APPLICATION OF PHASED ANTENNA ARRAY WITH DIGITAL BEAMFORMING TO ESTABLISH THE INTERNAL RADIO NETWORK OF THE DISTRIBUTED SATELLITE

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Abstract — The possibility of application of the Phased Antenna Array with Digital Beamforming (PAA-DB) to establish the Internal Radio Network (IRN) of the Distributed Satellite (DS) is shown, which allows increasing the efficiency of the IRN by using the spatial separation of information flows between Core and Edge Satellites. The issues considered are related to choosing the Reference Coordinate System, where it is proposed to measure the relative motion of the satellite of the DS. An Orbital Coordinate System of the Core Satellite is proposed as the Reference Coordinate System. It is shown how the information about the coordinates of the Edge Satellites in the Orbital Coordinate System of the Core Satellite enables one to obtain information about the coordinates of the Edge Satellites in the Orbital Coordinate System by recalculating method. It is shown that using modified 3GPP protocols enables the measurement of the inclination ranging between the Core and Edge satellites. To measure the Edge Satellites’ angular coordinates inside the Orbital Coordinate System of the Core Satellite, it is proposed to use the Phased Antenna Array with Digital Beamforming of the Core Satellite in combination with a modified monopulse radio direction-finding method. The modified monopulse method is based on predicting the received signal level of the Edge Satellite and matching it with the spatial frequency real response of the Phased Antenna Array. The difference between the predicted value and assessed spatial frequency enables us to estimate the angle of deviation of the emitter, the edge satellite, from the fixed beam orientation direction of the Phased Antenna Array. To determine the angular coordinates of the Edge Satellite, the values of several spatial frequencies of the Phased Antenna Array are used, which correspond to the level of the received signal in adjacent fixed ones.

Anotaciya — Показано можливість застосування цифрової антенної решітки (ЦАР) для побудови внутрішньої радіоміжної мережі (ВРМ) розподіленого супутника (РС), що дозволяє підвищити ефективність внутрішньої радіоміжної мережі за рахунок застосування просторового розподілу інформаційних потоків між кореневим і кінцевим супутниками. Розглянуто питання вибору опорної системи координат, в якій проводиться процедура вимірювання відносного руху космічних апаратів розподіленого супутника. Як опорну систему координат запропоновано використовувати з’єднану систему координат кореневого супутника. Показано, як інформація про координати кінцевих супутників у з’єднаній системі координат кореневого супутника дозволяє отримати інформацію про координати кінцевих супутників в орбітальній системі координат методом перерахунку. Показано, що використання модифікованих протоколів групи 3GPP дає змогу вимірювати похили багатоістотність між кореневим і кінцевим супутниками. Для вимірювання кутових координат кінцевих супутників у з’єднаній системі координат кореневого супутника запропоновано використовувати цифрову антенну решітку кореневого супутника в поєднанні з модифікованим монопульсним методом радіолокації. В основу модифікованого монопульсного методу покладено прогнозування рівня сигналу кінцевого супутника, що приймається, і порівняння його з реальним відходом просторової частоти цифрової антенної решітки. Різниця в прогнозах величини і оцінці просторової частоти дає змогу вичислити кут відхилення джерела випромінювання, яким є кінцевий супутник, від напрямку направленості цифрової антенної решітки. Для визначення кутових координат кінцевого супутника використовуються значення декількох просторових частот цифрової антенної решітки, які представляють рівень сигналу, що приймається, в суміжних фіксованих променях.

Introduction

The 20s of the XXI century have become rich for active implementation of several projects on creating the Low-Earth-Orbit (LEO) Satellite Telecommunication Systems. Today, at least two LEO Internet Broadband Access Satellite Systems, like Starlink and
OneWeb, are in the stage of active satellite constellations formation. 1,842 Starlink satellites have already been launched into an LEO with an altitude of 547 km. 358 OneWeb satellites have been launched into LEO at 1200 km altitude. Thus, at least two groups of telecommunication satellites, unprecedented in their numbers, are being formed on LEO. According to their plans, only these two satellite systems announced that more than 20 thousand satellites will be launched in the next ten years. The main goal of the Starlink and OneWeb orbital constellations being formed is to provide Internet Broadband Access Services for various groups of customers.

In parallel with the services and infrastructure development for Internet Access, services based on the protocols and infrastructure of Internet Networks are being developed. These services include the Internet of Things (IoT) and its industrial implementation, the Industrial Internet of Things (IIoT). The prospects for implementing IoT and IIoT services stimulate the growth of demand for bandwidth regardless of the telecommunications infrastructure development level at the service provider location. Terrestrial 4G and 5G Generation Mobile Networks do not provide total and continuous coverage, as, from the economic point of view, these networks’ deployment corresponds with the population density in the service area. Areas with low population density do not provide acceptable cost-effectiveness for 4G and 5G Networks. Using satellite telecommunications and information systems in LEO is a solution for expanding the IoT and IIoT Services Areas. During the past two years, operators of up-to-date systems, such as Iridium, and start-up projects, such as AFINO with forecasted launches of 72 satellites for the M2M IIoT services provision, the Swarm Technologies project, or Lacuna, entered the IoT services market.

The disadvantage of Satellite Broadband Systems (SBS) is its targeting of providing a multipurpose service that does not consider the IoT peculiarities. In paper [1], the current state analysis is presented, and the development prospects for the IoT systems in LEO and geostationary orbit (GEO) are indicated. The narrow-band specialization presents the disadvantage of highly dedicated satellite systems based on the narrow-band IoT protocols: NB-IoT and LoRaWAN. In paper [2] it was proposed the concept of building a multifunctional multipurpose IoT system based on the Distributed Satellite Architecture and LEO Constellation, which allows providing broadband-based services with a minimum Round Trip Time (RTT) for IoT Sensitive Delay Services and IoT Insensitive Delay Services based on narrow-band low-speed data transfer protocols.

I. Task statement

The term "Distributed satellite" (DS) defines a Micro-Constellation of Microsat and CubeSats that perform a functionality simultaneously and perform a group flight near one another [3, 4]. The implementation of DS technology enables the utilization of tiny spacecraft to perform functionality that, for various reasons, cannot be performed through one large spacecraft utilization. The DS peculiarities include the distribution of the payload over several spacecraft, which, in aggregate and during the interaction, makes it...
possible to ensure the efficient performance of a functional task. In this case, the spacecraft payloads inside the DS can be duplicated, or it can be unique for this DS.

There are two approaches in the DS Architecture: Centralized or Hierarchical, and Decentralized [2]. In the Centralized Architecture, the DS includes one central or Core, satellite, and several Edge Satellites. As a rule, the Core Satellite is a Microsatellite, and the Edge ones are nanosatellites or CubeSats. The DS Core Satellite provides flight control of all satellites involved in the DS, control of the Edge Satellites interaction to perform DS functional tasks, and interaction with other DS from the Satellite System. Edge Satellites perform limited functional tasks.

To ensure information interaction between satellites inside the DS, an Internal Radio Network (IRN) is established. The IRN goals are:

- providing information exchange between the Core and Edge Satellites as part of the DS for the satellite’s payloads to carry out tasks for the functional purpose of the System;
- ensuring the transmission of command and telemetry information between the Core and Edge Satellites from the DS and the Ground Control Complex (GCC) of the System;
- Edge Satellites positioning and movement measurements about the Core Satellite to ensure the flight safety of the DS subordinates.

IRN is built based on the mobile broadband standard proposed by the 3GPP research group, subject to its adaptation to the application peculiarities as part of the space system [2]. Among the advantages of these standards are the built-in mechanisms used to measure the radio network subscribers’ mutual range and ensure the subscribers’ operation synchronization. Paper [4] considers the possibility of the WiMAX protocol utilization for the IRN construction and suggests ways for modifying this protocol to adapt to the peculiarities of information exchange between DS subordinates.

Applying the multiple access method based on Orthogonal Frequency Division Multiplexing (OFDM) and MIMO (Multiple Input Multiple Outputs) technologies is a peculiarity of the 3GPP group protocols. MIMO technology increases the efficiency of network operation via spatial selectivity, reducing energy costs, reusing the allocated frequency band, and improving the quality and, consequently, the speed of information transmission in the network.

All satellites subordinates from the DS perform a group flight at a distance of no more than 1 km from one another. This distance is significantly less than the currently accepted flight safety standards for space objects. Therefore, when developing the DS technology, a particular concern is being given to measuring the relative motion parameters of satellite subordinates in the DS. The report [5] presents the results of two CubeSats’ mutual motion modeling on LEO and predicts their relative position changes based on the information utilization from the GPS. However, such a solution requires permanent GCC support for spacecraft functioning.

In paper [6], the method based on slant range measurement was proposed for the DS Edge Satellites’ relative motion parameters, supported by the built-in mechanisms of the IEEE 802.16 protocol family, subject to their adaptation to the satellite system’s
peculiarities. However, the proposed method also requires periodic support from the GCC.

The study task is to adapt IRN to provide measurements of the Edge Satellites’ relative motion in the Core Satellite Orbital Coordinate System (OCS) without the GCC facilities and equipment support.

II. Parameters of the Edge Satellite’s relative motion inside the Orbital Coordinate System of the Core Satellite

To ensure the DS spacecraft-subordinates’ flight safety, an important issue is the choice of a Reference Coordinate System in which the relative motion of the satellite will be measured. As the DS Reference Coordinate System, the Body Reference Coordinate System (BRCS) of the Core Satellite $Ox_0Y_0Z_0$ can be chosen, since the Core Satellite provides the movement control of the DS Edge Satellite. Figure 1 shows the Coordinates Determination Scheme for the Edge Satellites in the Core Satellite’s BRCS. The BRCS Center is located at the Center of Mass of the Core Satellite (point O in Fig. 1). The $Ox_0$ axis is directed along with the satellite movement, the $Oy_0$ axis is located in the orbital plane and is in the opposite direction from the Earth’s Center, and the $Oz_0$ axis complements the Coordinate System to the right [7]. The Core Satellite Orbital Coordinate System (OCS) $OxgYgZg$, which is used in Flight Dynamic Calculations, is uniquely associated with the Core Satellite’s BRCS. The OCS center coincides with the BRCS Center and is located in the Center of Mass of the Core Satellite. The BRCS and OCS axes deviation is determined by the Onboard Attitude Control System Sensors’ parameters of the Core Satellite. Information about the Edge Satellites’ coordinates within the Core Satellite’s $Ox0Y0Z0$ BRCS enables obtaining information about the Edge Satellites’ coordinates inside the $OxgYgZg$ OCS via the recalculation method.

![Diagram](image-url)

*Fig. 1. The Edge Satellites coordinate determination in the Body Reference Coordinate System of the Core Satellite*
The Edge Satellites’ location in the Core Satellite’s OX₀Y₀Z₀ BRCS is determined by three parameters (see Fig. 1): range \( r \), the angle \( \theta \) between the direction to the Edge Satellite and the OX₀ axis, and the angle \( \varphi \) between the direction to the Edge Satellite and the OZ₀ axis. The given parameters determine the \( \bar{R} \) vector length and orientation of the Edge Satellite location in the Core Satellite BRCS.

The slant range measuring technique, which assumes the 3GPP Group protocols adaptation to operating terms inside the DS, is given in the paper [6].

For the IRN Establish, the Core Satellite is equipped with a Phased Antenna Array with Digital Beamforming (PAA-DB). Radio-technical measurements, which are performed with the PAA-DB application, are being executed in OX₀Y₀Z₀ PAA-DB Coordinate System (Fig. 2). The PAA-DB CS Reference Point (see point O, Fig. 2) is located in the center of the PAA-DB Aperture Plane. The \( X_p \) and \( Y_p \) axes are located in the PAA-DB plane. The \( Z_p \) axis is located perpendicularly to the PAA-DB aperture plane.

![Fig. 2. Interaction between the BRCS of the Core Satellite and the PAA-DB coordinate system](image)

In the OX₀Y₀Z₀ PAA-DB CS, the \( \bar{R}_1 \) and \( \bar{R}_2 \) vectors’ directions, which characterize the Edge Satellites 1 and 2 (see Fig. 1) relative position, are determined by the pare following angular parameters: the angles of deviation of the \( \bar{R}_1 \) or \( \bar{R}_2 \) vectors from the OZ₀ axis, which correspond to angles \( \theta_1 \) and \( \theta_2 \) respectively (see Fig. 2) and by azimuth angles, i.e., deviations of the \( \bar{R}_1 \) and \( \bar{R}_2 \) vectors’ projections onto the PAA-DB OX₀Y₀ aperture plane from the direction of the OX₀ axis, which correspond to angles \( \alpha_1 \) and \( \alpha_2 \) respectively. An alternative option is to describe the \( \bar{R}_1 \) and \( \bar{R}_2 \) vectors orientations in the PAA-DB CS via two pairs of angles: the \( \alpha_1 \) azimuth angle which is the angle between the \( \bar{R}_1 \) vector’s projection onto the PAA-DB aperture plane and the OX₀ axis, and the \( \beta_1 \) elevation
angle which is the angle between the $\vec{R}_1$ vector and its projection onto the PAA-DB OX$_p$Y$_p$ aperture plane. In case of $\vec{R}_2$ vector, a pair of angles $\alpha_2$ and $\beta_2$ is used.

As can be seen from Fig. 1, the PAA-DB Center Point of origin location differs from the location of the center of mass of the root satellite. This difference is determined by the PAA-DB location within the Constructive Coordinate System (CCS) of the Core Satellite, which coincides with the BRCS, but differs in the direction of the axes and the point of origin of coordinates [8]. The PAA-DB Center Point of origin displacement and the deviation of the DAA OX$_p$Y$_p$Z$_p$ axes’ orientation from the OX$_0$Y$_0$Z$_0$ ASC are being determined during the design and manufacturing process of the satellite – Core Satellite at the stage of the Testing in the facilities of the Assembly Plant. These parameters are included in the Design Data Package of the Core Satellite’s PAA-DB and are registered in the form of constants into its test tag and into the database to consider when making measurements inside the DS and performing further flight dynamic calculations.

Thus, the 3GPP Group protocols utilized for IRN implementation in combination with PAA-DB installed in the Core Satellite make it possible to measure the relative location of the DS Edge Satellites within the Core Satellite’s BRSC. As a result of measurements for each Edge Satellite, a parameters cluster is formed that uniquely locates the Edge Satellite coordinates in a Rectangular Coordinate System: the $r$ slant range and two angles $\theta$ and $\phi$.

### III. Peculiarities of the PAA-DB beams forming and of their directional diagrams’ description

The peculiarity of the PAA-DB technology is the beams forming of the receiving or transmitting antenna array through digital techniques operations [9]. Digital techniques apply to both linear and planar arrays. In half-wavelength PAA-DB the distance $d = \lambda/2$ between elements, one-dimensional and two-dimensional Fast Fourier Transform (FFT) algorithms are widely used. The one- and two-dimensional matrix formed as a result of FFT operations performed determines the spatial frequencies of the PAA-DB, i.e., the response of the signal received in the generated fixed beam with the spatial orientation parameters given. The advantage of the PAA-DB technology results in spatial frequency signals simultaneously calculated of all the beams of either linear or planar PAA-DB being performed through one- or two-dimensional FFT calculating operations.

When using the FFT algorithm, the PAA-DB forms fixed beams. As shown in paper [9], the phase shift coefficient between the PAA-DB neighboring elements signals of dimension $N$ takes $N$ discrete values. In particular, for a linear PAA-DB of dimension $N$, the phase shift coefficient is

$$\frac{d \sin \alpha}{\lambda} = \frac{k}{N},$$

(1)
where \( d \) is a distance between the PAA-DB elements; \( \alpha \) is a deviation angle of the orientation direction of the PAA-DB beam from the normal to the aperture plane; \( \lambda \) is the wavelength; \( k \) is the number of the beam, \( k = 0, 1, 2, \ldots, (N-1) \).

It can be shown that for the half-wave linear PAA-DB with a step \( d = \lambda/2 \), the orientation of fixed beams with number \( k \) is calculated via the expression

\[
\alpha_k = \arcsin \frac{2k}{N}.
\] (2)

When analyzing the PAA-DB radio technical parameters, it is necessary to consider that digital processing replaces signal processing with the help of physical devices – phase shifters, and does not affect the envelope shape of the generated DAA radiation pattern (RP). Therefore, to estimate the parameters of the PAA-DB fixed beams, the models and descriptions used for PAA [10, 11] can be applied. In particular, the Beam Width (BW) of the fixed beam of a linear \( N \)-dimensioned half-wave PAA-DB at the level-3 dB \( \theta_3 \), expressed in degrees, is calculated as follows

\[
\theta_3(k) \approx \frac{\theta}{N \cos \alpha_k}.
\] (3)

The BW at the level of the first zeros \( \theta_0 \), expressed in degrees, is calculated by the relational expression

\[
\theta_0 \approx \frac{230}{N \cos \alpha_k}.
\] (4)

Table 1 presents the values of the deviation angle and the BW width at the -3dB level of fixed beams for linear PAA-DBs of \( N = 8, 16, 32 \) dimensions. When using Table 1, it should be considered that the number of the beam can take positive and negative values, which corresponds to the deviation of the beam from normal to the aperture plane to the left or the right.

For the PAA-DB description, the models of the normalized envelope function of the array coefficient used in [9-11] are valid. In particular

\[
(u_k) = \frac{\sin \left( \frac{1}{2} NKd \alpha_k \right)}{N \sin \left( \frac{1}{2} Kd \alpha_k \right)} = \frac{\sin \left( \frac{1}{2} N \pi \left( \sin \alpha - \sin \alpha_k \right) \right)}{N \sin \left( \frac{1}{2} \pi \left( \sin \alpha - \sin \alpha_k \right) \right)} = \frac{\sin(N \Psi_k)}{N \sin \Psi_k}.
\] (5)
where $u_k$ is the angular variable of the $k$-th beam, $u_k = \sin \alpha - \sin \alpha_k$; $\Psi_k$ is the generalized angle variable of the $k$-th beam, which is related to the angle variable $u_k$ as follows:

$$\Psi_k = \frac{Kd u_k}{2} = \frac{1}{2} Kd \left[ \sin \alpha - \sin \alpha_k \right] = \frac{1}{2} \pi \left[ \sin \alpha - \sin \alpha_k \right];$$

$K$ is a coefficient inversely proportional to the wavelength, $K=2\pi/\lambda$.

Table 1. Orientation of fixed beams of linear PAA-DB of $N=8, 16, 32$ dimension

| $k$ | $N=8$ | | | | $N=16$ | | | | | | $N=32$ | | |
|-----|--------|---------------|--------|---------------|--------|---------------|--------|---------------|--------|---------------|--------|---------------|
|     | $\alpha$ | $\theta_3$ | $A$ | $\theta_3$ | $\alpha$ | $\theta_3$ | $\alpha$ | $\theta_3$ | $\alpha$ | $\theta_3$ | $\alpha$ | $\theta_3$ |
| 0   | 0      | 12.688° | 0   | 6.344° | 0     | 3.172° | 0   | 3.172° | 0     | 3.172° |
| 4   | -      | -     | 30° | 7.325° | 14.77° | 3.28° |
| 5   | -      | -     | 38.682° | 8.127° | 18.21° | 3.339° |
| 6   | -      | -     | 48.59° | 9.591° | 22.024° | 3.421° |
| 7   | -      | -     | 61.045° | 13.104° | 25.944° | 3.528° |
| 8   | -      | -     | -     | -     | 30° | 3.663° |
| 9   | -      | -     | -     | -     | 34.229° | 3.836° |
| 10  | -      | -     | -     | -     | 36.682° | 3.955° |
| 11  | -      | -     | -     | -     | 43.433° | 4.368° |
| 12  | -      | -     | -     | -     | 48.59° | 4.795° |
| 13  | -      | -     | -     | -     | 54.341° | 5.441° |
| 14  | -      | -     | -     | -     | 61.045° | 6.552° |
| 15  | -      | -     | -     | -     | 69.636° | 9.115° |

IV. Method for measuring the angular coordinates of the Edge Satellite

PAA-DB is used in the IRN to improve its efficiency via spatial separation. Thus, the idea of MIMO technology, which has been developed in the networks of the 3GPP Group, is being implemented. The PAA-DB utilization allows applying a rough estimation of the relative position of the Edge Satellite since the information exchange between the Core and Edge Satellites is carried out inside the spatial domain limited by the PAA-DB fixed beam coverage. However, the determining accuracy of the Edge Satellites Angular Coordinates is limited by the coverage width of the PAA-DB fixed beams. As shown in Table 1, for a half-wave linear uniform array with 32 beams, the BW, depending on the beam orientation, is $3° \div 5°$. For a flat PAA-DB of dimension $32 \times 32$, the BW will be similar. Such accuracy in determining the current relative coordinates enables spatial separation for the information flows independent transmission, but is insufficient for predicting the relative movement of the Edge Satellites subordinates inside the DS.

The built-in mechanisms for providing MAC-level control of the 3GPP Group protocols, subject to their adaptation for use as part of the DS [6], make it possible to apply
in IRN the idea of monopulse measurement of the angular parameters of the radiation source, which is widely used in modern radar and GCC of space systems for spacecraft movement trajectory measurements [12, 13].

The basic principle of determining the angular coordinates via the monopulse method is to receive a signal from a radiation source with two beams shifted by at least half the BW, to form the sum and difference signals to determine the direction mismatch towards the radiation source, followed by adjustment to achieve both beams’ signals balance. As previously noted, the PAA-DB performs the formation of the fixed beams via digital methods. Thus, it is impossible to use the monopulse method as it is for measuring the angular terminal satellites’ coordinates via PAA-DB.

The utilization of the 3GPP Group protocols for IRN and the Core Satellite equipment with PAA-DB enables to apply the idea of a monopulse method for the Edge Satellites’ angular coordinates’ measurement. This offer is based on the following factors:

1. There exists a method for the relative range measuring between the Core and Edge Satellites.
2. 3GPP Group protocols provide control of the radiated power level via mobile devices. Thus, the Core Satellite controls the radiated power level of the Edge Satellites via MAC-layer commands and is aware of the radiated power level of the Edge Satellites.
3. The DS IRN operates in an outer space environment, where additional factors affecting the propagation of radio waves are lacking. The only factor is the Free Space Propagation Loss, which is easy to predict.
4. The PAA-DB forms fixed beams, the envelope of which corresponds to similar beams of the PAA, and the half-power cross-section has an elliptical shape, close to a circle, for beams with a deviation within ±45° [9, 10].

Based on these factors, the following mechanism for the DS Edge Satellites’ angular coordinates measuring inside the BRCS can be proposed.

The DAA forms a set of fixed beams that overlap in space. As can be seen from Table 1, the angular distance between the orientation direction of the fixed beams approximately corresponds to the beams’ width at half power level $\theta_3$. Therefore, the radio signal transmitted by the edge satellite is received in at least two adjacent beams with different levels of amplitude or power. The signal level being received depends on the angle magnitude of the direction deviation to the edge satellite from the orientation direction of the PAA-DB beam.

The Received Power of the $i$-th Edge Satellite signal, which is received in the PAA-DB $k$-th beam, is determined by the basic equation of the radio link for the propagation of radio signals in free space and has the following form

$$P_x = \left( \frac{P G(\theta_{ES})}{L_{FS_{ES}}} \right) G_{lp}(\Delta \theta_k).$$ (6)
where $P_t$ is the Radiated Power of the Edge Satellite; $G_t(\varphi_{ES})$ is the gain of the Edge Satellite Transmitting Antenna Gaine towards the Core Satellite; $G_{tp}(\Delta\theta_k)$ is the Antenna Gain of the $k$-th PAA-DB beam. The Antenna Gain is related to the Antenna Array Factor as follows \cite{10,11}:

$$G_k(\Delta\theta_k) = \sqrt{G_{tp}(\Delta\theta_k)} , \quad (7)$$

$L_{FS_{ES}}$ is the Free Space Propagation Loss of a radio signal during propagation from the Edge to the Core Satellite:

$$L_{FS_{ES}} = \left( \frac{4\pi R_i}{\lambda} \right)^2 , \quad (8)$$

where $R_i$ is the distance from the $i$-th Edge to the Core Satellite.

Certaine distance between the Edge and Core Satellites enables the calculate of the Predicted Received Power Level of the Edge Satellite Signal

$$P_{r_{\text{max}}} = \left( P_t G_t(\varphi_{ES}) \right) \left( \frac{1}{L_{FS_{ES}}} \right) G_{\text{max}k} , \quad (9)$$

where $G_{\text{max}k}$ is the maximum Antenna Gain of the $k$-th PAA-DB beam (provided that the direction towards the Edge Satellite coincides with the orientation direction of the $k$-th PAA-DB beam).

Thus, when receiving a signal from the Edge Satellite, in the $k$-th PAA-DB beam, a difference appears between the received signal and the predicted signal level

$$\Delta P_\lambda = P_{r_{\text{max}}} - P_t , \quad (10)$$

This difference results from the deviation of the direction to Edge Satellite from the orientation direction of the $k$-th PAA-DB beam. As appears from the relational expression for $P_\lambda$, the decrease in the power of the received signal depends on the decrease in the gain of the $k$-th PAA-DB beam in the direction towards the Edge Satellite. By estimating the magnitude of this decrease, it is possible to determine the magnitude of the Direction Deviation Angle to the Edge Satellite from the orientation direction of the $k$-th beam.
When solving several tasks, a simplified description of the PAA-DB Array Coefficient Envelope is of interest. To simplify direct and inverse calculations, a linear-piecewise approximation of the Array Coefficient Normalized Envelope can be proposed.

The discreteness of the envelope argument $\Delta \theta_N$ readings - the angle of deviation from the orientation direction of the $\alpha_k$, $k$-th PAA-DB beam, is selected from the condition:

$$\Delta \theta_N = \frac{\Delta \theta_1}{N \cos \alpha_k},$$

where $\Delta \theta_1$ is the angle reading step, $\Delta \theta_1 = 0.08$ rad; $N$ is the dimension of the linear equidistant half-wave PAA-DB.

The value of the normalized envelope of the PAA-DB Array constant $\hat{G}(\theta)$ at the angle of deviation $\Delta \theta$ value from the orientation direction of the $k$-th beam $\alpha_k$ can be approximated by the following relational expression:

$$\hat{G}(\theta) = \hat{G}_i - \Delta \hat{G}_i \cdot |x|,$$

where $i$ is the number of the discrete component of the approximation function argument $i = \left\lfloor \frac{\theta}{\Delta \theta_N} \right\rfloor$, $\lfloor \cdot \rfloor$ identifies the integer part of the division; $x$ is a residue of division:

$$x = \theta - i \Delta \theta_N = \theta - \left\lfloor \frac{\theta}{\Delta \theta_N} \right\rfloor,$$

$\hat{G}_i$ is a component that prosess the values in accordance with Table 2.

<table>
<thead>
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<th>$i$</th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>0.9359</td>
<td>0.9086</td>
<td>0.8768</td>
<td>0.8410</td>
</tr>
<tr>
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<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
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<td>16</td>
<td>17</td>
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<tr>
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<td>0.7585</td>
<td>0.7126</td>
<td>0.6642</td>
<td>0.6136</td>
<td>0.5614</td>
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<td>20</td>
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<td>23</td>
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<tr>
<td>$\hat{G}_i$</td>
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<td>0.2913</td>
<td>0.2386</td>
<td>0.1875</td>
<td>0.1381</td>
<td>0.0910</td>
<td>0.0464</td>
<td>0.0047</td>
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</tr>
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</table>

Value $\Delta \hat{G}_i$ is the coefficient of inclination of the straight line segment in the $i$-th approximation segment. This one prosess the value
\[ \Delta G_i = \Delta G_{0i} \cdot \cos \alpha_k, \]  

where \( \Delta G_{0i} \) is the approximation coefficient for the \( i \)-th section, which is selected in accordance with Table 3.

\[ G \]

\[ \begin{array}{cccccccc}
   i & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
   \Delta \hat{G}_{0i} & 0 & 0.0325 & 0.0979 & 0.1616 & 0.2238 & 0.2850 & 0.3421 & 0.3971 & 0.4479 \\
   i & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 \\
   \Delta \hat{G}_{0i} & 0.4946 & 0.5358 & 0.5738 & 0.6054 & 0.6321 & 0.6529 & 0.6683 & 0.6775 & 0.6808 \\
   i & 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 \\
   \Delta \hat{G}_{0i} & 0.6788 & 0.6708 & 0.6583 & 0.6396 & 0.6160 & 0.5892 & 0.5575 & 0.5216 \\
\end{array} \]

The given approximation of the PAA-DB coefficient can be used to determine the Antenna Gain value via the angle of deviation from the PAA-DB beam orientation direction and vice versa, to determine the deviation angle value via the certain Antenna Gain value in the fixed PAA-DB beam.

Using the presented Piecewise Linear Approximation method for the Normalized Envelope of the PAA-DB Array constant, it is possible to solve the inverse problem: the determination of the deviation angle \( \Delta \theta_k \) of the direction to the Edge Satellite from the orientation direction of the \( k \)-th PAA-DB beam. Taking into account the BW features of the PAA-DB beams, the estimate of the direction deviation to the Edge Satellite from the orientation of the neighboring \( k \pm 1 \)-th fixed PAA-DB beam \( \Delta \theta_{k \pm 1} \) can be similarly obtained. For a flat PAA-DB, at least three estimates will be obtained: an estimate for the \( k \)-th beam, for which the Edge Satellite locates within the half-power beamwidth zone; \( \Delta \theta_k \leq \theta_3 \), and two more estimates for two neighboring beams \( k \pm 1 \), for which the condition \( \theta_2 \leq \Delta \theta_k \leq \theta_1 \) is satisfied.

Fig. 3 shows an example of receiving the Edge Satellite Signal from linear uniform half-wave PAA-DB of the \( N=16 \) dimension. PAA-DB establishes 16 beams (see Table 1). Fig. 3 demonstrates the BW at Gain Level -3dB, which determines the BW of the fixed beams in terms of -3dB level. The Edge Satellite Signal is being received at an angle of 20° (in Fig. 3. the receiving angle of the Edge Satellite Signal is marked with a vertical straight line). As can be seen from Fig. 3, the Edge Satellite Signal is being received within the BW beam No +3 main part, i.e., within the beamwidth \( \theta_3 \), and is being received within the BW at the level of the first zeros \( \theta_0 \) of the +2 beam.
V. Calculation of the Edge Satellite angular coordinates based on the measurements results

To determine the Edge Satellite Angular Coordinates inside the BRCS of the Core Satellite, the ellipses or circles intersection method can be used. Fig. 4 shows a simplified model. The Edge Satellite is located in the visibility zone of the PAA-DB beams, which have a close location relati to the normal to the PAA-DB aperture plane. In this case, the cross-sections of the beams are practi of a circular shapecal. Fig. 4 shows cross-sections of adjacent beams’ radiation patterns at the -3dB level. The Edge Satellite is located within the beam A limits (see Fig. 4). The Received Signal Power Level determines the deviation angle from the A beam’s orientation direction and is represented by the ring inside beam A (the ring is highlighted in gray) on the BW cross-sectional plane.

The Edge Satellite Signal is also received inside two neighboring beams: beam B and beam C (see Fig. 4). However, the deviation angle of the direction towards the Edge Satellite in these beams goes beyond the -3dB area and on the cross-sectional plane defines two circles around beams B and C (red and green circles, respectively). The intersection point K of the three circles on the cross-sectional plane determines the edge satellite angular position inside the root satellite’s ACS.
Fig. 4. The placement of the DAA fixed beams’ sections when determining the angular parameters of the edge satellite’s relative position inside the root satellite’s ACS

The solution to this problem can be obtained through the Solving Triangles method [14]. The distance between the orientation directions of the fixed beams $\Delta r_{\pm 1}$ is known. In Fig. 4 they are shown as the segments $AB$ and $AC$, which correspond to the angles $\Delta r_{AB}$ and $\Delta r_{AC}$, respectively. The radii of the circles are known, which correspond to the angles $\Delta \theta_A$, $\Delta \theta_B$, $\Delta \theta_C$. It can be shown that the intersection points of two circles, which correspond to the angles $\Delta \theta_A$ and $\Delta \theta_B$ in a rectangular coordinate system centered at point $A$, are given by formulations:

$$y_{1,2} = \frac{2\Delta \alpha_{ABy} c}{2\Delta \alpha_{ABx} + \Delta \alpha_{ABy}} \pm \sqrt{\left(\frac{2\Delta \alpha_{ABy} c}{2\Delta \alpha_{ABx} + \Delta \alpha_{ABy}}\right)^2 - 4\left(\Delta \alpha_{ABy}^2 + \Delta \alpha_{ABx}^2\right)\left(c^2 - \Delta \theta_{A}^2 \Delta \alpha_{ABx}^2\right)}$$

$$x_{1,2} = \frac{c - y_{1,2} \Delta \alpha_{ABx}}{\Delta \alpha_{ABy}},$$

where $\Delta \alpha_{ABx}$ is the angle between the orientation direction of the beams $A$ and $B$ along the $X$ axis; $\Delta \alpha_{ABy}$ is the angle between the orientation direction of the beams $A$ and $B$ along the $Y$ axis; $c$ is a parameter that processes a value.
Two intersection points between circles at the points A and C, the radii of which correspond to the values $\Delta \theta_A$ and $\Delta \theta_C$, respectively, are determined by a similar method, and the shifting of the point C along the X and Y axes in a rectangular coordinate system with the origin point A corresponds to $\Delta \alpha_{AC}$ and $\Delta \alpha_{AB}$, respectively. The common point of intersection of three circles with centers at points A, B, C and radii $\Delta \theta_A$, $\Delta \theta_B$, $\Delta \theta_C$ is point K (see Fig. 4), the coordinates of which determine the angular position of the edge satellite relative to the orientation direction of the beam A. With the fixed beam A orientation known, which is determined by the angles $\alpha_{A_x}$ and $\alpha_{A_y}$ along the DAA aperture plane axes, X_ν and and Y_ν, respectively, it is possible to determine the angles $\theta$ and $\varphi$ that determine the orientation direction of the vector $\vec{R}$ to the edge satellite’s current position.

**Conclusion**

1. An important aspect of ensuring satellite flight safety in the framework of the Distributed Satellite is the relative satellite position and movement measurements, one relative to the other. Such measurements can be fulfilled inside the Body Reference Coordinate System of the Core Satellite. The recalculations of the obtained values into the Orbital Coordinate System of the Core Satellite can be carried out subject to the Attitude Control System of the Core Satellite. Obtaining information about the current coordinates of all the DS satellite subordinates makes it possible to solve tasks on predicting the orbit of each satellite, take measures to prevent collisions and dangerous flyby rendezvous, and ensure the safety of the flight of the entire DS micro-constellation outside the system GCC visibility zone.

2. To improve the efficiency of the Internal Radio Network of a Distributed Satellite, the Phased Antenna Array with Digital Beamforming may be applied via the Spatial Separation technique. In addition to the information exchange provision between the Core and Edge Satellites, the PAA-DB can provide the measurement of the angular position parameters of the Edge Satellites relatively to the Core Satellite as part of a Distributed Satellite to determine the current coordinates of the Edge Satellites in the Body Reference Coordinate System of the Core Satellite.

3. For measurement of the angular parameters of the Edge Satellites position in the BRCS of the Core Satellite, a modified monopulse positioning finding method for the Edge Satellite Signal can be applied. The necessity to modify the monopulse method called forth the fact that the PAA-DB forms fixed beams with unchangeable orientation. The modified method is based on the prediction of the Edge Satellite’s Received Signal Level based on the slant range measurement and the measurement of the received signal level in adjacent beams, the Radiation Patterns of which do not intersect at the -3dB Level, but intersect at
the first zeros level of the Radiation Pattern. The level decrease in the received signal relative to the predicted reception level is an indicator, based on which the angular deviation of the emitter position, the Edge Satellite, from the orientation direction of the PAA-DB fixed beam is determined.

4. To reduce the calculation complexity of the Deviation Angle of the emitter, the Edge Satellite, from the orientation direction of the PAA-DB fixed beam, the Piecewise Linear Approximation method is proposed. The proposed method makes it possible to approximate, with sufficient accuracy for practice, the envelope of the normalized lattice constant of dimension 8 or more.

5. A method is proposed for the determination of the Edge Satellite’s Angular Coordinates based on the use of a modified monopulse method through the measurement results application of the direction deviation angle to the Edge Satellite relative orientation directions of the PAA-DB fixed beams, obtained at least in three adjacent beams.

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