

ТЕХНІЧНІ НАУКИ

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THE METHOD OF JOINT DETERMINATION OF THE HYDRAULIC PERMEABILITY OF A POROUS STRUCTURE AND ITS CAPILLARY RADIUS

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У статті розроблено новий метод визначення гідравлічної проникності пористої структури і її капілярного радіуса на основі рівняння К. Л. Унарокова, що описує кінетику всмоктування рідини в капілярно-пористий зразок проти сил тяжіння. Визначення цих фізико-технічних характеристик пористої структури здійснюється на основі результатів двох вимірювань координати руху капілярного фронту і часу від початку процесу в рамках експерименту по всмоктуванню рідини в зразок. Обчислення гідравлічної проникності і капілярного радіуса пористої структури проводиться на основі отриманої в даній роботі точної апроксимації рівняння Унарокова біном Ньютона

Ключові слова: пориста структура, проникність, капілярний радіус, кінетика всмоктування, рівняння Унарокова, апроксимація

1. Introduction

Experimental methods for determining the physico-technical characteristics of a porous structure, such as hydraulic permeability and its capillary radius, are of great practical importance; therefore, in many cases they are standardized [1]. After all, artificial porous materials [2] are very widely used in the elements of modern technological equipment. In particular, they are used in various filters, textile materials, heat pipe wicks, etc. Therefore, it is very important to know the characteristics that are realized in the processes of filtration, impregnation, and also in the processes of absorption of liquid into porous structures against gravity. In addition, it is important knowledge and characteristics of porous materials that exist in nature, such as soils, porous rocks, etc. In this regard, the development of new methods for determining these characteristics is quite relevant. Especially in cases when it comes to the creation of such methods for the complex determination of physico-technical characteristics, as a result of the realization of which not one but several characteristics are obtained at once. Because it is known from the planning theory of an experiment that the statistical error of its results (variance) in this case is significantly lower than in the direct measurement methods [3]. This is especially important in cases where the measurement methods used in the joint determination method have high accuracy. The simplicity of the measurement methods used in such methods is also important. The method of joint determination (more precisely, a method of indirect measurement) of the physico-technical characteristics of the porous structure, which is developed in this article, belongs to complex methods.

2. Literature review

In the general case, porous materials are divided into materials with open porosity (porous structures) and materials with closed porosity (porous media). It is obvious that only porous structures have hydraulic and capillary characteristics. To measure the values of the physico-technical characteristics of porous structures in the framework of a one-dimensional experiment, samples are used in the form of porous plates (filter materials), cores (rocks) and one-dimensional samples of constant cross section (heat pipe wicks)

The existing methods for determining the hydraulic and capillary characteristics of porous structures are based on several regularities describing the interaction of liquids with porous structures in the processes of pumping, capillary impregnation and absorption against gravity [2].

French hydrologist Darcy H. P. G., 1856 established the linear law of filtration of a fluid in a porous structure, according to which the flow rate of pumped water is proportional to the pressure differential $Q=K \cdot (A/L) \cdot \Delta P$. Then, the expression of the filtration coefficient K ($K=k/\mu$, where k – the hydraulic permeability of the porous structure, m^2 ; μ – the dynamic viscosity of the fluid, Pa·s) [2].

Later, the American hydrogeologist Slichter C. S., 1899 established that the hydraulic permeability k is equal to the product of the square of the characteristic particle size d^2 of the porous structure and the function F (m) from its porosity m , whose expression is determined by the type of porous structure. And only recently a new concept of dimensionless permeability (Slichter number) was introduced into hydrology: $Sl=k/d^2=F(m)$ [5].

Currently, there are several methods for determining separately – capillary, and separately – the hydraulic characteristics of porous structures. The most well-known method for determining the capillary radius of the R_C is a method based on measuring the height of the capillary rise of fluid in a sample against gravity, in which the porous structure is modeled by a bunch of capillaries of the same radius. In this case, the capillary radius is determined on the basis of J. Jurin's formula (J. Jurin, 1717) [2], ($2\sigma R_C = \rho g h_{\max} \sin \alpha$, where R_C is the capillary radius, $R_C = R \cdot \cos \theta$, R – the meniscus radius, θ – the wetting angle of the surface of the porous structure). There are other known methods for determining R_C , for example, the method of mercury pushing into a porous sample [2].

The best-known method for the experimental determination of hydraulic permeability is the method of pumping fluid through a sample of a porous structure, based on Darcy's law: $W = -(k/\mu) \cdot \text{grad} P$ [5]. A method for determining the permeability of a porous structure in the process of fluid flow through a sample of a porous material under the action of gravity has also been patented [6]. The main disadvantage of the existing methods is that, in general, the porous structure is anisotropic, therefore, its permeability measurement must be carried out in the direction of the capillary transport of fluid. In the known methods for determining k , the implementation of this requirement is difficult.

The analysis of the special literature and patent sources showed that currently there are no known methods for the joint determination of the hydraulic and capillary characteristics of porous structures based on the analysis of the kinetics of sample absorption.

3. The aim of research

The aim of research is development of a method for jointly determining the capillary radius and hydraulic permeability of a porous structure.

To achieve the aim, the following tasks are set:

1. To analyze the possibility of experimentally determining two characteristics of a porous structure based on primary data on the absorption of liquid into a porous sample against gravity.

2. Within the framework of the Newton's binomial, which is reduced to a linear form, obtain an exact approximation for the nonlinear Unarokov equation, which describes the kinetics of the capillary absorption of a liquid.

3. To obtain analytical expressions for determining the capillary radius and hydraulic permeability of the porous structure based on the evaluation of the experimental data of the absorption curve for one experiment.

4. Materials and methods

The process of capillary absorption of fluid into a one-dimensional sample of a porous structure against gravity is described by the well-known Unarokov equation [7]:

$$\ln[1 - (x/x_0)]^{-1} - x/x_0 = t \cdot (k \rho g \cdot \sin \alpha) / (\mu \cdot m \cdot x_0) \quad (1)$$

where x – the movement of the front of the meniscus in the sample of the porous structure (m), α – the angle between the one-dimensional sample and the horizon (radian), x_0 – the maximum movement of the liquid in the sample of the porous sample (m), $x_0 = h_{\max} / \sin \alpha$, h_{\max} – the maximum rise of liquid vertically, $h_{\max} = 2\sigma / (\rho g R) = a^2 / R$, R – the meniscus radius (m), μ – the dynamic viscosity of the fluid ($\text{Pa} \cdot \text{s}$), g – the acceleration of gravity (9.8 m/s^2), $t(c)$ – absorption duration.

Since the x coordinate is represented implicitly in the Unarokov equation, equation (1) can't be used directly for the joint determination of the capillary radius R_C and the hydraulic permeability of the k porous structure. Therefore, it is necessary to find an approximation of this formula, on the basis of which it would be possible to solve the problem posed of determining a pair of unknown absorption kinetics of R_C and k .

Thus, the main task of this article is development of an approximation of the Unarokov equation, describing the process of liquid absorption into a porous sample against gravity, which would be sufficiently accurate as a basis for a method of experimentally determining such physico-technical characteristics of a porous structure as hydraulic permeability and capillary radius.

5. Research results

Since in this case the problem is not exactly solved, it is possible to solve it approximately. To obtain an approximate solution of the problem, the logarithmic function can be expanded into a power series and use the two or three first terms of the obtained decomposition.

$$x^2 \left[1 + \frac{2}{3} (x/x_0) + \frac{2}{4} (x/x_0)^2 + \frac{2}{5} \times \right. \\ \left. \times (x/x_0)^3 + \dots \right] = t \cdot (k \rho g \cdot x_0 \cdot \sin \alpha) / (\mu m) \quad (2)$$

However, the resulting power series “converges” very slowly and it is not possible to use its fragments for the approximate solution of the Unarokov equation with respect to the unknown x/x_0 .

To obtain a more accurate correlation than on the basis of series fragments, and explicitly, it is possible to use the well-known approximation method for the sum of power series, such as approximation by rational functions [8] (the universal Pade method). But in this case it turned out to be more convenient to use another method of approximation of the power series (according to its first three members), developed by the author of the article [9], for example, within the framework of such a simple function as binomial theorem.

By approximating the sum of the power series \sum in square brackets with the Newton's binomial, let's obtain the following formula:

$$x^2 \cdot [1 - x/(1,2x_0)]^{-0,8} = t \cdot (k \rho g \cdot x_0 \cdot \sin \alpha) / (\mu m) \quad (3)$$

This formula coincides with the Unarokov equation at $x =$ and is a fairly accurate approximation for $x > 0$. In this case, the approximation gives a slightly overestimated result, and the approximation error increases with increasing variable. For $(x/x_0) = 0.5$, the error will be only

0.4 %, for $(x/x_0)=2/3$ it will be 1.5 %, and only at $(x/x_0)=3/4$ will it reach 3 %, but here it should be taken into account that the range of change of x is limited: $(x/x_0)_{\max} \leq 1$.

If the resulting expression $x^2 \cdot [1-x/(1,2x_0)]^{-0,8} = t \cdot (k \cdot \rho g \cdot x_0 \cdot \sin \alpha) / (\mu m)$ "flip", the left and right parts, respectively, and move x^2 in the right side of the equation, let's obtain:

$$[1-x/(1,2x_0)]^{0,8} = (x^2/t) \cdot [(\mu m)/(k \cdot \rho g \cdot x_0 \cdot \sin \alpha)]$$

And if both parts of this expression are raised to a power of 5/4 and multiplied by $1,2 x_0$, and also the term $1,2 x_0$ is moved from the left side to the right side of the equation, let's obtain the following:

$$x = 1,2x_0 - 1,2x_0 \cdot (x^2/t)^{1,25} \cdot \times [(\mu m)/(k \cdot \rho g \cdot x_0 \cdot \sin \alpha)]^{1,25} \quad (4)$$

After carrying out additional transformations let's finally obtain:

$$x = 1,2x_0 - 1,2 \cdot (x^2/t)^{1,25} \cdot x_0^{0,25} \times [(\mu m)/(k \cdot \rho g \cdot \sin \alpha)]^{1,25} \quad (5)$$

Let's introduce a new notation for a complex variable that includes the time: $z = (x^2/t)^{1,25}$. In this case, the ratio obtained above will be written in the form of a linear equation, which will allow to separate the capillary and hydraulic characteristics of the porous structure:

$$x = c_1 - c_2 \cdot z \quad (6)$$

where the coefficients are $c_1 = 1,2x_0$, and $c_2 = 1,2 \cdot x_0^{0,25} \cdot [(\mu m)/(k \cdot \rho g \cdot \sin \alpha)]^{1,25}$.

If the experimental data on the kinetics of liquid absorption into a capillary-porous structure, i.e., the points $x_i = f(z_i)$, are presented in the coordinate system $x-z$, where the function x is the ordinate axis and the variable z is the abscissa axis, then they should lie "on the straight line, which will cut off the $1.2 x_0$ segment on the ordinate axis, and on the abscissa axis the segment $z_0 = c_1/c_2 = [(k \cdot \rho g \cdot x_0 \cdot \sin \alpha) / (\mu m)]^{1,25}$.

This result can be obtained in another way. It is known [7] that for small values of x/x_0 and t there is a linear dependence $x^2 \sim t$. Indeed, if rewrite the result obtained above (2) in the form:

$$x^2/t = k \cdot \rho g \cdot x_0 \cdot \sin \alpha / \{(\mu m) \times \times [1 + \frac{2}{3} (x/x_0) + \frac{2}{4} (x/x_0)^2 + \frac{2}{5} (x/x_0)^3 + \dots]\}, \quad (7)$$

and use the well-known L'Hôpital rule regarding the left-hand side of expression (7), it is possible to set the limit of the relation $x^2/t = (z_0)^{0,8}$, subject to the condition $x \rightarrow 0$:

$$\lim(x^2/t) |_{x \rightarrow 0} = k \cdot \rho g \cdot x_0 \cdot \sin \alpha / (\mu m). \quad (8)$$

The hydraulic permeability of the porous structure k can also be determined on the basis of the value of z_0 obtained above directly from the exact Unarokov formula:

$$k = t \cdot (\rho g \cdot \sin \alpha) / \{x_0 \cdot \ln[1 - (x_i/x_0)]^{-1} - x_i/x_0\}, \quad (9)$$

where x_i – the displacement of the front of the meniscus in the porous structure, which must meet the condition $x_i/x_0 \leq 2/3$.

The curve, which describes the Unarokov equation, and the straight line, which describes its approximation – approximation by the Newton's binomial, in the $x-z$ coordinate system have the form (Fig. 1):

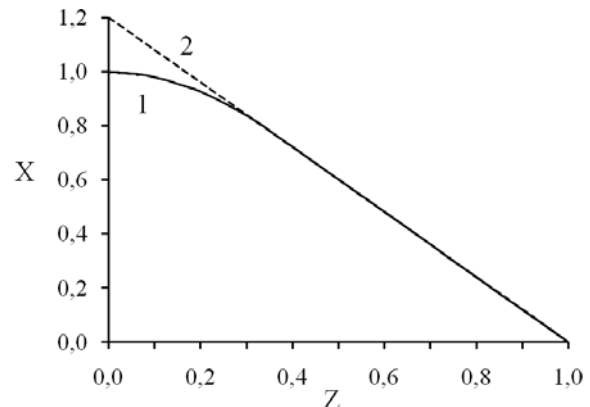


Fig. 1. Curves the Unarokov equation (1) and its approximation (2) with the Newton's binomial in $x-z$ coordinates: 1 – Unarokov equation; 2 – linear approximation

After determining from the graph (Fig. 1) the value of x_0 from the ratio obtained above, it is possible to easily calculate the value of the capillary radius from the analytical expression

$$x_0 = a^2 / (R_C \cdot \sin \alpha): \\ R_C = a^2 / (x_0 \cdot \sin \alpha). \quad (10)$$

The value of the hydraulic permeability k of the porous structure can be calculated on the basis of the values of the segments x_0 and z_0 obtained from the graph from the formula:

$$k = (\rho g \cdot x_0 \cdot \sin \alpha) / [(\mu m) \cdot (z_0)^{0,8}]. \quad (11)$$

By the way, the capillary radius and hydraulic permeability of the porous structure can be easily determined only by two points of the straight line $x = f(z)$: (x_1, z_1) and (x_2, z_2) .

Writing equations for two points of a straight line, let's obtain a system of two equations with two unknowns:

$$\begin{aligned} [1 - x_1/(1,2x_0)]^{0,8} &= (x_1^2/t_1) \cdot [(4k)/(\rho g \cdot x_0 \cdot \sin \alpha)]; \\ [1 - x_2/(1,2x_0)]^{0,8} &= (x_2^2/t_2) \cdot [(4k)/(\rho g \cdot x_0 \cdot \sin \alpha)]. \end{aligned} \quad (12)$$

This system is quite easily solved with respect to the unknown ho by dividing the first equation by the second – the right and left sides, respectively:

$$[1-x_1/(1,2x_0)]^{0,8}/[1-x_2/(1,2x_0)]^{0,8}=(x_1^2t_2)/(x_2^2t_1) \quad k=t_1 \cdot (gR_c^2 \cdot \sin\alpha) / \{x_0 \cdot \ln[1-(x_1/x_0)]^{-1} - x_1/x_0\} \quad (14)$$

Raising both sides to the 1.25 power gives the following:

$$[1-x_1/(1,2x_0)]/[1-x_2/(1,2x_0)]=[(x_1^2t_2)/(x_2^2t_1)]^{1,25}$$

Completion of transformations leads to the expression:

$$[1-x_1/(1,2x_0)] \cdot (x_2^2t_1)^{1,25} = [1-x_2/(1,2x_0)] \cdot (x_1^2t_2)^{1,25}$$

From this relationship, it is already possible to obtain an expression for the unknown x_0 :

$$x_0 = \frac{x_1 \cdot (x_2^2t_1)^{1,25} - x_2 \cdot (x_1^2t_2)^{1,25}}{[1,2 \cdot \{(x_2^2t_1)^{1,25} - (x_1^2t_2)^{1,25}\}]} \quad (13)$$

And now, after calculating the x_0 value from the obtained relation, let's express the capillary radius R_c from the expression $x_0 = a^2 \cdot \cos\theta / (R_c \cdot \sin\alpha)$:

$$R_c = \sqrt{(x_0 \cdot R \sin\alpha / a^2 \cdot \cos\theta)},$$

as well as on the basis of the obtained height value x_0 , the hydraulic permeability k of the porous structure can be determined directly from the Unarokov exact formula:

where x_0 – the above maximum possible displacement of the front of the menisci against gravity in a porous medium,

x_1 и t_1 – movement of the meniscus (m), time (s) since the start of the process of absorption into the capillary-porous structure to the first measurement, respectively,

x_1 и t_1 – movement of the front of the meniscus (m) and time (s) since the beginning of the process of absorption into the capillary-porous medium to the second dimension, respectively,

R_c – the capillary radius (m), g – the acceleration of gravity (9.8 m/s^2),

θ и α – the contact angle of wetting and the angle of inclination of the capillary to the horizon, respectively (degree).

Let's calculate the capillary and hydraulic characteristics of a porous structure in the process of joint determination of capillary radius and hydraulic permeability based on experimental data on the absorption of ethyl alcohol (96 %) into a porous sample ($m=22-28$ %) made of Бр0Ф10-1 bronze powder [11].

Experimental data describing the kinetics of ethanol absorption in a porous sample against gravity are presented in Table 1.

Table 1

The points of the ethanol absorption curve (96 %) against gravity in a porous sample of Бр0Ф10-1 bronze powder ($m=22-25$ %) at a temperature of 20°C .

H, mm	10.5	21.5	30.5	40.0	50.5	61.0	70.5	80.0
τ , s	3.0	7.5	13.0	19.5	31.5	52.5	77.0	111.5

As shown by a preliminary analysis, the data of the absorption curve No. 1 in the range of $40-80$ mm "falls" on the straight line $x=x_0 - \text{const} \cdot (x^2/t)$. Therefore, to determine the desired characteristics of the porous structure, two experimental points can be used: the first – ($x_1=40$ mm, $t_1=19.5$ s) and the second – ($x_2=80$ mm, $t_2=111.5$ s). These two points are necessary and sufficient to solve a system of two equations with two unknowns (definitions of capillary radius R_c and hydraulic permeability k).

As a result of solving the system of two equations, the expression x_0 is obtained in advance, on the basis of which the values of R_c and k are calculated.

Assuming that the sample is installed vertically (in this case $\sin\alpha=1$), and also that the average porosity is $m=25$ %, let's calculate the x_0 value.

Substituting the coordinates of the points of the kinetic curve (1) into the expression for x_0 , let's obtain $x_0=125.9$ mm. Here it must be said that h_0 is not the height of the equilibrium lifting of the liquid in the porous structure, because in the process of soaking the liquid into the sample, it is not the equilibrium contact angle that is realized, but the non-equilibrium "leakage angle". To determine the unknown unknowns, both of these angles must be determined additionally.

According to the [12], one can find the physical properties of liquid ethanol (96 %) at a temperature of

20°C and a pressure of 1 atm: its density is $\rho=808$ (kg/m^3), the dynamic viscosity of ethanol is $\mu=1.198 \cdot 10^{-3}$ ($\text{n}\cdot\text{s/m}^2$), and its surface tension $\sigma=22.8 \cdot 10^{-3}$ (n/m).

First, we determine the value of the capillary radius, and then we calculate the hydraulic permeability value from the original Unarokov equation. $R_c=a^2/(h_0 \cdot \sin\alpha)$.

We get $R=46 \cdot 10^{-6}$ m (46 microns).

On the basis of reference data, as well as the value $x_0=125.9$ mm and the value R , let's calculate the hydraulic permeability of the porous structure k from the original Unarokov equation:

$$\ln[1-(x_1/x_0)]^{-1} - x_1/x_0 = t_1 \cdot (kg \cdot \sin\alpha) / (x_0 \cdot \mu\text{m}) \quad (15)$$

Substituting the values into the formula, let's find that the hydraulic permeability is $k=1,58 \cdot 10^{-8} \text{ m}^2$.

But here it is necessary to bear in mind that the angle θ is not a static edge angle occurring in thermodynamically equilibrium conditions, but a dynamic angle θ^* obtained in non-equilibrium conditions of fluid flowing onto the surface as it is absorbed into the porous structure. Therefore x_0 differs from the equilibrium height of the liquid in the sample of the porous structure against gravity and should be determined additionally.

6. Conclusions

1. A new method for jointly determining at once two physico-technical characteristics of a porous structure is developed: hydraulic permeability k and capillary radius R_C , in which only one experiment is necessary and sufficient to absorb a liquid into a sample of a porous structure against gravity.

2. Sufficiently accurate approximation of the exact Unarokov equation, which describes the process of fluid absorption by the Newton's binomial, makes it possible to obtain an analytical solution of the problem, and

therefore, in essence, is the mathematical basis of the proposed method for determining k and R_C .

3. The developed method of jointly determining the physico-technical characteristics of a porous structure provides an accurate determination of its hydraulic permeability and capillary radius based on the results of just two measurements of the displacement of the front of the menisci in the framework of a single experiment on fluid absorption into a porous sample against gravity depending on the time from the start of the process .

References

1. DSTU/GOST 3816: 2009 (ISO 811-81). Polotna tekstil'nye. Metody opredeleniya gigroskopicheskikh i vodoottalkivayushchikh svoystv. URL: http://online.budstandart.com/ru/catalog/doc-page?id_doc=73715
2. Fizicheskaya entsiklopediya. In 5 vols. / ed. by Prokhorov A. M. Moscow: Izd. BSE, 1988. 3456 p.
3. Hamraoui A., Nylander T. Analytical Approach for the Lucas–Washburn Equation // Journal of Colloid and Interface Science. 2002. Vol. 250, Issue 2. P. 415–421. doi: <http://doi.org/10.1006/jcis.2002.8288>
4. Nalimov V. V., Golikova T. I. Logicheskie osnovaniya planirovaniya eksperimentov. Moscow: Metallurgiya, 1976. 128 p.
5. Rukovodstvo po opredeleniyu koeffitsienta fil'tratsii vodonosnykh porod metodom opytной otkachki. P-717-80. Moscow: Energoizdat, 1981. 161 p.
6. Sposob opredeleniya pronitsaemosti poristykh materialov: A. S. No. 744286. Semena M. G. et. al. 1980. BI No. 24.
7. Unarokov K. L. Kapillyarnyy pod'em zhidkosti v poristoy srede // Zhurnal fizicheskoy khimii. 1979. Vol. LIII, Issue 3. P. 588–591.
8. Baker G. A., Graves-Morris P. Pade Approximants. Addison-Wesley Publishing Company, Inc., 1981. 502 p. doi: <http://doi.org/10.1017/cbo9780511530074>
9. Ludanov K. Method of Obtaining Approximate Formulas // EUREKA: Physics and Engineering. 2018. Vol. 2. P. 72–78. doi: <http://doi.org/10.21303/2461-4262.2018.00589>
10. Sposib spil'nogo viznachennya kapilyarnikh ta hidravlichnikh kharakteristik poristoi strukturi: Pat. No. 86182 UA. MPK: G01N 13/02 / Ludanov K. I. No. a 2013 08517. declared: 08.07.2013; published: 25.12.2013. Bul. No. 24.
11. Kostornov A. H. Strukturnaia hydrodynamika porystikh metallycheskykh materyalov. IV Rozdil. Funktsionalni materialy. Vol. 2 // Prohresyvni materialy i tekhnolohii. Kyiv: Akadempriodyka, 2003. 663 p.
12. Vargaftik N. B. Spravochnik po teplofizicheskim svoystvam gazov i zhidkostey. Moscow: Nauka, 1973. 720 p.

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