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## ELECTRON ACCELERATOR DOSIMETRY IN RADIATION THERAPY: PHOTON BACKSCATTERING

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*На сьогодні 52 % онкологічних пацієнтів отримують радіаційну терапію. Розроблено експериментальну методику визначення фактору оберненого розсіювання для лінійного прискорювача Siemens Oncor Impression Plus. Експерименти були виконані за допомогою водяного фантому. Даний підхід також може бути використано для інших типів прискорювачів.*

*Ключові слова: фактор оберненого розсіювання, багатопелюстковий коліматор, лінійний прискорювач електронів, моніторна камера, радіаційна терапія, рак.*

*Today 52 % of patients with a cancer get radiation therapy. It is developed the experimental procedure for determining the backscatter factor for Siemens Oncor Impression Plus linear accelerator. The experiments were carried out based on water phantom. This technique can be also used for other kinds of accelerators.*

*Keywords: backscatter factor, multi-leaf collimator, linear electron accelerator, monitor chamber, radiation therapy, cancer.*

### 1. Introduction

The appearance of a multi-leaf collimator (MLC) in a linear electron accelerator (LEA) made it indispensable in the radiotherapy. The MLC is an important tool for radiation therapy dose delivery. Originally introduced as a substitute for alloy block field shaping, it is now recognized that this device can also be used for intensity modulated radiotherapy. In either case, it is important to view this equipment as a sophisticated device that requires a number of distinct steps for introduction and continued use in the clinic. Firstly, it is necessary to organized and carried out a series of

acceptance tests for a new accelerator with collimator. Secondly, additional commissioning measurements are needed to model the collimator for treatment planning.

MLC configurations may be categorized as to whether they are total or partial replacements of the upper jaws, the lower jaws, or else are tertiary collimation configurations (Fig. 1). The particular configuration along with other collimator design aspects, such as whether the wedge is internal or external, creates a number of x-ray beam collimation and control configurations. MLC machines may place the tertiary block trays and the gantry housing closer to the patient

than non-MLC machines. In some cases, wedges and compensating filter assemblies are also placed undesirably close to the patient. This limits the extent of some non-coplanar treatment techniques.

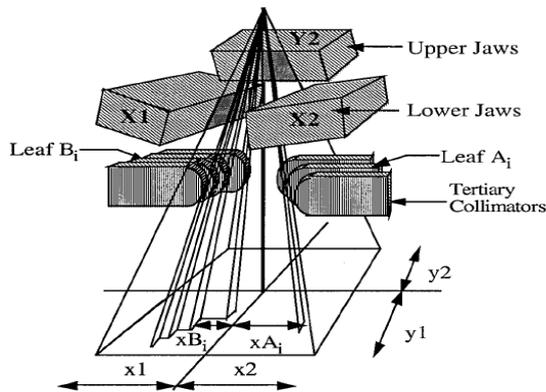


Fig. 1. Flow-chart of a photon collimation system with upper and lower jaws and a tertiary MLC. The Y1 jaw has been omitted for clarity. At the plane the field dimensions are indicated from isocenter

Upper Jaw configuration entails splitting the upper jaw into a set of leaves. Currently the Elekta MLC is designed in this manner. In the Philips design, the MLC leaves move in the y-direction (parallel to the axis of rotation of the gantry). A “back-up” collimator located beneath the leaves and above the lower jaws augments the attenuation provided by the individual leaves. The back-up diaphragm is essentially a thin upper jaw that can be set to follow the leaves if they are being ganged together to form a straight edge or else set to the position of the outermost leaf if the leaves are forming a shape.

The primary advantage of the upper jaw replacement configuration is that the range of motion of the leaves required to traverse the collimated field width is smaller, allowing for a shorter leaf length and therefore a more compact treatment head diameter. The disadvantage of having the MLC leaves so far from the accelerator isocenter is that the leaf width must be somewhat smaller and the tolerances on the dimensions of the leaves as well as the leaf travel must be tighter than for other configurations.

The lower jaws can be split into a set of leaves as well. The Siemens and the General Electric (GE) MLC options use this configuration. The GE MLC system is no longer being sold. In both the Siemens design, the leaf ends are straight and are focused on the x-ray source. The Siemens design uses 41 opposed leaf pairs. The inner 41 leaf pairs project to a dimension of 1.0 cm at the plane at isocenter.

All leaves can travel from the full open position (projecting to a field half-width of 20 cm) to 10 cm across the central axis. All the leaves are independently controlled and travel with a speed of up to 1.5 cm/sec. The leaves may be manually positioned with an MLC hand control and these leaf-settings can be uploaded to an information management Record and Verify (R&V) system. The leaf ends as well as the leaf sides match the beam divergence, making the configuration double-focused.

The Varian MLC is an example of a tertiary collimator system (Fig. 2). This device is positioned just below the level of the standard upper and lower adjustable jaws. The major disadvantage of placing the MLC below the standard jaw system is the added bulk. Clearance to the mechanical isocenter is an additional, but minor, problem. Clearance for the Varian MLC depends on the exact combination of beam modifiers used for a particular treatment situation. When the MLC is fitted and a block support tray is added for additional field shaping, clearance to the isocenter is the same as the non-MLC treatment head. Of course, there is no change in clearance when the dynamic wedge feature is used.

In addition to the question of clearance, the diameter of the head at the level of the secondary and tertiary collimator system is increased. Moving the MLC farther from the x-ray target requires an increase in the size of the leaves and a longer travel distance to move from one side of the field to the other. The end result is that a tertiary system decreases the collision free zone. For example, if a blocking tray holder is retained, patients whose treatment positions call for their elbows to extend laterally, such as in breast cancer, may not clear unless the blocking tray holder is removed.

## 2. Measurement description and problem formulation

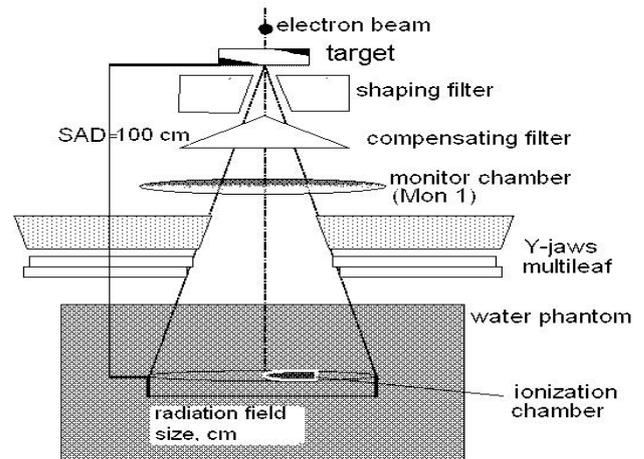


Fig. 2. Scheme of "head" of LEA Siemens Oncor and measurement units (water phantom, ionization chamber)

However, the use of the MLC requires measurement of a lot of dosimetric variables that affect the "primary" dose and increase the time of clinical dosimetry.

High-energy electron hits the target with a material with a high atomic number  $Z$  and produces high-energy X-rays. This process is occurring into the "main part" of LEA consisting of a tungsten target, shaping filter, compensating filter, monitor chamber (Mon 1), "curtains" collimator (Y-jaws), and collimator blades (multi-leaf). The scheme of the device is shown at Fig. 2. A therapeutic beam first enters into shaping filter, located just behind the target, and then – to the compensating filter, which provides a homogeneous beam shape to the ray. Finally, the compensated X-ray emission is arrived

at monitor chamber [1].

Monitor chamber collects an ion charge generated within its air volume. When the total charge assembled in the chamber, corresponds to a given dose (accelerators calibrate in a way that 1 monitor unit (MU) corresponds to a dose of 1 cGy (centi Gray), the LEA finishes the radiation process [2]. The charge is formed mainly by direct scattering of photons from the central section of the filters. Whereas the monitor chamber is near the collimator plates, the inverse scattering from plates to the monitor chamber affects the formation of the charge [3–5].

It was shown [6–12] that the monitor chamber registers an additional charge, formed by the photons and electrons reflected from the upper and lower collimator plates. The presence of the inverse scattering of photons and electrons from materials with of a high atomic number was experimentally investigated for the photons with an energy of 8 MV (conventionally, the energy of diagnostic and therapeutic gamma- and X-rays is expressed in kilovolts or megavolts (kV or MV), whilst the energy of the therapeutic electrons is expressed in terms of megaelectronvolts (MeV).

In the first case, this voltage is the maximum electric potential used by a linear accelerator to produce the photon beam. The beam is produced by a spectrum of energies: the maximum energy is approximately equal to the beam's maximum electric potential multiplied to the electron charge. Thus a 1 MV beam is produced by photons of no more than about 1 MeV, which is about 20 % for 15x15 cm<sup>2</sup> field exposure [10]. However, it should be noted that this study was conducted at a time when LEA mostly used only for research, but since construction of collimators were significantly improved.

Earlier studies by placing a 0.3 mm copper foil between the collimator jaws and the monitor chamber in a Therac-20 linear accelerator (AECL Chalk River, Ottawa, Canada) have shown an increase of 10 % in charge collection by the monitor chamber when the jaw openings were changed from 0x0 cm to 40x40 cm [8].

Patterson & Shragge [3] suggested to disengage a dose rate feedback control so as to maintain constancy in the beam current on the target as well as the photon output. The above method was adopted in several type of linear accelerators [11], and no significant difference in beam delivery time was observed. From the study it was concluded that the beam scattering from the collimator jaws is negligible for Mevatron-VI, Mevatron-XII, and Mevatron-77 (Siemens Medical Systems, Iselin, N.J.) and Varian Clinac-4 (Varian Associates, Palo Alto, California) accelerators.

Also some studies [12, 13] indicated that the backscatter effect from the collimators is negligible for the Varian Clinac-18 accelerator, possibly due to the absorption of the backscattered photons by the finite thickness of the aluminum exit window. Duzenli et al [14] have reported a reduction in dose delivery for photon beams from a Clinac-2100C Varian accelerator equipped with Kapton beam monitor chambers. However, they have reported negligible backscatter effect for the 6 MV beam from Clinac-600C equipped with Mica monitor chambers.

The contribution of the backscattering dose also depends on the design of collimators LEA. For example, LEA Varian collimator comprises two pairs of “curtain“ X and Y, located directly near the monitor chamber, and set of multi-leafs. In this collimator design, the scattered radiation will be more fall to the monitor chamber than in case of the collimator using in LEA Siemens Oncor.

Modern methods of cancer treatment such as intensity modulated radiotherapy (IMRT), stereotactic beam radiotherapy (SBRT) based on the use of “small” radiation fields. In means that the number of backscattered photons and electrons will increase and affect the final dose. It is therefore important to know the percentage in which the monitor chamber will underestimate the final dose, which may lead to an inadequate exposure of patients.

In articles [15–17] we have previously discussed some aspects of optimization of LEA parameters and methods of their use in radiation therapy. The purpose of this paper is an experimental study of backscattering factor for a linear accelerator Siemens Oncor Impression Plus for photon energies of 6 and 18 MV with asymmetric radiation fields.

### 3. Materials and methods

LEA Siemens Oncor Impression Plus has two photon energies (6 and 18 MV) and six electron energies (6, 9, 12, 15, 18, 21 MeV). The upper “curtain“ (Y-jaws) and lower lobe (multi-leaf) are placed at 22.47 cm and 30.27 cm from the monitor chamber, respectively. This LEA is used for three-dimensional conformal radiotherapy (3D CRT), IMRT and radiation electrons of surface cancer diseases.

To evaluate the backscattering factor it is used the PTW PinPoint 31014 ionization chamber, applied for dosimetry of “small” fields, PTW MP3 water phantom and electrometer PTW UNIDOS. The measurements were carried out by the SAD method (source - axis - rotation distance). In this method, an ionization chamber is places at 100 cm from the radiation source at 10 cm depth in the center of the irradiation field, size of which is show at Fig. 1.

The electrometer records the output charge at the ionization chamber for different radiation fields when 100 MV is applied to LEA. Initially the fields values are measured when the multileaf X-jaws is changing and Y-jaws is stable, so that 1x10, 2x10 ... 10x10 cm<sup>2</sup>, after measurements were performed vice versa. Output charges for each field normalized to 100 %, that corresponded to 10x10 cm<sup>2</sup> field (in this case we believe that the contribution of the scattered field into a dose can be omitted).

Since the Y-jaws are closer to the monitor chamber, their contribution into a dose from scattering will be bigger, that is why the output difference at changing the X-jaws and Y-jaws is a factor of backscattering. The measurements were carried out for photons with energies of 6 and 18 MV.

### 4. Results of experimental study of backscattering

The measurement results are introduced into

Tables 1 and 2. Table 1 presents the data of backscattering factor for 6 MV, Table 2 – for 18 MV. For normalized values graphs are constructed in order to be seen clearly that due to changing the field size by "curtain" of collimator (Y-jaws), the charge, which is collected by ionization chamber, is decreased (Fig. 3, 4). One can see that the factor of backscattering is negligible, averaging 0.7 % for photons with both 6 MV and 18 MV energy and increasing radiation field is

generally minimal. But for photons with the energy of 6 MV and field of  $1 \times 10 \text{ cm}^2$ , this value is reached 1.46 %.

Backscattering factor is smaller in LEA Siemens Oncor due to collimator design. Single pair of "curtain" Y-jaws (contrary to two pairs of "curtain" in LEA Varian, i. e. multi-leaf and Y-jaws), gives possibility to place a "curtain" below the monitor chamber, and thus reduce the backscattering.

Table 1

Backscattering factor for 6 MV photon energy

Field, cm	6 MV				Scattering factor, %
	data from chamber		normalized data, %		
	Out (X)	Out (Y-jaws)	Out (X)	Out (Y-jaws)	Out (X-Y)
1x10, 10x1	234.5	229.5	68.467	67.007	1.460
2x10, 10x2	285.5	284	83.358	82.920	0.438
3x10, 10x3	303	300	88.467	87.591	0.876
4x10, 10x4	312.5	310	91.241	90.511	0.730
5x10, 10x5	320	317.5	93.431	92.700	0.730
6x10, 10x6	326.5	324.5	95.328	94.745	0.584
7x10, 10x7	331.5	330	96.788	96.350	0.438
8x10, 10x8	336	335	98.102	97.810	0.292
9x10, 10x9	339.5	339.5	99.124	99.124	0
10x10	342.5	342.5	100	100	0

Table 2

Backscattering factor for 18 MV photon energy

Field, cm	18 MV				Scattering factor, %
	data from chamber		normalized data, %		
	Out (X)	Out (Y-jaws)	Out (X)	Out (Y-jaws)	Out (X-Y)
1x10, 10x1	255.5	281.5	62.469	68.742	-6.273
2x10, 10x2	337.5	344.5	82.518	84.127	-1.609
3x10, 10x3	370	367	90.465	89.621	0.843
4x10, 10x4	383.5	381	93.765	93.040	0.725
5x10, 10x5	391	388.5	95.599	94.872	0.727
6x10, 10x6	397	394.5	97.066	96.337	0.729
7x10, 10x7	401	399	98.044	97.436	0.608
8x10, 10x8	404.5	403	98.900	98.413	0.487
9x10, 10x9	407.5	407	99.633	99.390	0.244
10x10	409	409.5	100	100	0

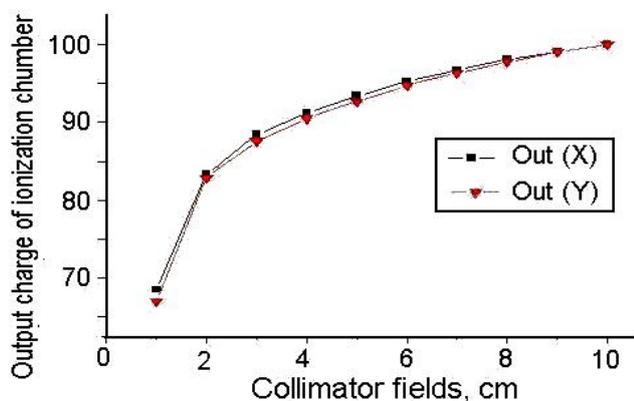


Fig. 3. Standardized data outputs for 6 MV photon energy: Out (X) – different configurations of radiation fields formed by collimator leaves; Out (Y) – different configurations of radiation fields formed by collimator "curtains"

Backscattering factor is smaller in LEA Siemens Oncor due to collimator design. Single pair of "curtain" Y-jaws (contrary to two pairs of "curtain" in LEA Varian, i. e. multi-leaf and Y-jaws), gives possibility to place "curtain" below the monitor chamber, and thus reduce the backscattering.

A different pattern is observed for photons of energy 18 MV, which is clearly seen in Fig. 4. When the radiation field size is changed by "curtain" collimator (Y-jaws), then ionization chamber accumulates a charge whose value is larger than when the field size is formed by collimator multi-leaf for fields  $1 \times 10 \text{ cm}^2$  and  $2 \times 10 \text{ cm}^2$ .

In this case, the backscattering factors are equal to  $-6.27 \%$  and  $-1.6 \%$ , respectively, due to the fact that the PinPoint ionization chamber overestimates the value of the absolute dose of a high-energy photons for "small" fields.

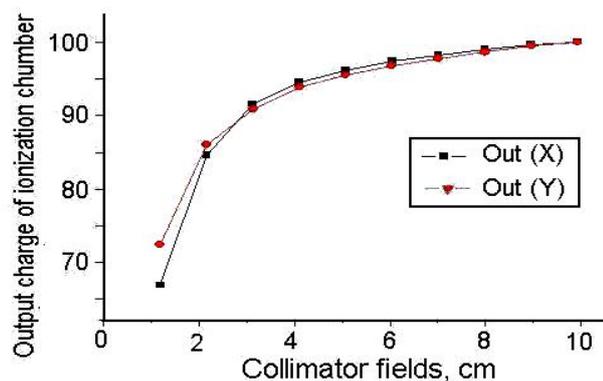


Fig. 4. Standardized data outputs for 18 MV photon energy: Out (X) – different configurations of radiation fields formed by collimator leaves; Out (Y) – different configurations of radiation fields formed by collimator "curtains"

### 5. Conclusions

Based on the experimental studies it can be concluded that the factor of backscattering in LEA Siemens Oncor Impression Plus does not exceed 0.7 % for photons with energies of 6 and 18 MV because "curtain" Y-jaws are not too close to the monitor chamber.

These results show that for dosimetry of LEAs, which use IMRT and SBRT techniques to treat cancer, must be determined the backscattering factor, especially for accelerators, which contain several pairs of "blind" because of their contribution to the final dose can be significant.

The developed method for determining the backscattering factor can be applied to any medical linear accelerator. Obtained values should be taken into account and included in the planning system for correct treatment planning.

Neglecting the backscattering causes the error downward when the therapeutic dose is calculated that may lead to insufficient exposure of patients.

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## МОДЕЛЬ КОНКУРЕНЦИИ В СИСТЕМАХ ТИПА “ПРОИЗВОДИТЕЛЬ-ПЕРЕКУПЩИК”

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*Построены математические модели конкурентных процессов в экономике с использованием известных универсальных моделей, описывающих поведение контрагентов на рынке. На основе математической модели Лотки-Вольтерра и дальнейшего её развития создана математическая модель “производитель-перекупщик”, получена её модифицированная версия, проведены исследования моделей с помощью математического пакета Mathcad. Выявлены неустойчивость поведения контрагентов, и перспективы дальнейшего усовершенствования моделей.*

*Ключевые слова: математическая модель, экономика, конкуренция, модификация, модель Лотки-Вольтерра, производитель, перекупщик, Mathcad, неустойчивость.*

*Mathematical models of competitive processes in the economy using known universal models describing the behavior of counterparties in the market are built. The mathematical model of “producer-second-hand dealer” on the basis of mathematical model by Lotka-Volterra and its further development is created. Its modified version is obtained and model analyses using mathematical package Mathcad is investigated. The behavior instability of the counterparties and some prospects for further improvements of the model are identified.*

*Keywords: mathematical model, economy, competition, modification, Lotka-Volterra model, producer, second-hand dealer, Mathcad, instability.*