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MATHEMATICAL MODELING OF BENDING OF ISOTROPIC AND ANISOTROPIC PLATES WITH ELLIPTICAL AND LINEAR INCLUSIONS

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The bending problem of an infinitely large thin anisotropic plate with an elliptical or linear elastic inclusion inserted into a hole without initial tension and under perfect mechanical contact with the plate matrix is solved. The plate at infinity is subjected to constant bending moments. The solution is obtained by employing the formalism of generalized complex potentials, expansions of functions into Laurent series and Faber polynomials, as well as conformal mapping techniques to transform the exterior of the unit circle into the exterior of an ellipse. An exact analytical solution for the case of an elliptical inclusion, providing expressions for bending moments and transverse forces both in the plate matrix and in the inclusion, is presented. For the case when the elliptical inclusion reduces to a line, formulas for calculating the moment intensity factors (MIF) at its ends are derived. This approach accurately captures the singular behavior of bending moments and identifies conditions under which MIF values are significant. Numerical studies were conducted for plates made of isotropic material (CAST–V) and anisotropic material (skew-wound glass-fiber-reinforced plastic) under various values of the inclusion's relative stiffness and axis ratio. It was found that decreasing the inclusion's stiffness leads to an increase in bending moments in certain contact zones, with higher moment concentrations in anisotropic plates compared to isotropic ones. For linear inclusions, significant MIF values arise only for substantially stiff or soft inclusions; when the stiffnesses of the plate and inclusion differ by less than a few times, MIF values are negligible, and it is inappropriate to discuss bending moment singularities. Isotropic plates are treated as a special case of anisotropic ones, enabling the extension of these results to a broad class of engineering problems involving composites and structures with embedded elements.

Keywords: *thin plate, bending, mathematical modeling, numerical methods, holes, inclusions, complex potentials.*

Introduction

Despite significant progress in the development of the applied theory of bending of thin anisotropic plates [1, 2, 3], as of now, few problems have been solved for plates with foreign inclusions. This is especially accurate for cases with linear inclusions, when it is necessary to study the singular behavior of the main characteristics at their ends. For multi-connected plates, such a statement is still a difficult mathematical problem, while for a plate with a single inclusion, an exact solution can be obtained by considering the linear inclusion as a limiting case of an elliptical one, when one of the semiaxes is zero.

A mathematical modeling of the bending process of an anisotropic plate with an elastic inclusion is given in the paper. The solution of the problem of bending of an anisotropic plate with an elliptical, including linear, elastic inclusion, obtained by applying the methods of conformal mappings; the expansion of functions into Laurent series and Faber polynomials, as well as the formula for calculating the moment intensity factors (MIFs) are presented. The results of numerical calculations, which allowed to identify the influence of the stiffness of the elastic inclusion, the ratio of the semiaxes of the inclusion and the anisotropy of the materials of the plate and the inclusion on the values of the bending moments arising in the plate, are given.

Problem statement and solution method

We consider an infinite anisotropic matrix plate with an elliptical hole L_1 with center at the origin of the coordinate system Oxy and semiaxes a_1, b_1, located along the coordinate axes (Fig. 1). An elastic inclusion of another material is inserted into the hole. The plate and the inclusion are in ideal mechanical contact. Bending moments of constant magnitude M_x^∞, M_y^∞, H_{xy}^∞ act on the plate-matrix at infinity.

To solve the problem, we will use the complex potentials of Lekhnitskii's theory of bending of anisotropic plates [1, 2]. The region occupied by the plate-matrix will be denoted by S, and the region of elastic inclusion – by S^1.

When using complex potentials, solving the problem reduces to finding the derivatives of functions of generalized complex variables $W_k'(z_k)$ for the matrix plate and $W_k'^1(z_k^1)$ for inclusion from the appropriate boundary conditions.

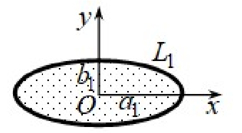

Fig. 1. Infinite plate with elliptic inclusion

Derivatives of complex potentials for a plate-matrix $W_k'(z_k)$ are functions of generalized complex variables

$$z_k = x + \mu_k y, \tag{1}$$

where μ_k are roots of the characteristic equation [4]

$$D_{22}\mu^4 + 4D_{26}\mu^3 + 2(D_{12} + 2D_{66})\mu^2 + 4D_{16}\mu + D_{11} = 0; \tag{2}$$

$D_{ij} = B_{ij}D_0$ are material stiffness parameters for which

$$B_{11} = (a_{22}a_{66} - a_{26}^2)/\Delta, B_{12} = (a_{16}a_{26} - a_{12}a_{66})/\Delta, B_{16} = (a_{12}a_{26} - a_{16}a_{22})/\Delta;$$

$$B_{22} = (a_{11}a_{66} - a_{16}^2)/\Delta, B_{26} = (a_{12}a_{16} - a_{26}a_{11})/\Delta, B_{66} = (a_{11}a_{22} - a_{12}^2)/\Delta;$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{vmatrix},$$

where a_{ij} are deformation coefficients for the plate material; $D_0 = 2h^3/3$; h is the half-thickness of the plate.

Functions $W_k'(z_k)$ are defined in the regions S_k, which are obtained from the region S by affine transformations (1) and have the following form [2]

$$W_k'(z_k) = \Gamma_k z_k + \sum_{n=1}^{\infty} \frac{a_{k1n}}{\zeta_{k1}^n}, \tag{3}$$

where Γ_k are constants found from the system of equations

$$2Re\sum_{k=1}^{2}\Gamma_k = A_{11}M_x^\infty + A_{21}M_y^\infty + A_{31}H_{xy}^\infty, 2Re\sum_{k=1}^{2}\mu_k\Gamma_k = A_{12}M_x^\infty + A_{22}M_y^\infty + A_{32}H_{xy}^\infty;$$

$$2Re\sum_{k=1}^{2}\mu_k^2\Gamma_k = A_{13}M_x^\infty + A_{23}M_y^\infty + A_{33}H_{xy}^\infty, 2Re\sum_{k=1}^{2}\frac{1}{\mu_k}\Gamma_k = 0;$$

$$A_{11} = (2D_{22}D_{66} - 2D_{26}^2)/\Delta_1; \quad A_{21} = (2D_{16}D_{26} - 2D_{12}D_{66})/\Delta_1; \quad A_{31} = (2D_{12}D_{26} - 2D_{12}D_{22})/\Delta_1;$$

$$A_{12} = (D_{12}D_{26} - D_{16}D_{22})/\Delta_1; \quad A_{22} = (D_{12}D_{16} - D_{11}D_{26})/\Delta_1; \quad A_{32} = (D_{11}D_{22} - D_{12}^2)/\Delta_1;$$

$$A_{13} = (2D_{16}D_{26} - 2D_{12}D_{66})/\Delta_1; \quad A_{23} = (2D_{11}D_{66} - 2D_{16}^2)/\Delta_1; \quad A_{33} = (2D_{12}D_{16} - 2D_{11}D_{26})/\Delta_1;$$

$$\Delta_1 = \begin{vmatrix} D_{11} & 2D_{16} & D_{12} \\ D_{12} & 2D_{26} & D_{22} \\ D_{16} & 2D_{66} & D_{26} \end{vmatrix}; \quad A_i = \sum_{j=1}^{3} A_{ij}/D_{11},$$

where a_{k1n} are unknowns; ζ_{k1} are variables that are defined from the conformal mapping of the exterior of the unit circle $|\zeta_{k1}| \geq 1$ on the appearance of ellipses L_{k1}:

$$z_k = R_{k1}\left(\zeta_{k1} + \frac{m_{k1}}{\zeta_{k1}}\right),$$

where

$$R_{k1} = \frac{a_1 - i\mu_k b_1}{2}; \quad m_{kl} = \frac{a_1 + i\mu_k b_1}{2R_{k1}}. \tag{4}$$

If the functions $W_k'(z_k)$ are determined, then the bending moments and transverse forces at all points of the plate are calculated using the following formulas:

$$\left(M_x, M_y, H_{xy}\right) = -2Re\sum_{k=1}^{2}\left(p_k, q_k, r_k\right)W_k''(z_k)\,;\quad \left(N_x, N_y\right) = -2Re\sum_{k=1}^{2}\left(\mu_k s_k, -s_k\right)W_k''(z_k)$$

in which

$$p_k = D_{11} + 2D_{16}\mu_k + D_{12}\mu_k^2\,;\qquad q_k = D_{12} + 2D_{26}\mu_k + D_{22}\mu_k^2\,;$$

$$r_k = D_{16} + 2D_{66}\mu_k + D_{26}\mu_k^2\,;\qquad s_k = -D_{16} - (D_{12} + 2D_{66})\mu_k - 3D_{26}\mu_k^2 - D_{22}\mu_k^3\,.$$

For bending moments on an arbitrary surface with a normal n we have

$$M_n = M_x\cos^2 nx + M_y\cos^2 ny + 2H_{xy}\sin nx\cos nx\,.$$

Derivatives of complex potentials for inclusion $W_k'^1(z_k^1)$ are also functions of generalized complex variables

$$z_k^1 = x + \mu_k^1 y\,, \tag{5}$$

where μ_k^1 are roots of the characteristic equation of the form (2), in which the coefficients D_{ij} are replaced by the appropriate stiffness parameters D_{ij}^1 for inclusion. These functions are defined in the areas S_k^1, obtained from the region S^1 by affine transformations (5). In these finite simply connected holomorphic domains, they can be expanded into series in Faber polynomials, which after transformations can be written as power series as follows:

$$W_k'^1(z_k^1) = \sum_{n=0}^{\infty} a_{kn}^1\left(\frac{z_k^1}{R_k^1}\right)^n\,. \tag{6}$$

Here a_{kn}^1 are unknowns; R_k^1 are constants, which are calculated by analogy with formulas (4).

Let's find the unknowns a_{kln} and a_{kn}^1 from the boundary conditions on the contact contour of the inclusion with the matrix-plate [1, 2]

$$2Re\sum_{k=1}^{2}\left(g_{k1i}W_k'(z_k) - g_{ki}^1 W_k'^1(z_k^1)\right) = 0\,, \tag{7}$$

where $g_{k11} = 1$; $g_{k1}^1 = 1$; $gg_{k12} = \mu_k, g_{k2}^1\mu_k^1$; $g_{k13} = p_k/\mu_k$; $g_{k3}^1 = p_k^1/\mu_k^1$; $g_{k14} = q_k$; $g_{k4}^1 = q_k^1$.

We substitute functions (3) and (6) into boundary conditions (7) and use the series method. Taking into account that on the inclusion contour $\zeta_{k1} = \sigma$, we obtain $a_{k1n} = a_{kn}^1 = 0$ at $n \ge 2$, and to define a_{k11} and a_{k1}^1 we have a system of algebraic equations

$$\sum_{k=1}^{2}\left(a_{k11} - m_k^1 a_{k1}^1 - \bar{a}_{k1}^1\right) = -\sum_{k=1}^{2}\left[{}_k R_{k1}m_{k1} + {}_k \bar{R}_{k1}\right];\quad \sum_{k=1}^{2}\left(\mu_k a_{k11} - \mu_k^1 m_k^1 a_{k1}^1 - \bar{\mu}_k^1\bar{a}_{k1}^1\right) = -\sum_{k=1}^{2}\left[\mu_{k\,k} R_{k1}m_{k1} + \bar{\mu}_{k\,k}\bar{R}_{k1}\right];$$

$$\sum_{k=1}^{2}\left(\frac{p_k}{\mu_k}a_{k11} - \frac{p_k^1}{\mu_k^1}m_k^1 a_{k1}^1 - \frac{\bar{p}_k^1}{\bar{\mu}_k^1}\bar{a}_{k1}^1\right) = -\sum_{k=1}^{2}\left[\frac{p_k}{\mu_k}{}_k R_{k1}m_{k1} + \frac{\bar{p}_k}{\bar{\mu}_k}{}_k \bar{R}_{k1}\right];$$

$$\sum_{k=1}^{2}\left(q_k a_{k11} - q_k^1 m_k^1 a_{k1}^1 - \bar{q}_k^1\bar{a}_{k1}^1\right) = -\sum_{k=1}^{2}\left[q_{k\,k} R_{k1}m_{k1} + \bar{q}_{k\,k}\bar{R}_{k1}\right].$$

Thus, the derivatives of complex potentials have the form

$$W_k'(z_k) = {}_k\, z_k + \frac{a_{k11}}{\zeta_{k1}}, W_k'(z_k) = a_{k1}^1\frac{z_k^1}{R_k^1}\,,$$

and for the moments we get the following expressions

$$\left(M_x, M_y, H_{xy}\right) = -2Re\sum_{k=1}^{2}\left(p_k, q_k, r_k\right)\left({}_k - \frac{a_{k11}}{R_{k1}\left(\zeta_{k1}^2 - m_{k1}\right)}\right),\quad \left(M_x^1, M_y^1, H_{xy}^1\right) = -2Re\sum_{k=1}^{2}\left(p_k^1, q_k^1, r_k^1\right)\frac{a_{k1}^1}{R_k^1}\,.$$

If the hole passes into a straight section, and the inclusion – into an elastic line, respectively, then it is also possible to calculate the moment intensity factors (MIFs) k_{m1} (for moments M_y) and k_{m2} (for moments H_{xy}). Similarly to the case of a flat problem [5], for the MIF we obtain the expressions

$$\left(k_{m1}, k_{m2}\right) = \frac{1}{\sqrt{a_1}} 2Re \sum_{k=1}^{2} \left(q_k, r_k\right) a_{k11} .$$

Based on MIF and thanks to expressions [1]

$$\sigma_y = \frac{3M_y}{2h^3} z ; \quad \tau_{xy} = \frac{3H_{xy}}{2h^3} z$$

it is possible to find the maximum values of the MIF (at $z=h$)

$$\left(k_1, k_2\right) = \frac{3}{2h^2} \left(k_{m1}, k_{m2}\right).$$

Numerical studies

Numerical studies of bending moments for isotropic CAST-V material (material M1) and anisotropic skew-wound glass-fiber-reinforced plastic (material M2) were conducted. Their deformation coefficients are given in Table 1. The deformation coefficients for the inclusion material were chosen as follows: $a_{ij}^{(l)} = \lambda^{(l)} a_{ij}$, where $\lambda^{(l)}$ is the relative stiffness parameter.

Table 1. Material constants

Material	$a_{11}\times10^{-4}$, MPa^{-1}	$a_{22}\times10^{-4}$, MPa^{-1}	$a_{12}\times10^{-4}$, MPa^{-1}	$a_{66}\times10^{-4}$, MPa^{-1}
M1	72.100	72.100	-8.600	161.500
M2	10000	2.800	-0.770	27.000

For a plate with a circular radius inclusion a_1 ($b_1=a_1$) (Fig. 2) under the influence of bending moments $M_y^{\infty} = m_y$ for different values of the relative stiffness parameter $\lambda^{(1)}$ accurate to a constant factor m_y, the values of the moments M_s at the points of contact of the plate with the inclusion on the areas perpendicular to the inclusion contour are shown in Table 2. Here θ – central angle of the hole, which is counted from the positive direction of the axis Ox counterclockwise. The values of $\lambda^{(1)}$, which are equal to 0 and ∞, correspond to the cases of a plate with an absolutely hard and an absolutely soft inclusion (a hole).

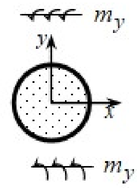

Fig. 2. Infinite plate with circular inclusion

Analyzing the data in Table 2, it can be seen that with a decrease in the stiffness of the inclusion (i.e., with an increase in $\lambda^{(1)}$) moment values M_s at points near θ=0 are growing, at points near θ=π/2 they first fall (at $\lambda^{(1)}>1$), then increase (at $\lambda^{(1)}<1$). At $\lambda^{(1)}<10^{-2}$ inclusion can be considered absolutely rigid, at $\lambda^{(1)}>10^2$ – absolutely soft (hole). The concentration of moments in an anisotropic plate is higher than in an isotropic one.

Table 2. Moment values M_s at the plate contact points

Material	$\lambda^{(1)}$	θ, rad						
		0	π/12	π/6	π/4	π/3	5π/12	π/2
M1	0	–0.164	–0.128	–0.029	0.107	0.242	0.341	0.377
	10^{-2}	–0.124	–0.092	–0.005	0.114	0.235	0.319	0.351
	10^{-1}	0.159	0.161	0.166	0.172	0.178	0.183	0.184
	0.5	0.720	0.671	0.539	0.359	0.179	0.046	–0.002
	2	1.248	1.169	0.951	0.653	0.355	0.137	0.058
	10	1.590	1.497	1.245	0.899	0.554	0.301	0.209
	10^2	1.704	1.608	1.346	0.989	0.631	0.370	0.274
	∞	1.718	1.622	1.359	1.000	0.641	0.378	0.283
M2	0	–0.222	–0.254	0.007	0.464	0.511	0.303	0.190
	10^{-2}	–0.185	–0.215	0.034	0.463	0.496	0.288	0.176
	10^{-1}	0.078	0.056	0.215	0.461	0.403	0.192	0.090
	0.5	0.653	0.616	0.568	0.476	0.275	0.074	–0.006
	2	1.369	1.257	0.925	0.534	0.256	0.098	0.046
	10	2.032	1.820	1.212	0.608	0.312	0.208	0.184
	10^2	2.311	2.053	1.328	0.641	0.344	0.266	0.253
	∞	2.348	2.084	1.343	0.646	0.349	0.274	0.263

Analyzing the data in Table 2, it can be seen that with a decrease in the stiffness of the inclusion (i.e., with an increase in $\lambda^{(1)}$) moment values M_s at points near θ=0 are growing, at points near θ=π/2 they first fall (at $\lambda^{(1)}>1$), then increase (at $\lambda^{(1)}<1$). At $\lambda^{(1)}<10^{-2}$ inclusion can be considered absolutely rigid, at $\lambda^{(1)}>10^2$ – absolutely soft (hole). The concentration of moments in an anisotropic plate is higher than in an isotropic one.

The values of the moments at the points of the plate near the inclusion, as well as the values of MIF (at $b_1/a_1=10^{-5}$) are given in Table 3 for different ratios of semiaxes b_1/a_1 of elliptical inclusion and relative stiffness parameter $\lambda^{(1)}$.

From Table 3 it is seen that with decreasing ratio b_1/a_1 moment M_s values near the ends of the bigger semiaxis rapidly increase in modulus; at $b_1/a_1<10^{-3}$ the inclusion can be considered linear and the MIF can be calculated for it. In this case, the MIF occurs if the relative stiffness of the inclusion material is large or small.

Table 3. Moment and MIF values depending on $\lambda^{(1)}$ and b_1/a_1

$\lambda^{(1)}$	b_1/a_1						KIM
	1	0.5	10^{-1}	10^{-2}	10^{-3}	10^{-4}	$k_{m1}^{\pm}$
M1							
0	–0.16	–0.19	–0.42	–3.02	–29.00	–288.69	–0.136
10^{-2}	–0.12	–0.16	–0.33	–0.97	–1.32	–1.38	–0.001
0.5	0.72	0.63	0.53	0.50	0.49	0.49	0.000
2	1.25	1.40	1.77	1.97	1.99	1.99	0.000
10^2	1.70	2.40	7.62	42.10	86.81	97.28	0.013
∞	1.72	2.44	8.18	72.78	718.83	7178.87	1.000
M2							
0	–0.22	–0.27	–0.69	–5.37	–52.18	–520.13	–0.170
10^{-2}	–0.19	–0.24	–0.53	–1.51	–1.99	–2.06	–0.001
0.5	0.65	0.58	0.51	0.49	0.49	0.49	0.000
2	1.37	1.55	1.86	1.97	1.99	1.99	0.000
10^2	2.31	3.59	12.71	57.10	91.35	97.22	0.007
∞	2.35	3.70	14.48	135.77	1348.63	13475.67	1.000

The MIF graphs ($k_{m1}^{\pm}$) depending on the stiffness of the inclusion (parameter $\lambda^{(1)}$) are shown in Fig. 3. We see that for a linear elastic inclusion the influence of the parameter $\lambda^{(1)}$ is the same as for an elastic circular core: at $\lambda^{(1)}<10^{-3}$ inclusion can be considered absolutely rigid, and when $\lambda^{(1)}>10^3$ – absolutely soft (cracked). When $10^{-4}<\lambda^{(1)}<10^4$ the values of the MIF are quite small and can be neglected. Therefore, the MIF for linear elastic inclusions can be considered only if the stiffness of the inclusion differs from the stiffness of the plate by no less than 10^3 times.

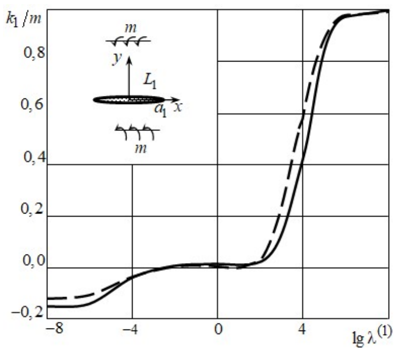

Fig. 3. Dependence of the MIF on the relative stiffness of a linear inclusion

Conclusions

The problem of bending of an infinite anisotropic plate with an elliptical elastic inclusion was solved using complex potentials. An approach for calculating the intensity coefficients of the moments of the MIF was proposed.

The mathematical modeling of the bending process made it possible to estimate the influence of the relative stiffness of the inclusion on the magnitude of the bending moments at the points of contact of the plate with the inclusion, and also to find out in which cases the inclusion can be considered absolutely soft and absolutely rigid.

The influence of the ratio of the semiaxes of the elliptical inclusion on the magnitude of the bending moments and on the MIF was studied. It was established that the inclusion can be considered linear at $b_1/a_1<10^{-3}$. It was found that MIF can be considered in cases where the inclusion material differs from the plate material by no less than 10^3 times.

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Математичне моделювання згину ізотропних та анізотропних плит з еліптичними та лінійними включеннями

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Розв'язано задачу теорії згину тонких плит для нескінченної анізотропної плити з еліптичним або лінійним пружним включенням, вставленим в отвір без попереднього натягу й з умовами ідеального механічного контакту з плитою-матрицею. Для отримання розв'язку використано апарат узагальнених комплексних потенціалів, розклади функцій у ряди Лорана й за многочленами Фабера, а також метод конформних відображень для переходу від зовнішності одиничного кола до зовнішності еліпса. У роботі наведено точне аналітичне розв'язання задачі для випадку еліптичного включення й отримано вирази для згинальних моментів і поперечних сил як у плиті-матриці, так і у включенні. Для випадку, коли еліптичне включення вироджується у лінійне, виведено формули для обчислення коефіцієнтів інтенсивності моментів (КІМ) у його кінцях. Запропонований підхід дозволяє коректно описати сингулярну поведінку згинальних моментів й оцінити умови, за яких КІМ мають істотні значення. Проведено числові дослідження для плит з ізотропного (КАСТ–В) й анізотропного (склопластик косокутного намотування) матеріалів за різних значень відносної жорсткості включення і співвідношення його півосей. Встановлено, що зменшення жорсткості включення призводить до зростання згинальних моментів у певних зонах контакту з плитою, причому концентрація моментів в анізотропних плитах вища, ніж в ізотропних. Показано, що для лінійного включення великі значення КІМ спостерігаються лише у випадках суттєво жорстких або м'яких включень; при близьких жорсткостях плити і включення (менш ніж у декілька разів) КІМ майже зникають, а отже, вести мову про сингулярності моментів у таких випадках некоректно. Ізотропні плити розглянуто як окремий випадок анізотропних, що дозволяє поширити отримані результати на великий клас технічних задач механіки композитів і конструкцій із вставними елементами.

***Ключові слова**: тонка плита, згин, математичне моделювання, числові методи, отвори, включення, комплексні потенціали.*

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