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VISCOELASTIC STATE OF AN ANISOTROPIC PLATE WITH A SINGLE ELLIPTIC INCLUSION

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The linear viscoelasticity problem for an infinite anisotropic plate with an elliptical elastic inclusion under ideal mechanical contact conditions is solved. To obtain the solution, the small parameter method is applied, where the variation of Poisson's ratios over time is chosen as the parameter, effectively reducing the time-dependent problem to a sequence of analogous boundary value problems in the theory of elasticity. The construction of the solution is based on the complex potentials apparatus, conformal mapping methods, and Laurent series expansions. Boundary conditions at the contact interface are satisfied using the generalized least squares method, ensuring high accuracy of the unknown constants at any given moment. Analytical expressions for bending moments and shear forces in the plate are derived, explicitly incorporating viscoelastic time operators. For the case where the elliptical inclusion degenerates into a straight elastic line, formulas for calculating moment intensity factors at its endpoints are provided. The proposed approach allows for a correct description of the evolution of singular moment behavior and an evaluation of the material properties' influence on their temporal variation. Numerical studies were conducted for materials with various relaxation properties and different relative inclusion stiffnesses. It is established that the most intensive redistribution of moments occurs during the initial stage of the viscoelastic process, after which the stress state of the plate approaches a stationary phase. It is proven that moment concentration depends non-linearly on inclusion stiffness, being minimal at intermediate stiffness values and increasing sharply for holes or perfectly rigid inclusions. Isotropic plates are treated as a special case of anisotropic ones, allowing the results to be extended to a wide range of problems in composite mechanics and long-term strength prediction.

Keywords: viscoelasticity, bending, mathematical modeling, numerical methods, inclusions, complex potentials, small parameter method.

Introduction

Despite the great practical importance of knowledge about the change in the stress state of multi-connected viscoelastic plates with time after the application of a load, such studies have not been carried out in sufficient volume so far [1]. In this paper, the viscoelasticity problem for plates, thanks to the small parameter method, is reduced to a sequence of similar problems of the theory of elasticity, which are solved using complex potentials and the generalized least squares method. The problem is solved for a plate with an elliptical (in some cases linear) elastic inclusion, in some cases with holes or rigid inclusions. The results of numerical calculations, which allowed to identify the influence of time on the values of bending moments arising in the plate, are given.

Problem statement and solution method

An infinite anisotropic plate with an elliptical hole L_1 centered at the origin of the coordinate system Oxy and semi-axes a_1, b_1 , located along the coordinate axes is considered (Fig. 1). An elastic inclusion of another material is inserted into this hole without prior tension, which is in ideal mechanical contact with the matrix plate. It is assumed that bending moments of constant magnitude $M_x^\infty, M_y^\infty, H_{xy}^\infty$ are applied to the plate at infinity.

This problem in the elastic formulation can be solved using the complex potentials of the bending theory. In this case, the problem is reduced to finding the functions of the complex variables $W'_k(z_k)$ from the corresponding boundary conditions [2]. They are functions of generalized complex variables

$$z_k = x + \mu_k y, \quad (1)$$

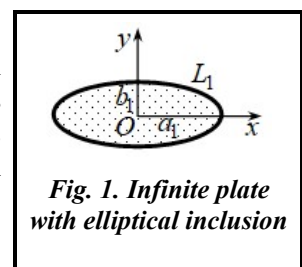


Fig. 1. Infinite plate with elliptical inclusion

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where μ_k are roots of the corresponding characteristic equation [2], which depends on the stiffness of the plate material D_{ij} .

All the main characteristics of the elastically deformed state of the plate are calculated through the functions

$$(M_x, M_y, H_{xy}, N_x, N_y) = -2 \operatorname{Re} \sum_{k=1}^2 (p_k, q_k, r_k, \mu_k, s_k, -s_k) W_k''(z_k),$$

where

$$\begin{aligned} p_k &= D_{11} + 2D_{16}\mu_k + D_{12}\mu_k^2; & q_k &= D_{12} + 2D_{26}\mu_k + D_{22}\mu_k^2; \\ r_k &= D_{16} + 2D_{66}\mu_k + D_{26}\mu_k^2; & s_k &= -D_{16} - (D_{12} + 2D_{66})\mu_k - 3D_{26}\mu_k^2 - D_{22}\mu_k^3. \end{aligned} \quad (2)$$

When finding the values of the studied quantities in time, one can use Volterra principle [3], that is, solve the problem without taking into account the properties of viscoelasticity, and then in this solution replace the elastic constants with time operators and determine their action in time. However, for multi-connected domains, it is unrealistic to obtain such solutions of the problems of the theory of elasticity. Therefore, direct application of Volterra principle to the analysis of the stress-strain state of bodies is impossible.

In this regard, there is a need to obtain such solutions that would explicitly contain elastic constants. This can be done using the small parameter method, for which one can take the change in time of Poisson's ratios ν_{12} . Let's write it in the form

$$\nu_{12} = \nu_{12}^0 + \lambda, \quad (3)$$

where ν_{12}^0 is the instantaneous elastic coefficient value ν_{12} ; λ is a small parameter.

For orthotropic materials, the strain rates are related to the engineering constants by the following relations:

$$a_{11} = \frac{1}{E_1}; \quad a_{22} = \frac{1}{E_2}; \quad a_{12} = -\frac{\nu_{21}}{E_1} = -\frac{\nu_{12}}{E_2}; \quad a_{66} = \frac{1}{G_3},$$

where E_i, G_3 – the corresponding Young's moduli and shear modulus.

Taking into account (3), for a_{12} we obtain

$$a_{12} = a_{12}^0 - \lambda a_{11},$$

where $a_{12}^0 = -\nu_{12}^0 a_{11}$.

Expanding all quantities in terms of a small parameter λ and taking into account that for real materials $\left| \frac{\lambda}{\lambda_1} \right| < 1, \left| \frac{\lambda}{\lambda_2} \right| < 1$, the following form of quantities is obtained (2):

$$p_k = \sum_{j=0} p_{jk} \lambda^j; \quad q_k = \sum_{j=0} q_{jk} \lambda^j; \quad r_k = 2D_0 b_{66} \mu_k,$$

and complex potentials take shape

$$W_k'(z_k) = \sum_{j=0} \lambda^j W_{jk}'(z_k), \quad (4)$$

where $W_k'(z_k) = \Gamma_{jk} z_k + W'_{j0k}(z_k)$; Γ_{jk} are determined from the system of equations

$$\begin{aligned} 2 \operatorname{Re} \sum_{k=1}^2 \left(1, \mu_k, \mu_k^2, \frac{1}{\mu_k} \right) \Gamma_{0k} &= \left(-\frac{a_{11} M_x^\infty}{D_0} - \frac{a_{12} M_y^\infty}{D_0}, -\frac{a_{66} H_{xy}^\infty}{2D_0}, -\frac{a_{12}^0 M_x^\infty}{D_0} - \frac{a_{22} M_y^\infty}{D_0}, 0 \right); \\ 2 \operatorname{Re} \sum_{k=1}^2 \left(1, \mu_k, \mu_k^2, \frac{1}{\mu_k} \right) \Gamma_{1k} &= \left(\frac{a_{11} M_y^\infty}{D_0}, 0, \frac{a_{11}^0 M_x^\infty}{D_0}, 0 \right); \quad \Gamma_{ij} = 0 \quad (j \geq 2); \end{aligned}$$

$D_0 = 2h^3/3$; h is the half-thickness of the plate; $W'_{j0k}(z_k)$ are functions that are holomorphic in domains S_k bounded by contours L_{k1} under affine transformations (1) of the domain S .

Derivatives of complex potentials for inclusion are similarly given in the following form:

$$W_k^{(1)}(z_k^{(1)}) = \sum_{j=0}^{\infty} \lambda^j W_{jk}^{(1)}(z_k^{(1)}). \quad (5)$$

Here $W_k^{(1)}(z_k^{(1)})$ are functions that are holomorphic in the domains $S_k^{(1)}$.

Taking this into account, the solution of the viscoelasticity problem is reduced to finding the derivatives of the complex potentials of the approximations $W_k'(z_k)$ from the corresponding boundary conditions.

The functions $W_k'(z_k)$ are defined in the domains S_k , which are obtained from the given domain by affine transformations (1). Using the methods of conformal mappings and expansions of functions into Laurent series, for the derivatives of the complex potentials of the approximations we obtain the expressions

$$W_{jk}'(z_k) = \Gamma_{jk} z_k + \sum_{n=1}^{\infty} \frac{a_{jk1n}}{\zeta_{k1}^n}, \quad (6)$$

where a_{jk1n} are unknown; ζ_{k1} are variables that are determined from relations

$$z_k = R_{k1} \left(\zeta_{k1} + \frac{m_{k1}}{\zeta_{k1}} \right); \quad R_{k1} = \frac{a_1 - i\mu_k b_1}{2}; \quad m_{k1} = \frac{a_1 + i\mu_k b_1}{2R_{k1}}. \quad (7)$$

The derivatives of the complex potentials for inclusion $W_k^{(1)}(z_k^{(1)})$ are expanded into series in Faber polynomials and after transformations we obtain

$$W_{jk}^{(1)}(z_{jk}^{(1)}) = \sum_{n=0}^{\infty} a_{jkn}^{(1)} \left(\frac{z_k^{(1)}}{R_k^{(1)}} \right)^n, \quad (8)$$

where $R_k^{(1)}$ are calculated according to formulas (7); $a_{jkn}^{(1)}$ are unknown.

The complex potentials for the matrix plate and the inclusion must satisfy the boundary conditions at the contact points. These conditions have the form

$$2 \operatorname{Re} \sum_{k=1}^2 \left(g_{k1i} W_k'(z_k) - g_{ki}^{(1)} W_k^{(1)}(z_k^{(1)}) \right) = 0, \quad (9)$$

where $g_{k11} = 1$; $g_{k1}^{(1)} = 1$; $g_{k12} = \mu_k$; $g_{k2}^{(1)} = \mu_k^{(1)}$; $g_{k13} = \frac{p_k}{\mu_k}$; $g_{k3}^{(1)} = \frac{p_k^{(1)}}{\mu_k^{(1)}}$; $g_{k14} = q_k$; $g_{k4}^{(1)} = q_k^{(1)}$.

If we substitute the expressions for functions (4) and (5) into the boundary conditions (9) and equate the coefficients at the corresponding powers of the parameter λ , we obtain the following recurrent sequence of boundary conditions for finding these approximations:

$$2 \operatorname{Re} \sum_{k=1}^2 \left(g_{0k1i} W_k'(z_k) - g_{0ki}^{(1)} W_k^{(1)}(z_k^{(1)}) \right) = f_{j1i}(t), \quad (i = \overline{1, 4}), \quad (10)$$

$$g_{0k11} = 1; \quad g_{0k1}^{(1)} = 1; \quad g_{0k12} = \mu_k; \quad g_{0k2}^{(1)} = \mu_k^{(1)}; \quad g_{0k13} = \frac{p_{0k}}{\mu_k}; \quad g_{0k3}^{(1)} = \frac{p_{0k}^{(1)}}{\mu_k^{(1)}}; \quad g_{0k14} = q_{0k}; \quad g_{0k4}^{(1)} = q_{0k}^{(1)},$$

$$f_{j11} = -(1 - \delta_j^0) 2 \operatorname{Re} \sum_{k=1}^2 \sum_{i=0}^{j-1} [W_{ik}'(z_k) - W_{ik}^{(1)}(z_k^{(1)})];$$

$$f_{j12} = -(1 - \delta_j^0) 2 \operatorname{Re} \sum_{k=1}^2 \sum_{i=0}^{j-1} [\mu_k W_{ik}'(z_k) - \mu_k^{(1)} W_{ik}^{(1)}(z_k^{(1)})];$$

$$f_{j13} = -(1 - \delta_j^0) 2 \operatorname{Re} \sum_{k=1}^2 \sum_{i=0}^{j-1} \left[\frac{p_{j-i,k}}{\mu_k} W_{ik}'(z_k) - \frac{p_{j-i,k}^{(1)}}{\mu_k^{(1)}} W_{ik}^{(1)}(z_k^{(1)}) \right];$$

$$f_{j14} = -(1 - \delta_j^0) 2 \operatorname{Re} \sum_{k=1}^2 \sum_{i=0}^{j-1} [q_{j-i,k} W_{ik}'(z_k) - q_{j-i,k}^{(1)} W_{ik}^{(1)}(z_k^{(1)})].$$

To find the unknown factors in formulas (4), we will use Rabotnov time operators

$$\lambda = D_1 \cdot \mathcal{D}_\alpha^* (-\beta_1^* - \delta_1^*) + D_2 \cdot \mathcal{D}_\alpha^* (-\beta_2^*),$$

where $D_1 = \frac{\delta_1^*}{4} \left[\frac{E_1^0}{E_2^0} \frac{\delta_2^*}{\beta_1^* - \beta_2^* + \delta_1^*} + 1 - 4\nu_{12}^0 \right]$; $D_2 = \frac{\delta_2^* E_1^0}{4 E_2^0} \frac{\beta_1^* - \beta_2^*}{\beta_1^* - \beta_2^* + \delta_1^*}$; E_i^0 are instantaneous elastic values of Young's moduli E_i ; δ_i^* , β_i^* are rheological constants of the plate material associated with changes E_i in time [4]. If we raise λ to a power, we obtain

$$\lambda^j = \sum_{k=0}^j C_j^k D_1^{j-k} D_2^k \frac{\mathcal{D}_\alpha^{*k} (-\beta_2^*) - \mathcal{D}_\alpha^{*j-k} (-\beta_1^* - \delta_1^*)}{-\beta_2^* + \beta_1^* + \delta_1^*}. \tag{11}$$

Calculating λ^j by formula (11) and substituting them into expressions (4), we will find the functions $W'_k(z_k)$ and their derivatives, and therefore, the bending moments and shear forces at any time. To determine the unknown constants a_{jk1n} , $a_{jk1n}^{(l)}$ of functions (6) and (8), we will use the generalized least squares method and satisfy the boundary conditions (10) in differential form. To do this, on the contact contour of the plate with inclusions, we will select a set of points $t_{1m}(x_{1m}, y_{1m})$ ($m = \overline{1, M}$) and satisfy the conditions differentiated along the arc of the contour in them

$$2 \operatorname{Re} \sum_{k=1}^2 \sum_{n=1}^{\infty} [g_{0k1i} \delta_k \varphi'_{k1n}(t_{k1m}) a_{jk1n} - g_{0ki}^{(l)} \delta_k^{(l)} \varphi'_{kn}^{(l)}(t_{km}^{(l)}) a_{jkn}^{(l)}] = \frac{df_{j1i}(t_m)}{ds} - 2 \operatorname{Re} \sum_{k=1}^2 g_{0k1i} \delta_k \Gamma_{jk}, \quad (i = \overline{1, 4}; m = \overline{1, M}),$$

where $\delta_k = \frac{dz_k}{ds}$; $\delta_k^{(l)} = \frac{dz_k^{(l)}}{ds}$; $t_{k1m} = x_{1m} + \mu_k y_{1m}$; $t_{km}^{(l)} = x_{1m} + \mu_k^{(l)} y_{1m}$; $\varphi'_{k1n} = -\frac{n}{\zeta_{k1}^{n-1} R_{k1} (\zeta_{k1}^2 - m_{k1})}$, $\varphi_{kn}^{(l)} = -\frac{n(z_k^{(l)})^{n-1}}{R_{k1} (R_k^{(l)})^n}$.

If the inclusion becomes an elastic line, then we can calculate the moment intensity factors (MIF) k_{1M}^\pm (for moments $M_y^{(l)}$) and k_{2M}^\pm (for moments $H_{xy}^{(l)}$) [5]. For them we have the following formulas

$$k_{1M}^\pm = 2 \operatorname{Re} \sum_{k=1}^2 q_k G_k; \quad k_{2M}^\pm = 2 \operatorname{Re} \sum_{k=1}^2 r_k G_k$$

in which $G_k = \pm \frac{\sqrt{a_1}}{2R_1} \sum_{n=1}^{\infty} (\pm 1)^n n a_{k1n}$; the upper signs correspond to the right and left ends of the linear inclusion.

Numerical studies

Numerical studies of the values of the quantities for the aluminum plate (material M1) [6] and epoxy (material M2) [7] were carried out. The deformation coefficients and rheological constants for these materials are given in Table 1. The deformation constants for the inclusion material were chosen in proportion to the coefficients for the plate: $a_{ij}^{(l)} = \lambda^{(l)} a_{ij}$, where $\lambda^{(l)}$ is the relative stiffness parameter.

Table 1. Material constants

Material	$a_{11} \times 10^{-4}$, MPa ⁻¹	$a_{22} \times 10^{-4}$, MPa ⁻¹	$a_{12} \times 10^{-4}$, MPa ⁻¹	$a_{66} \times 10^{-4}$, MPa ⁻¹	α , s ^{-0.5}	$\beta_1^* \times 10^3$, s ^{-0.5}	$\beta_2^* \times 10^3$, s ^{-0.5}	$\delta_1^* \times 10^3$, s ^{-0.5}	$\delta_2^* \times 10^3$, s ^{-0.5}
M1	0.1408	0.1458	-0.0352	0.3521	0.500	0.00050	0.00049	0.00615	0.00614
M2	0.4347	0.6250	-0.0478	3.2467	0.846	0.15700	0.27450	0.03230	0.12950

The obtained results showed that over time the values of the moments in the plate change. In this case, large changes occur only in the first 50 hours after the load is applied, and after 200 hours they practically do not change, that is, a stationary state is established in the plate.

During the research, the number of approximations J by powers of a small parameter λ increased until the next approximation did not change the values of the moments by less than 0.01%. In the considered cases, under such conditions, it was necessary to leave from 6 to 10 powers of the parameter λ . The results for an infinite plate with a circular inclusion, on which the moments $M_y^\infty = m$, $M_x^\infty = H_{xy}^\infty = 0$ act, are given. All results are given with an accuracy of a factor of m/D_0 .

Fig. 2 shows the graphs of the distribution of moments M_s in the initial and stationary states for the limiting cases $\lambda^{(1)}$ ($\lambda^{(1)}=0$ and $\lambda^{(1)}=\infty$) depending on the central angle θ , which is counted from the positive direction of the axis Ox counterclockwise. The solid lines show the moments for the initial state, and the dotted lines show the moments for the stationary state. Curves 1, 3 correspond to the material M2, and curves 2, 3 – to the material M1. The absolutely soft inclusion ($\lambda^{(1)}=\infty$) is shown by lines 1, 2, while the absolutely rigid one ($\lambda^{(1)}=0$) is shown by lines 3, 4.

As can be seen from Fig. 2, the values of bending moments change significantly when transitioning to the stationary state. The most significant changes occur at the points corresponding to the angles $\theta=0$ and $\theta=\pi/2$, and the largest concentration of moments is observed in the first case for a plate with an absolutely soft inclusion (hole). The most noticeable changes in the values of moments over time occur for a plate with an absolutely rigid inclusion (more than three times compared to the stationary state), and the smallest ones in the case of a hole (about 3.5%).

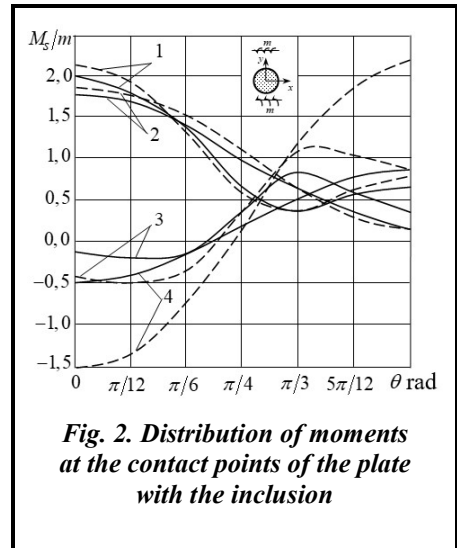


Fig. 2. Distribution of moments at the contact points of the plate with the inclusion

In addition, a study was conducted on the change in the MIF depending on the stiffness of the inclusion material in the initial and stationary states. It was found that the influence of relative stiffness on the values of moments for a linear inclusion is the same as for a circular one. The inclusion can be considered absolutely rigid at $\lambda^{(1)} < 10^{-3}$ and absolutely soft at $\lambda^{(1)} > 10^3$. In other cases, the MIF values are quite small, and therefore they can be neglected.

The results obtained using the least squares method were identical to the results obtained using the exact solution.

Conclusions

Using the small parameter method and the complex potential apparatus, the bending problem of an infinite viscoelastic anisotropic plate with an elliptical elastic inclusion was solved. An approach was proposed in which the viscoelasticity problem is reduced to a sequence of problems of the theory of elasticity.

Mathematical modeling of the bending process for plates made of different materials made it possible to study the change in time of the values of bending moments at the points of contact of the plate with the inclusion, as well as to establish the influence of the relative stiffness of the inclusion material on the viscoelastic state of the plate.

The influence of the relative stiffness of the inclusion on the MIF over time was studied. The limits at which the inclusion can be considered absolutely soft or absolutely rigid were determined.

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В'язкопружний стан анізотропної плити з поодиноким еліптичним включенням**¹ А. О. Кошкін, ^{1,2} О. О. Стрельнікова**¹ Харківський національний університет радіоелектроніки,
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Розв'язано задачу лінійної в'язкопружності для нескінченної анізотропної плити з еліптичним пружним включенням, що перебуває в умовах ідеального механічного контакту з плитою-матрицею. Для отримання розв'язку застосовано метод малого параметра, де як параметр обрано зміну коефіцієнтів Пуассона у часі, що дозволило звести часову задачу до послідовності аналогічних крайових задач теорії пружності. Побудова розв'язку ґрунтується на використанні апарату комплексних потенціалів, методів конформних відображень і розкладів функцій у ряди Лорана. Для задоволення граничних умов на контурі контакту використано узагальнений метод найменших квадратів, що забезпечує високу точність визначення невідомих сталих у будь-який момент часу. У роботі наведено аналітичні вирази для згинальних моментів і перерізувальних сил у плиті, що явно містять часові оператори в'язкопружності. Для випадку, коли еліптичне включення вироджується у прямолінійну пружну лінію, виведено формули обчислення коефіцієнтів інтенсивності моментів у його кінцях. Запропонований підхід дозволяє коректно описати еволюцію сингулярної поведінки моментів й оцінити вплив властивостей матеріалу на їхню зміну у часі. Проведено числові дослідження для плит із матеріалів із різними релаксаційними властивостями за різних значень відносної жорсткості включення. Встановлено, що найбільш інтенсивний перерозподіл моментів відбувається на початковому етапі в'язкопружного процесу, після чого напружений стан плити наближується до стаціонарного. Доведено, що концентрація моментів нелінійно залежить від жорсткості включення: вона мінімальна при середніх значеннях жорсткості і різко зростає у випадках отворів або абсолютно жорстких включень. Ізотропні плити розглянуто як окремий випадок анізотропних, що дозволяє поширити отримані результати на великий клас задач механіки композитів і прогнозування їхньої довготривалої міцності.

Ключові слова: в'язкопружність, згин, математичне моделювання, числові методи, включення, комплексні потенціали, метод малого параметра.

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