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INVESTIGATION OF THE INTERPOLATION REPRESENTATION OF RANDOM PROCESSES WITH NON-EQUIDISTANCE INTERPOLATION KNOTS

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The article deals with some interpolation representations of random processes with non-equidistance interpolation knots. Research is based on observations of the process and its derivatives of the first, second and third orders at some types of knots and observations of the process and its derivatives of the first and second orders at another types of knots
Keywords: *random, process, interpolation, representations, series, knot, non-equidistance, separability, convergence, probability*

У статті розглядаються питання інтерполяційних представлень випадкових процесів за нерівновіддаленими вузлами інтерполяції. Дослідження базується на значеннях процесу та його похідних першого, другого та третього порядку у вузлах інтерполяції одного типу, а також значеннях процесу та його похідних першого та другого порядку у вузлах інтерполяції другого типу
Ключові слова: *випадковий, процес, інтерполяція, представлення, серія, вузол, нерівновіддалений, сепарабельність, збіжність, ймовірність*

1. Introduction

Interpolation representations of a class of random process with non-equidistance interpolation knots are investigated. The necessary results from the theory of entire functions of complex variable are formulated. The function bounded on any bounded region of the complex plane is considered. The estimation of the residual of the interpolation series is obtained. The interpolation formula that uses the value of the process and its derivatives at the knots of interpolation is proved. Considering the separability of the process and the convergence of a row, it is obtained that the interpolation row converges to the random process uniformly over in any bounded area of parameter changing. The convergence with probability 1 of the corresponding interpolation series to a random process in any bounded domain of parameter changes is proved.

2. Literature review

The one of the fundamental results in the Theory of Information Transmission is a theorem of the function expression with a bounded spectre of values in the periodic sequence of initial moments. The significance of that fact was first introduced by Kotel'nikov;V.;A. in [1]. Further, Shannon;C. was investigated these problems in [2, 3]. The Theorem of Kotel'nikov-Shannon is generally well-known [4]. Later, the work, which summarized the Theorem of Kotel'nikov-Shannon, is appeared [5]. In the present time, the investigations related to the construction of interpolation polynomials are attracting significant interest. Many problems concerning the construction of a spline approximation as well as a representation of a motion in 3D-modelling with help of interpolation and approximation [6] and the modern theory of signal transmission [7] are based on the Kotel'nikov-Shannon theorem. The problems of constructing interpolation polynomials with non-equidistance interpolation knots are interested. The present work is concerned on the problems stated above.

3. Aim and research problems

The aim of research – to construct the interpolation representations of stochastic processes along non-equidistant interpolation knots. Two types of interpolation knots are considered as the basis. For the first type, the interpolation formula includes the value of the process and its derivatives of the first, second and third orders. For a second type of knots, the interpolation formula includes the value of the process and its derivatives of first and second orders.

Research problems: proof of the interpolation formula and convergence of the corresponding series with probability 1.

4. Assumptions and methods of research

Let's consider the interpolation representation of random processes [8, 9] on non-equidistance interpolation knots of the type

$$t_{n0} = n\frac{7\pi}{\alpha},$$

$$t_{n1}=n\frac{7\pi}{\alpha}+\frac{\pi}{\alpha}, n\in \mathrm{Z}$$

based on observations of the process and its derivatives of the first, second and third orders at knots $t_{n0}, n\in \mathrm{Z}$ and observations of the process and its derivatives of the first and second orders at knots $t_{n1}, n\in \mathrm{Z}$.

Let's formulate the necessary results from the theory of entire functions of complex variable.

Lemma. Let $f(z)$ be an entire bounded on the real axis function of exponential type with indicator *σ*.

Then, for any *α*, *α*>*σ*, the representation holds true

$$f(z)=\sum_{n=-\infty}^{\infty}\left(-\frac{f(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (z-t_{n0})^4}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\right.$$
$$-\frac{f'(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (z-t_{n0})^3}\times\frac{1}{\sin^3\frac{\pi}{7}}+\frac{f''(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 \pi(z-t_{n0})^2}\times\frac{1}{2\sin^3\frac{\pi}{7}}-$$
$$-\frac{f'''(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (z-t_{n0})}\times\frac{1}{\sin^3\frac{\pi}{7}}-\frac{f\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(z-t_{n0}-\frac{\pi}{\alpha}\right)^2}\times\frac{1}{\sin^4\frac{\pi}{7}}+$$
$$+\frac{f'\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(z-t_{n0}-\frac{\pi}{\alpha}\right)}\times\frac{1}{\sin^4\frac{\pi}{7}}+$$
$$\left.+\frac{f''\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{5}\right)^3\left(z-t_{n0}-\frac{\pi}{\alpha}\right)}\times\frac{1}{2\sin^4\frac{\pi}{7}}\right)\times$$
$$\times\sin^4\frac{\alpha}{7}(z-t_{n0})\sin^3\frac{\alpha}{7}\left(z-t_{n0}-\frac{\pi}{\alpha}\right), \quad (1)$$

where $t_{n0}=n\frac{7\pi}{\alpha},\ n\in Z,$ provided that the interpolation series (1) converges uniformly in any bounded region of the complex plane.

Proving Lemma, as in [10–12] we obtain estimate of the residual of the interpolation series (1), which has the following form

$$|R_n(z)|\le LG(z)C_f\frac{\alpha}{\alpha-\sigma}\frac{1}{n}, \quad (2)$$

where L is a constant,

$$C_f=\sup_{t\in R}|f(t)|,\ G(z)=\left|\sin^4\frac{\alpha}{7}z\times\sin^3\frac{\alpha}{7}(z-\frac{\pi}{\alpha})\right|$$

is a function bounded on any bounded region of the complex plane.

5. Results of the research

Consider a random $\xi(t)$, $t\in R$ with $\mathrm{M}\xi(t)=0$ and covariance function, which representation is

$$B(t,s)=\int_{\Lambda\times\Lambda} f(t,\lambda)\overline{f(s,\mu)}F(d\lambda,d\mu), \quad (3)$$

where Λ is a set of parameters, $F(A_1,A_2)$ is a positive definite additive complex function on $\Lambda\times\Lambda$ such that

$$\int_{\Lambda\times\Lambda}|F(d\lambda,d\mu)|<+\infty. \quad (4)$$

The function $f(t,\lambda)$ relating to *t* is an entire function of exponential type with indicator $\sigma(\lambda)$ such that

$$\sup_{\lambda\in\Lambda}\sup_{-\infty<t<+\infty}|f(t,\lambda)|=C_f<+\infty, \quad (5)$$

$$\sup_{\lambda\in\Lambda}\sigma(\lambda)=\sigma<+\infty. \quad (6)$$

The following theorem holds true.

Theorem 1. Let $\xi(t)$ is a separable random process that satisfies conditions (3)–(6). Then for any $\alpha,\ \alpha>\sigma$ with probability 1, the following representation holds true

$$\xi(t)=\sum_{n=-\infty}^{\infty}\left(-\frac{\xi(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (t-t_{n0})^4}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\frac{\xi'(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (t-t_{n0})^3}\times\right.$$
$$\times\frac{1}{\sin^3\frac{\pi}{7}}+\frac{\xi''(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (t-t_{n0})^2}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\frac{\xi'''(t_{n0})}{\left(\frac{\alpha}{7}\right)^4 (t-t_{n0})}\times\frac{1}{\sin^3\frac{\pi}{7}}-$$
$$-\frac{\xi\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(t-t_{n0}-\frac{\pi}{\alpha}\right)^2}\times\frac{1}{\sin^4\frac{\pi}{7}}+\frac{\xi'\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(t-t_{n0}-\frac{\pi}{\alpha}\right)}\times$$
$$\left.\times\frac{1}{\sin^4\frac{\pi}{7}}+\frac{\xi''\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{5}\right)^3\left(t-t_{n0}-\frac{\pi}{\alpha}\right)}\times\frac{1}{2\sin^4\frac{\pi}{7}}\right)\times$$
$$\times\sin^4\frac{\alpha}{7}(t-t_{n0})\sin^3\frac{\alpha}{7}\left(t-t_{n0}-\frac{\pi}{\alpha}\right). \quad (7)$$

Proof: according to the theorem about spectral representation of random processes [9], we will write the process $\xi(t)$ as follows:

$$\xi(t)=\int_{\Lambda} f(t,\lambda)Z(d\lambda), \quad (8)$$

where $Z(d\lambda)$ is a random measure on Λ, such that $\mathrm{M}Z(A_1)\cdot\overline{Z(A_2)}=F(A_1,A_2)$. For any natural *n* consider a process $\xi_n(t)$, which we will define as a partial sum with a number n of row (7).

$$\xi_n(t)=\sum_{k=-n}^{n}\left(-\frac{\xi(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})^4}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\frac{\xi'(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})^3}\times\right.$$
$$\times\frac{1}{\sin^3\frac{\pi}{7}}+\frac{\xi''(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})^2}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\frac{\xi'''(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})}\times\frac{1}{\sin^3\frac{\pi}{7}}-$$
$$\left.-\frac{\xi\left(t_{k0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(t-t_{k0}-\frac{\pi}{\alpha}\right)^2}\times\frac{1}{\sin^4\frac{\pi}{7}}+\frac{\xi''\left(t_{k0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{5}\right)^3\left(t-t_{k0}-\frac{\pi}{\alpha}\right)}\times\frac{1}{2\sin^4\frac{\pi}{7}}\right)\times$$
$$\times\sin^4\frac{\alpha}{7}(t-t_{k0})\sin^3\frac{\alpha}{7}\left(t-t_{k0}-\frac{\pi}{\alpha}\right).$$

Using the representation (8) and the statement of the lemma, we will write $\xi_n(t)$ as follows:

$$\xi_n(t)=\sum_{k=-n}^{n}\int_{\Lambda}\left(-\frac{f(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})^4}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\frac{f'(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})^3}\times\frac{1}{\sin^3\frac{\pi}{7}}+\right.$$
$$+\frac{f''(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})^2}\times\frac{1}{2\sin^3\frac{\pi}{7}}-\frac{f'''(t_{k0})}{\left(\frac{\alpha}{7}\right)^4(t-t_{k0})}\times\frac{1}{\sin^3\frac{\pi}{7}}-$$
$$-\frac{f(t_{k0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0}-\frac{\pi}{\alpha})^2}\times\frac{1}{\sin^4\frac{\pi}{7}}+$$
$$\left.+\frac{f'(t_{k0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0}-\frac{\pi}{\alpha})}\times\frac{1}{\sin^4\frac{\pi}{7}}+\frac{f''\left(t_{k0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{5}\right)^3\left(t-t_{k0}-\frac{\pi}{\alpha}\right)}\times\frac{1}{2\sin^4\frac{\pi}{7}}\right)\times$$
$$\times\sin^4\frac{\alpha}{7}(t-t_{k0})\times\sin^3\frac{\alpha}{7}\left(t-t_{k0}-\frac{\pi}{\alpha}\right)\times Z(d\alpha). \qquad (9)$$

Then, based on the representation (1), (8), (9) and the estimation (2), we obtain

$$\mathrm{M}\,|\xi(t)-\xi_n(t)|^2\le R_n^2(t)\int_{\Lambda\times\Lambda}\left|F\left(d\lambda,,d\mu\right)\right|=$$
$$=L^2G^2(t)C_f^2\left(\frac{\alpha}{\alpha-\sigma}\right)^2\times\frac{1}{n^2}\int_{\Lambda\times\Lambda}\left|F\left(d\lambda,d\mu\right)\right|. \qquad (10)$$

From the inequality (10) and considering the condition (4), we obtain the following: an interpolation row (7) converges to $\xi(t)$ in the mean square.

Considering the separability of the process $\xi(t)$ and the convergence of a row

$$\sum_{n=-\infty}^{\infty}\left|\xi(t)-\xi_n(t)\right|^2,$$

obtain that the interpolation row (7) converges to the random process $\xi(t)$ almost surely uniformly over t in any bounded area of changing of t.

We obtain that the interpolation series (7) converges with probability 1 to a random process $\xi(t)$ in any bounded domain of changes of parameter t.

Consider the interpolation representation of random processes [8] on non-equidistance interpolation knots of the type

$$t_{n0}=n\frac{7\pi}{\alpha},\ t_{n1}=n\frac{7\pi}{\alpha}+\frac{\pi}{\alpha},$$
$$t_{n2}=n\frac{7\pi}{\alpha}+\frac{2\pi}{\alpha},\ n\in\mathrm{Z}$$

based on observations of the process and its derivatives of the first and second orders at knots $t_{n0},t_{n1},n\in\mathrm{Z}$ and observations of the process at knots $t_{n2},n\in\mathrm{Z}$.

Let's formulate the necessary results from the theory of entire functions of complex variable.

Lemma. Let $f(z)$ is an entire bounded on the real axis function of exponential type with indicator σ.

Then for any α, α> σ, the representation holds true

$$f(z)=\sum_{n=-\infty}^{\infty}\left(\left[\frac{f(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(z-t_{n0})^3}+\frac{f'(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(z-t_{n0})^2}+\right.\right.$$
$$\left.+\frac{f''(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(z-t_{n0})}\frac{1}{2}\right]\times\frac{1}{\sin^3\frac{\pi}{7}\sin^2\frac{2\pi}{7}}-$$
$$-\left[\frac{f\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(z-t_{n0}-\frac{\pi}{\alpha}\right)}+\frac{f'\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(z-t_{n0}-\frac{\pi}{\alpha}\right)^2}+\right.$$
$$\left.+\frac{f''\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(z-t_{n0}-\frac{\pi}{\alpha}\right)}\right]\times\frac{1}{\sin^4\frac{\pi}{7}}+$$
$$\left.+\frac{f\left(t_{n0}+\frac{2\pi}{\alpha}\right)}{\frac{\alpha}{7}\left(z-t_{n0}-\frac{2\pi}{\alpha}\right)}\times\frac{1}{2\sin^3\frac{\pi}{7}\sin^3\frac{2\pi}{7}}\right)\times$$
$$\times\sin^3\frac{\alpha}{7}(z-t_{n0})\times\sin^3\frac{\alpha}{7}\left(z-t_{n0}-\frac{\pi}{\alpha}\right)\times$$
$$\times\sin\frac{\alpha}{7}\left(z-t_{n0}-\frac{2\pi}{\alpha}\right), \qquad (11)$$

where

$$t_{n0} = n\frac{7\pi}{\alpha},\ n \in Z,$$

provided that the interpolation series (11) converges uniformly in any bounded region of the complex plane.

Proving Lemma, we obtain estimation of the residual of the interpolation series (11), which has the following form

$$|R_n(z)| \le$$

$$\le LG(z)C_f\,\frac{\alpha}{\alpha-\sigma}\frac{1}{n}, \quad (12)$$

where L is a constant,

$$C_f = \sup_{t\in R}|f(t)|,$$

$$G(z) =$$

$$= \left|\sin^3\frac{\alpha}{7}z \times \sin^3\frac{\alpha}{7}(z-\frac{\pi}{\alpha}) \times \sin\frac{\alpha}{7}(z-\frac{2\pi}{\alpha})\right|$$

is a function bounded on any bounded region of the complex plane.

Consider a random $\xi(t)$, $t\in R$ with $\mathrm{M}\xi(t)=0$ and covariance function, which representation is

$$B(t,s) = \int\limits_{\Lambda\times\Lambda} f(t,\lambda)\overline{f(s,\mu)}F(d\lambda,d\mu), \quad (13)$$

where Λ is a set of parameters, $F(.,.)$ is a positive definite additive complex function on $\Lambda\times\Lambda$ such that

$$\int\limits_{\Lambda\times\Lambda} |F(d\lambda,d\mu)| < +\infty. \quad (14)$$

The function $f(t,\lambda)$ relating to t is an entire function of exponential type with indicator $\sigma(\lambda)$ such that

$$\sup_{\lambda\in\Lambda}\ \sup_{-\infty<t<+\infty}|f(t,\lambda)| = C_f < +\infty, \quad (15)$$

$$\sup_{\lambda\in\Lambda}\sigma(\lambda) = \sigma < +\infty. \quad (16)$$

The following theorem holds true.

Theorem 2. Let $\xi(t)$ is a separable random process that satisfies conditions (13)–(16). Then for any α, $\alpha>\sigma$ with probability 1 the following representation holds true

$$\xi(z) = \sum_{n=-\infty}^{\infty}\left(\left[\frac{\xi(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0})^3} + \frac{\xi'(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0})^2} + \frac{\xi''(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0})}\frac{1}{2}\right]\times\right.$$

$$\times\frac{1}{\sin^3\frac{\pi}{7}\sin^2\frac{2\pi}{7}} - \left[\frac{\xi\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(t-t_{n0}-\frac{\pi}{\alpha}\right)^3} + \frac{\xi'\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(t-t_{n0}-\frac{\pi}{\alpha}\right)^2} + \frac{\xi''\left(t_{n0}+\frac{\pi}{\alpha}\right)}{\left(\frac{\alpha}{7}\right)^3\left(t-t_{n0}-\frac{\pi}{\alpha}\right)}\right]\times$$

$$\left.\times\frac{1}{\sin^4\frac{\pi}{7}} + \frac{\xi\left(t_{n0}+\frac{2\pi}{\alpha}\right)}{\frac{\alpha}{7}\left(t-t_{n0}-\frac{2\pi}{\alpha}\right)}\times\frac{1}{2\sin^3\frac{\pi}{7}\sin^3\frac{2\pi}{7}}\right)\times$$

$$\times\sin^3\frac{\alpha}{7}(t-t_{n0})\times\sin^3\frac{\alpha}{7}\left(t-t_{n0}-\frac{\pi}{\alpha}\right)\times\sin\frac{\alpha}{7}\left(t-t_{n0}-\frac{2\pi}{\alpha}\right). \quad (17)$$

We obtain that the interpolation series (17) converges with probability 1 to a random process $\xi(t)$ in any bounded domain of changes of parameter *t*.

6. Conclusions

The work is devoted to investigation of interpolation representations of a class of random processes. The main results are theorems on the convergence of interpolating series to a random process with probability 1. The following results were obtained:

1. We constructed two types of representation knots' groups.

2. For the first type of knots, the interpolation formula includes the value of the process and its derivatives of the first, second and third orders. For the second type of knots, the interpolation formula includes the value of the process and its derivatives of first and second orders. The interpolation formula that uses the value of the process and its derivatives at the knots of interpolation is constructed.

3. We have proved the convergence of the interpolation series to the considered stochastic process with probability 1.

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Ganna Verovkina, PhD, Associate Professor, Department of Mathematical Physics, Taras Shevchenko National University of Kyiv, Volodymyrska str., 64/13, Kyiv, Ukraine, 01601
E-mail: ganna.verov@gmail.com