherical from steel sheets by hydroforming is proposed [4]. It is a relatively new case of metal processing by pressure, which has many advantages over traditional cold blanking processes, in particular, it allows to create more complex components in fewer operations. For certain geometric shapes, hydroforming technology allows one to create parts that have less weight, increased yield strength of the material, are cheaper to manufacture and can be made from a smaller number of workpieces, due to which there is less material waste.

In addition, the hydroforming process has a number of advantages over other forming processes, which has helped it find wide application in various industries. The advantages of the technology include: the possibility of creating rotating elements, walls thinning [5], improved mechanical properties, better surface treatment [6], a smaller number of components required for the structure assembling [7], as well as a smaller number of necessary modifications due to the creation of a geometry, which is closer to the required final shape [8]. These advantages are primarily related to the ability of the working fluid to evenly exert pressure over the entire surface of the material, as well as the ability of the equipment to change the fluid pressure during the molding cycle based on the optimized loading trajectory.

Hydroforming is used in many manufacturing industries [8–10], including the automotive and aerospace industries, to produce components that would otherwise be difficult or impossible to form [11–13]. Fluid pressure allows the material to be deformed more uniformly and to create pressure in directions other than the forming direction, which opens up additional industrial opportunities. Hydroforming thus fills a manufacturing gap by providing higher material formability than conventional cold forming [8] at lower cost and cycle times than other specialized and non-traditional sheet metal forming technologies, such as superplastic forming or creep forming [14].

Formulation of the problem of modeling the manufacturing of spherical vessels from sheet steel by hydroforming

First, it is proposed to produce two elliptical bottoms with a ratio of diameter D to depth H equal to four, which are most common in the chemical industry and the production technology of which is well established, by blanking. Then, a strip with a width of approximately 0.415 D and a length of πD is cut from a steel sheet with a thickness of h, then a cylindrical shell with a diameter of D, which is welded along the height with one seam, is produced by forge rolling. The height of the cylindrical shell is selected by calculations in such a way that the vessel acquires a spherical shape at the lowest possible pressure during hydroforming, which protects the welds from destruction. Both bottoms are welded to the cylindrical shell on both sides, after which the welds are cleaned of irregularities. The welds must be of high quality and, to ensure their strength, performed in an inert gas (argon) environment. A hole is drilled in the center of one of the bottoms and a nozzle, through which grease will be supplied under high pressure so that the vessel workpiece acquires a spherical shape, is welded.

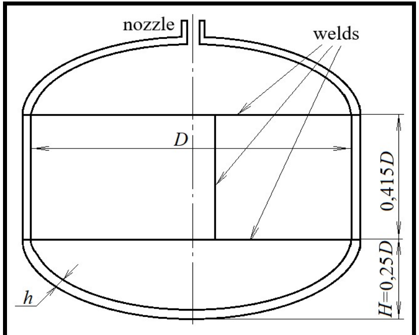

Fig. 1. Meridional cross-section of a welded vessel workpiece

When pumping grease, it is needed to ensure that air does not get into the vessel workpiece. In this case, the possible destruction of the vessel during hydroforming will not be accompanied by the scattering of fragments.

The meridional section of the manufactured vessel workpiece is shown in Fig. 1. Before hydroforming, the workpiece should be maintained at a high temperature to achieve relaxation of residual stresses from blanking and welding.

Results of computer modeling of hydroforming of spherical vessels

The hydroforming process will be modeled by solving the elastic-plastic problem for an axisymmetric calculation scheme of a symmetrical half of the workpiece.

For this purpose, the calculation methodology and software developed using the finite element method [15, 16], which allow to consider the kinetics of the thermally stressed state of structures of various levels of complexity [17, 18] with variable material properties taking into account real operating loading conditions [19, 20] and plastic deformations [21, 22], are used.

When developing calculation schemes, the system for specifying the initial data is based on a topologically regular decomposition of the body into elements in the form of arbitrary hexagons, the geometry and external influences on which can be specified in different coordinate systems, arbitrarily oriented relative to the global Cartesian system. Discretization into the simplest finite elements is performed automatically by the program according to the specified information [17, 18].

The properties of the materials from which the structure is made can depend on the temperature and are given as tables for its fixed values. For other values of temperature, they are determined by linear or quadratic interpolation.

An arbitrary number of types of boundary conditions, distributed on the sides of the elements, can be specified in the problem. The number of boundary conditions is specified for each subdomain. The components of the boundary conditions can vary in coordinates and time and are specified using special functions for fixed moments of time. For the problem of thermal conductivity, boundary conditions of the 2nd or 3rd kind, as well as heat transfer by radiation, can be specified. For the problem of mechanics, the components of stresses or displacements in the global or local coordinate systems are specified. The initial conditions can be specified by constant values or obtained by solving a stationary problem under the specified boundary conditions.

In the calculations, it will be assumed that the thickness of the steel sheet is h=1 cm, the diameter of the middle surface of the shell is D=200 cm, and its height is 83 cm. The meridional section of the elliptical bottom is approximately replaced by torus and spherical subregions. Fig. 2 shows an axisymmetric calculation scheme of half of the welded vessel workpiece, consisting of subregions of three types: cylindrical, torus, and spherical.

The geometry of the subdomains is given by the coordinates of their vertices r and z, and for the torus and sphere, the coordinates r and z of the points in the middle of the arc are also given in the coordinate array. The number 1E7 is a sign that later in the coordinate array the values of r and z for the curved surface will be given. The subdomains are divided in thickness into 4 finite elements so that the extreme ones are two times smaller than the average ones. Along the cylinder meridian, the torus and spherical subdomains are evenly divided into 10, 7 and 20 finite elements.

On the symmetric axial section of the calculation scheme, the boundary conditions of symmetry u_z=0, τ_{rz}=0 are set. The load is applied in the form of a stress normal to the inner surface and varies from 4 MPa in 22 steps according to the load with an interval of 1 MPa, depending on the deformation of this surface. The physical properties of the material were taken as follows: the modulus of elasticity of steel is E=2×10^5 MPa, Poisson's ratio is ν=0.3. When solving the elastoplastic problem, the deformation theory of plasticity was used [23]. The steel deformation diagram is shown in Fig. 3.

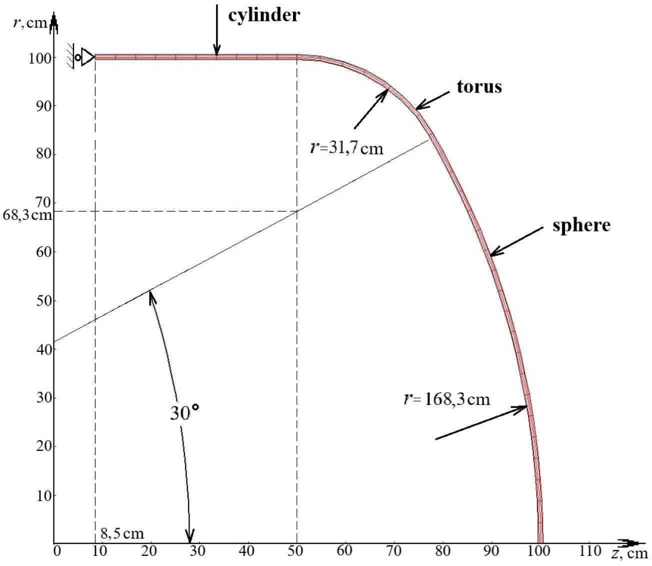

Fig. 2. Axisymmetric design diagram of half of a welded vessel workpiece

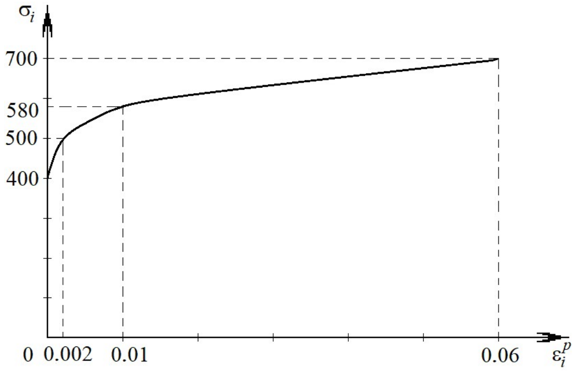

Fig. 3. Diagram of plastic deformation of steel

At each step, 9 iterations were set for the convergence of the problem solution, after which the geometry of the deformed computational model was determined, i.e. the logarithmic measure of deformation was used, because it more correctly describes the real process when modeling hydroforming. The pressure load along the normal to the inner surface requires recalculation of the deformed vessel geometry at each step, without which it is impossible to obtain the correct result.

Table 1 shows the dimensions of the vessel in the radial and axial directions relative to the center depending on the action of internal pressure P.

At a pressure of P=24 MPa, the shape of the vessel most closely resembles a sphere, despite the difference of 1.4 cm in the radial and axial directions. With a further increase in pressure to 26 MPa, despite the fact that the body has almost the same dimensions in the radial and axial directions, the meridional cross-section of the vessel resembles a circle less. At an angle of about 36° to the z-axis, the radius of the vessel is maximum and is approximately 113 cm.

Similar calculations were performed for the case when the workpiece walls thickness is h=0.6 cm. The load was applied starting from 2.5 MPa with a step of 0.62 MPa. The vessel acquired a spherical shape at a pressure of about 14.9 MPa. Table 2 shows the radial and axial dimensions of the vessel depending on the pressure P.

The geometry of the vessel model made of a steel sheet with a thickness of h=1 cm at a pressure of P=24 MPa is given in Fig. 4, and the geometry of the one made of a sheet with a thickness of h=0.6 cm at a pressure of P=14.9 MPa – in Fig. 5.

The stress intensity σ_i in the walls of the vessel model made of a sheet with a thickness of h=1 cm at a pressure of P=24 MPa is within 555.8–703.6 MPa, the strain intensity ε_i does not exceed 6.15%, and the accumulated intensity of plastic deformations $\varepsilon^p{}_i$ does not exceed 5.85%.

The stress intensity σ_i in the vessel model made of a sheet with a thickness of h=0.6 cm at a pressure of P=14.9 MPa is within 568.3–725.5 MPa, the strain intensity ε_i has values less than 7.07%, and the accumulated intensity of plastic deformations $\varepsilon^p{}_i$ does not exceed 6.75%.

Table 1. Dimensions of the vessel in the radial and axial directions relative to the center

P, MPa	0	23	24	25	26
l_r, cm	100.0	104.7	105.5	106.3	107.4
l_z, cm	91.5	102.6	104.1	105.5	107.2
(l_r–l_z), cm	8.5	2.1	1.4	0.8	0.2

Table 2. Radial and axial dimensions of the vessel depending on the pressure P

P, MPa	0	14.3	14.9	15.5	16.1
l_r, cm	100.0	105.3	106.2	107.2	108.6
l_z, cm	91.5	104.0	105.5	107.1	109.0
(l_r–l_z), cm	8.5	1.3	0.7	0.2	-0.4

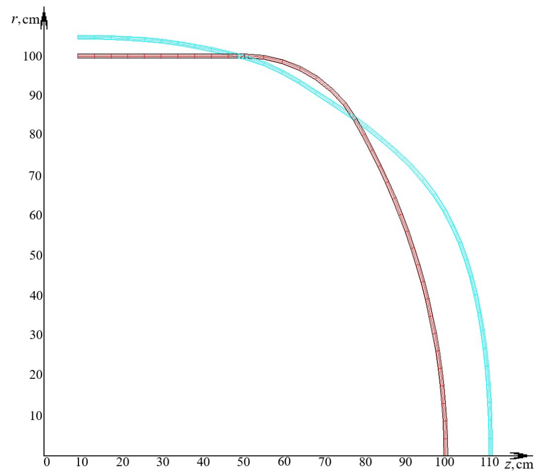

Fig. 4. Geometry of a vessel model made of a steel sheet with a thickness of h=1 cm

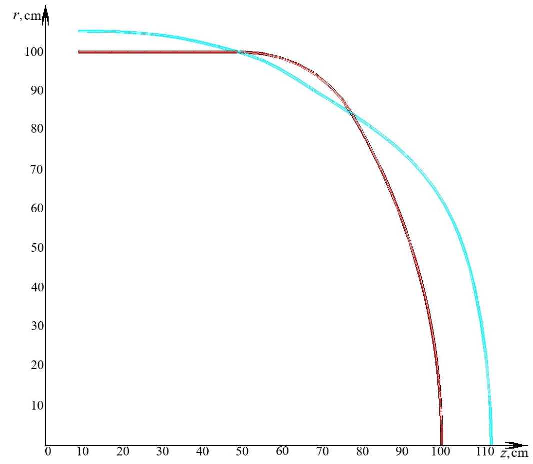

Fig. 5. Geometry of a vessel model made of a sheet steel with a thickness of h=0.6 cm

Conclusion

Hydroforming technology provides the ability to form hollow parts of complex shape from individual initial parts in combination with improved rigidity and strength characteristics due to a reduction in the number of welds and reduced costs for parts assembling. Spherical vessels have the lowest metal content. The minimum surface area in relation to the volume of the vessel minimizes heat exchange with the environment, which is very important during their operation.

The vessel samples obtained in the work by computer modeling of the hydroforming process are elastically deformed when reloaded to a pressure of 24 MPa and 14.9 MPa at sheet thicknesses of h=1 cm and h=0.6 cm, respectively, therefore they will not be damaged by low-cycle fatigue. They can be used in aerospace engineering as launch vehicles fuel tanks for liquefied oxygen or fluorine and hydrogen. For better thermal insulation, they can be covered from the outside with a layer of fine-pored foam. In addition, such fuel tanks will have a minimum weight, which is important during rocket flight. There are many other options for using hydroforming in structural elements of the aerospace industry, the parts of which are high-cost and quite sensitive to weight.

The considered models of vessels before hydroforming had a volume of 4120 liters, and after their deformation to a spherical shape – 4960 liters, i.e. the increase in volume occurred by almost 20.4%, which is also of practical importance. When filling such vessels with natural gas under high pressure, they can be transported by road, and after being equipped with a special device with a pressure gauge, they can be used to charge ordinary household gas cylinders in places where there is no centralized gas supply.

In the chemical industry, loading structures beyond the operational parameters allows, in places of stress concentration, to raise the yield strength of the material and avoid damage from low-cycle fatigue, thereby increasing the service life of the equipment. In addition, this technique is used to reduce the metal content of structures.

Computer modeling of the hydroforming process allows to select the height of the shells and obtain the parameters of vessels of other sizes and from other materials quickly and cheaply, which allows to avoid multiple experimental studies in order to achieve an acceptable result. Reducing the thickness of the shell wall even within one millimeter makes it possible to obtain better vessel parameters during hydroforming.

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Комп’ютерне моделювання процесу виготовлення посудин сферичної форми з листової сталі шляхом гідроформування

П. П. Гонтаровський, Н. В. Сметанкіна, Н. Г. Гармаш, І. І. Мележик, Т. В. Протасова

Інститут енергетичних машин і систем ім. А. М. Підгорного НАН України,
61046, Україна, м. Харків, вул. Комунальників, 2/10

Надійна і безпечна експлуатація посудин, наповнених газом або рідиною під високим тиском, потребує виконання певних вимог до їх міцності. Важливим також є зниження ваги й матеріалоємності. Численні галузі промисловості, серед яких автомобілебудування, хімічне машинобудування, ракетно-космічна промисловість, що масово поставляють продукцію, для виготовлення компонентів ефективно використовують технологію гідроформування, що являє собою процес обробки металів, при якому конструкції складної форми створюються за допомогою тиску рідини й обмеження переміщень замість традиційних механічних навантажень (або у поєднанні з ними). Успішне впровадження цієї технології стає можливим завдяки перевагам, які має гідроформування порівняно з традиційними методами, такими, як збірка штамповок за допомогою зварювання. У даній роботі пропонується моделювання виготовлення посудин сферичної форми з листової сталі шляхом гідроформування. Використовується розроблене на основі методу скінченних елементів програмне забезпечення, що дозволяє розв’язувати пружнопластичні задачі термомеханіки шляхом кроків за часом або навантаженням у поєднанні з ітераційним процесом на кожному з них, у ході якого уточнюється геометрія деформованої деталі. Для опису напружено-деформованого стану застосовується логарифмічна міра деформацій, яка дає змогу відобразити реальні процеси, що відбуваються у заготовці. Пластичні деформації враховуються за допомогою деформаційної теорії. Завдяки комп’ютерному моделюванню технології гідроформування одержано моделі сферичної форми, які мають найменшу металоємність при високому тиску. Отримані моделі посудин деформуються пружно при повторному навантаженні за рахунок підвищення границі текучості матеріалу, тому не зазнаватимуть пошкоджень від малоциклової втоми. Вони можуть бути використані в аерокосмічній техніці як паливні баки для зрідженого кисню чи фтору та водню. Комп’ютерне моделювання процесу гідроформування дозволяє швидше і дешевше встановити параметри посудин різноманітних розмірів і з різних матеріалів, а також, не вдаючись до багаторазових експериментальних спроб, отримати прийнятний результат.

***Ключові слова**: гідроформування, пружнопластична задача, метод скінченних елементів, логарифмічна міра деформацій.*

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