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ANALYSIS OF THE STRUCTURAL MODELS OF COMPETENCIES IN PROJECT MANAGEMENT

Виконано аналіз структурних моделей, які відображають топологію процесів управління проектами за допомогою орієнтованих графів. Показано, що сутність аналізу орієнтованих графів пов'язана з визначенням замкнених циклів. Доведена можливість структурного аналізу орієнтованих графів завдяки специфічним властивостям матриць суміжності та матриць досяжності, що дозволяє автоматизувати структурний аналіз схем управління на основі компетентнісного подходу.

Ключові слова: *компетентнісний підхід, орієнтований граф, матриця суміжності, замкнені цикли, аналітичний пошук.*

1. Introduction

The Leonard Euler method for finding cycles in graphs uses the procedure of sequential search of vertices in combination with the reception of the coloring of those edges of the graph that have already been traversed [1]. Determination of the cycles in graphs by this algorithm actually implements the scheme of a complete search of all possible variants with the presence of a heuristic component, introduces a certain uncertainty in the case of formalization for the automated solution of the problem. Determination of the cycles in directed graphs reflecting the topology of projects is an urgent task for solving a number of problems.

To solve the problem of analysis of structural schemes of projects, it is proposed to use the method of analytical determination of cycles in complex management schemes. Unlike the well-known method of Leonard Euler, the cycle is determined as a result of an analytical calculation, rather than a heuristic search. The basis for the analytical solution of the problem is the use of the characteristic properties of the adjacency matrix [2].

2. The object of research and its technological audit

Directed graphs are used in various industries for structuration of knowledge and reflection of internal relations between elements of systems. In project management, this includes both project management schemes and a competency model. The properties of these objects have not yet been fully studied.

The object of this research is the competency model in the field of professional project management, proposed by the International Association for Project Management [3]. Beginning with version 3.0, it is not only the representation of the structure of the competences themselves (in this version 3.0 «technical», «behavioral» and «contextual» and their elements), but also interaction between the elements of competences [4]. Such representation of competences as a system of interrelated interdependent elements makes it possible to apply graph theory for their analysis to more accurately determine the topology of such system with the identification of the most stable components.

One of the most problematic places in the application of such systems is at the same time a strong side, according to the intention of its creators – universality. Taking into account, all the same, the intended industry influence, in particular, as evidenced by the presence of «Industry extensions» for the project management standard of the PMBOK [5]. «Industry extensions» are created on the basis of an appropriate standard that defines the requirements for competences from the American Institute for Project Management [6]. It is necessary to take into account the specifics of the activity, in particular, possible additional elements of competencies, or changes in the relations in the basic «universal» structure. The requirements for the assessment of competencies, and, on the other hand, for the system of training specialists in the field of project management, it would be logical to harmonize with industry specifics, which the universal model does not offer. Therefore, in this research, it is proposed to resolve this contradiction on the basis of an analysis of the competence system as a directed graph.

3. The aim and objectives of research

The aim of research is to improve the method of analytical isolation of closed cycles in directed graphs of complex topological structures of project systems.

To achieve the aim, the following tasks are indicated:

1. To investigate the properties of the degrees of contiguity matrices of directed graphs.

2. To develop a methodology for identifying cycles in graphs based on the formation of reachability matrix with subsequent transposition.

4. Research of existing solutions of the problem

As known, a system that unites the sets of some entities, for example:

$S\{s_1,\ s_2,\ ...,\ s_m\}$,

which are vertices of the directed graph, connected by directed arcs and:

$G\{g_1,\ g_2,\ ...,\ g_r\}$,

can be displayed using an adjacency matrix:

$[c_{ij}]_S=[i,\ j]$,

each line of which shows the connections of one vertex to other vertices of the graph [7]. The element $c_{ij}=1$ reflects the arc between the vertices S_i and S_j. If $c_{ij}=0$, then there is no arc directly between the vertices of the graph i and j.

The connections between the elements of the sets $S\{s_1,\ s_2,\ ...,\ s_m\}$ and $G\{g_1,\ g_2,\ ...,\ g_r\}$ can also be described as an incidence matrix:

$[h_{ij}]_{S},\ g=[i,\ j]$,

the rows of which correspond to the vertices, and the columns to the arcs of the directed graph. Moreover, the h_{ij}-th element is equal to +1 if S_i is the initial vertex of the arc and (–1) if S_i is the finite vertex of the arc [2].

For the analysis of structures, an adjacency matrix is used that has specific properties [7]. In the case of successive construction of an adjacency matrix of degree $n=2$, 3 ... elements of the n-th degree $(c_{ij})^n$ show the path containing n arcs between the i-th and j-th vertices of the graph.

Multiplication of matrices is carried out according to the usual rule [2]:

$$\left\|c_{ij}^{n+1}\right\|=\begin{Bmatrix} c_{1.1}^n & c_{1.2}^n & \cdots & c_{1.m}^n \\ c_{2.1}^n & c_{2.2}^n & \cdots & c_{2.m}^n \\ \cdots & \cdots & \cdots & \cdots \\ c_{m.1}^n & c_{m.2}^n & \cdots & c_{m.m}^n \end{Bmatrix}\times\begin{Bmatrix} c_{1.1}^1 & c_{1.2}^1 & \cdots & c_{1.m}^1 \\ c_{2.1}^1 & c_{2.2}^1 & \cdots & c_{2.m}^1 \\ \cdots & \cdots & \cdots & \cdots \\ c_{m.1}^1 & c_{m.2}^1 & \cdots & c_{m.m}^1 \end{Bmatrix}=$$

$$=\begin{Bmatrix} \sum_{k=1}^{m} c_{1k}^n c_{k1}^1 & \sum_{k=1}^{m} c_{1k}^n c_{k2}^1 & \cdots & \sum_{k=1}^{m} c_{1k}^n c_{km}^1 \\ \sum_{k=1}^{m} c_{2k}^n c_{k1}^1 & \sum_{k=1}^{m} c_{2k}^n c_{k2}^1 & \cdots & \sum_{k=1}^{m} c_{2k}^n c_{km}^1 \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{k=1}^{m} c_{mk}^n c_{k1}^1 & \sum_{k=1}^{m} c_{mk}^n c_{k2}^1 & \cdots & \sum_{k=1}^{m} c_{mk}^n c_{km}^1 \end{Bmatrix}, \quad (1)$$

where n – degrees of the adjacency matrix; $n=1$, 2, ...; m – total number of vertices in the scheme.

To show the connections between elements of complex circuits, let's use such simplification: the presence of a constraint, determined from (1), will be denoted by the value of the matrix element:

$$[c^n_{ij}]=1. \quad (2)$$

In the absence of a connection – $[c^n_{ij}]=0$. That is, let's perform the operations of multiplication (1) with respect to all the rules accepted in mathematics, and at the stage of reflection of the results let's perform the transformations:

$$\left\|c_{ij}^{n+1}\right\|=\begin{cases} 1, \text{if } \sum_{k=1}^{m} c_{ik}^n c_{kj}^1 > 0 \text{ for } \{\forall i,j\in 1,2,...,m\}; \\ 0, \text{if } \sum_{k=1}^{m} c_{ik}^n c_{kj}^1 = 0 \text{ for } \{\forall i,j\in 1,2,...,m\}. \end{cases} \quad (3)$$

Structural analysis of complex systems is applied in various fields of knowledge. In published works on the structural analysis of complex schemes, recommendations, often without proof, are given in the form of algorithms for the search for cycles [8]. Using structural analysis of NCB competencies, it is shown that the cycles in the NCB competency matrix are the basis for the formation of knowledge kernels [7]. In [9], a number of issues related to the assessment and management of complexity in projects that demonstrate the obvious significance of structural analysis are considered. Structural analysis has become the basis for studying the relations between individuals and the exchange of knowledge in the organizations of the building project [10]. It is theoretically proved that there is an influence of fundamental strategic changes in the choice of the project and the organizational structure [11]. The authors of [12] modeled possible modifications of the algorithm for controlling the topology of the system and the environment, using the rules for transforming the graph. The publication [13] analyzes the combination of traditional and competence approach to learning and evaluation of results. On the basis of graph theory, a model for constructing the learning trajectory is proposed in [14]. The development of the Markov model for changing the states of a project-driven organization is performed on the basis of a structural analysis of the system [15]. A study of the structure of value indicators in projects is carried out in [16]. Modeling of team, environment and project interaction in the management structure is given in [17]. The paper [18] is devoted to the study of the ergodicity of directed graph of a project management system. A conceptual model for classification of the content structure of documents is studied in [19]. This work shows the search for incomplete duplicates in the content structure of documents.

These examples show that the theoretical justification of methods for analyzing management structures and sets of competences is promising in project management, since the structures of project systems and information communications in them significantly affect the results of activities

5. Methods of research

Let's perform research of the methods for representing various structures using an adjacency matrix. Let's consider the properties of adjacency matrices and its degrees from the point of view of applying these properties for structural analysis of design systems.

Lemma 1. Two arcs, one of which enters and the other leaves one vertex, make up two elements in the adjacency matrix. These elements are shifted from the main diagonal by 1 column in the direction of the arcs. This can be if and only if three vertices of the directed graph are represented by adjacent columns.

Proof. By the rule of reflection of directed graphs in the adjacency matrix, the line numbers correspond to the number of the vertex of the graph from which the arc leaves. And the numbers of the columns are the number

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of the vertex into which the arc enters. Let's note that in any directed graph (having a contour) one can separate a linear part of the contour in the direction of the arcs of the directed graph and an inverse (arc), which forms a cycle.

The numbers of the vertices of the directed graph play more the role of vertex identifiers and do not determine the mandatory order in the adjacency matrix. They do not affect the structure of the connections between the vertices. Therefore, let's assume that the vertices of the directed graph can be numbered arbitrarily. Therefore, let's impose the condition on assigning numbers to the vertices of the graph: in the linear subgraph $s \in S$ the vertices are numbered in the direction of the arcs of the digraph.

Let's consider a directed graph from such vertices: *a, b, c, d, e, f, g*. Let the vertices of the graph be connected by the links: $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow g$. Since the vertices of the directed graph can be numbered arbitrarily, let's take the following numbering:

$a \rightarrow \{i\}$;

$b \rightarrow \{i+1\}$;

$c \rightarrow \{i+2\}$;

$d \rightarrow \{i+3\}$;

$e \rightarrow \{i+4\}$;

$f \rightarrow \{i+5\}$;

$g \rightarrow \{i+6\}$. (4)

In this case, in the adjacency matrix, due to (4), the rows and columns corresponding to the vertices *a, b,..., g*, will be arranged sequentially, and the values of the corresponding elements of the adjacency matrix will be:

$c_{i,i+1} = c_{a,b} = 1$;

$c_{i+1,i+2} = c_{b,c} = 1$;

...

$c_{i+5,i+6} = c_{f,g} = 1$. (5)

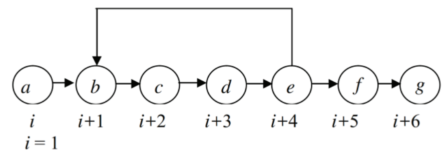
The elements of the adjacency matrix defined in (5) are shifted by one column from the main diagonal. That is, the arcs of the linear part of the directed graph are reflected in the adjacency matrix by a diagonal parallel to the principal graph. It is shifted by 1 column, provided that the vertices are located in the contiguity matrix sequentially along the direction of the arcs of the directed graph (Fig. 1).

Thus, an arc that does not form a diagonal, which is parallel to the principal one, does not belong to the linear part of the arcs of the directed graph. For example, in the case of the existence of a contour formed by an arc between the vertices $e \rightarrow b$, in the adjacency matrix the element $[c_{eb}]=1$, or taking into account the numeration (4), obtain $[c_{i+4,i+2}]=1$ (Fig. 2).

As can be seen from Fig. 2, reflection of the arc between the vertices $e \rightarrow b$ in the adjacency matrix is carried out through the value of the element $[c_{i+4,i+2}]=1$. This element forms a «triangle» with a linear part of the contour of the directed graph.

		To the top						
		a	b	c	d	e	f	g
From the top	a	0	1	0	0	0	0	0
	b	0	0	1	0	0	0	0
	c	0	0	0	1	0	0	0
	d	0	0	0	0	1	0	0
	e	0	0	0	0	0	1	0
	f	0	0	0	0	0	0	1
	g	0	0	0	0	0	0	0

Fig. 1. The adjacency matrix of a fragment of the linear part of the digraph



a

		To the top $i + ...$						
		a	b	c	d	e	f	g
From the top	a	0	1	0	0	0	0	0
	b	0	0	1	0	0	0	0
	c	0	0	0	1	0	0	0
	d	0	0	0	0	1	0	0
	e	0	1	0	0	0	1	0
	f	0	0	0	0	0	0	1
	g	0	0	0	0	0	0	0

b

Fig. 2. The adjacency matrix with an arc $e \rightarrow b$ forming a cycle: *a* – directed graph; *b* – the adjacency matrix of the digraph *a*

This property of reflection of cycles by means of an adjacency matrix is the basis for structural analysis.

Lemma 2. The elements of all columns of the contour, except the last, of the adjacency matrix of degree n are shifted to the degree of $n+1$ by one column in the direction of the edges of the directed graph.

Proof. Let's use the property of arbitrarily numbering of the vertices. In this case, the non-zero elements of the adjacency matrix of degree $n=1$:

$$c^1_{i,i+1}=1,\ i=k,\ k+1,\ ...,\ m-1;\ k \in 1,\ ...,\ m-1; \quad (6)$$
$$c^1_{m,k}=1,$$

where k, m – initial and final vertices entering the contour, $k<m$.

From (2) let's find elements of the adjacency matrix of degree $n+1$:

$$c^{n+1}_{ij} = \sum_{h=k}^{m} c^n_{ih} c^1_{hj}, \quad j=1,2,\cdots,m; \quad i=1,2,\cdots,m. \quad (7)$$

Let's calculate the values of the elements of one of the rows $s\{1,\ 2,\ ...\ m\}$ of the adjacency matrix of degree $n+1$:

$$c^{n+1}_{s,1} = c^n_{s,1} \cdot c^1_{1,1} + c^n_{s,2} \cdot c^1_{2,1} + c^n_{s,3} \cdot c^1_{3,1} + \cdots + c^n_{s,m} \cdot c^1_{m,1};$$
$$c^{n+1}_{s,2} = c^n_{s,1} \cdot \left(c^1_{1,2}\right) + c^n_{s,2} \cdot c^1_{2,2} + c^n_{s,3} \cdot c^1_{3,2} + \cdots + c^n_{s,m} \cdot c^1_{m,2};$$
$$c^{n+1}_{s,3} = c^n_{s,1} \cdot c^1_{1,3} + c^n_{s,2} \cdot \left(c^1_{2,3}\right) + c^n_{s,3} \cdot c^1_{3,3} + \cdots + c^n_{s,m} \cdot c^1_{m,3};$$
$$\vdots$$
$$c^{n+1}_{s,m} = c^n_{s,1} \cdot c^1_{1,m} + c^n_{s,2} \cdot c^1_{2,m} + \cdots + c^n_{s,m-1} \cdot \left(c^1_{m-1,m}\right) + c^n_{s,m} \cdot c^1_{m,m}, \quad (8)$$

where m – the number of the element in the row.

The non-zero elements $c_{i,i+1}=1$ ($i=1, 2, ..., m-1$) of the adjacency matrix are distinguished by brackets. Discarding the remaining elements, obtain in the general case that the value of the element of the row $s\{1, 2, ... m\}$ for the linear part of the graph will be determined by the first multiplier:

$$c^{n+1}_{s,h}=c^{n}_{s,h-1};\quad h=2,3,\cdots,m. \tag{9}$$

For example, from (9) for $n=1$ and $s=1$:

$c^2_{1,h}=c^1_{1,h-1}$.

This means that the element of the first row $c^1_{1,2}=1$ will move from the second column to the third column $c^2_{1,3}=1$. By analogy, for the 1st and subsequent rows, the elements reflecting the linear part of the graph will shift by one column in the direction of the arcs of the graph in the case of increasing to the next degrees.

Graphical interpretation of the proof in the example of calculation of the matrix element $[c^2_{2,4}]$ is shown in Fig. 3.

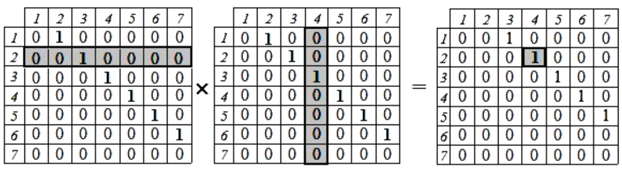


Fig. 3. Shift scheme of the element $[c^1_{2,3}]=1$ by one column in the element of the matrix $[c^2_{2,4}]=1$, which is the result of multiplying the matrices

To determine the value of the element $[c^2_{2,4}]$ from (2), multiply the elements of row 2 and column 4 and determine the sum. As can be seen, only two elements of the second row – $[c^1_{2,3}]=1$ and the fourth column – $[c^1_{3,4}]=1$, have a value other than zero. It is they which, according to (2), will give the value $[c^2_{2,4}]=[c^1_{2,3}]\times[c^1_{3,4}]=1$. In the general case, for example in Fig. 3, obtain:

$$[c^2_{1,3}]=[c^1_{1,2}];$$

$$[c^2_{2,4}]=[c^1_{2,3}];$$

$$[c^2_{3,5}]=[c^1_{3,4}];$$

$$[c^2_{4,6}]=[c^1_{4,5}];$$

$$[c^2_{5,7}]=[c^1_{5,6}]. \tag{10}$$

It is proved that the elements of all columns of the linear part of the directed graph, except for the last one, of the adjacency matrix of degree n are shifted to the degree of $n+1$ by one column in the direction of the edges of the directed graph.

Lemma 3. In the degree $n+1$ of the adjacency matrix, the elements of the last column of the contour of degree n shift to the first column of the contour.

Proof. Let a directed graph with a contour, reflected by an adjacency matrix with the conditions adopted in Lemma 2 (Fig. 2).

Let's consider the formation of any column k of an adjacency matrix of degree $n+1$. Elements of column k are calculated by (2):

$$c^{n+1}_{1,k}=c^n_{1,1}\cdot c^1_{1,k}+c^n_{1,2}\cdot c^1_{2,k}+\cdots+c^n_{1,m-2}\cdot c^1_{m-2,k}+c^n_{1,m-1}\cdot c^1_{m-1,k}+c^n_{1,m}\cdot c^1_{m,k};$$
$$c^{n+1}_{2,k}=c^n_{2,1}\cdot c^1_{1,k}+c^n_{2,2}\cdot c^1_{2,k}+\cdots+c^n_{2,m-2}\cdot c^1_{m-2,k}+c^n_{2,m-1}\cdot c^1_{m-1,k}+c^n_{2,m}\cdot c^1_{m,k};$$
$$c^{n+1}_{3,k}=c^n_{3,1}\cdot c^1_{1,k}+c^n_{3,2}\cdot c^1_{2,k}+\cdots+c^n_{3,m-2}\cdot c^1_{m-2,k}+c^n_{3,m-1}\cdot c^1_{m-1,k}+c^n_{3,m}\cdot c^1_{m,k};$$
$$c^{n+1}_{4,k}=c^n_{4,1}\cdot c^1_{1,k}+c^n_{4,2}\cdot c^1_{2,k}+\cdots+c^n_{4,m-2}\cdot c^1_{m-2,k}+c^n_{4,m-1}\cdot c^1_{m-1,k}+c^n_{4,m}\cdot c^1_{m,k};$$
$$\vdots$$
$$c^{n+1}_{m,k}=c^n_{m,1}\cdot c^1_{1,k}+c^n_{m,2}\cdot c^1_{2,k}+\cdots+c^n_{m,m-2}\cdot c^1_{m-2,k}+c^n_{m,m-1}\cdot c^1_{m-1,k}+c^n_{m,m}\cdot c^1_{m,k}. \tag{11}$$

For the system of equations (11), the initial conditions must be introduced: the numbers of the beginning k and the end of the cycle r. For example, for the scheme in Fig. 2 such data will be $k=b=2$ and $r=e=5$. Under such conditions, the element of the second multiplier $[c^1_{5,2}]=1$. And the penultimate element of the contour in the first matrix multiplier $[c^{(n-1)}_{4,5}]=1$, as a consequence of Lemma 1 on the parallelism by the main matrix diagonal of the elements of the linear part of the cycle.

Discarding the zero elements from (11), and taking the values of elements known from the initial conditions, obtains for $n=1$:

$$c^2_{4,2}=c^1_{4,5}\cdot c^1_{5,2}. \tag{12}$$

Graphical interpretation of the proof of Lemma 3 on the calculation of the element of the resulting matrix $[c^2_{4,2}]$ is shown in Fig. 4.

To determine the value of the element $[c^2_{4,2}]$ from (2), multiply the elements of row 4 and column 2 and determine the sum. Elements of the 4th row and the 2nd column and the multiplication result are highlighted in Fig. 4. They will, in accordance with (2), give the following values: $[c^2_{4,2}]=[c^1_{4,5}]\times[c^1_{5,2}]=1$.

The second multiplier does not change and always $[c^1_{r,k}]=1$. Therefore, in the general case, in the case of ascent to the next steps, the elements will jump from the penultimate column ($r-1$) to the first column k of the contour. This is true for all columns of elements that reflect the linear part of the directed graph.

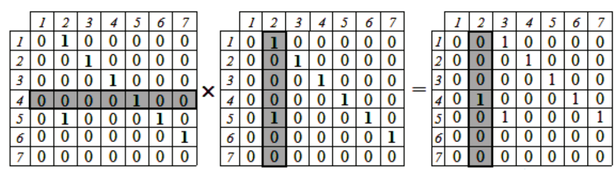


Fig. 4. The scheme of the jump of the element $[c^1_{4,5}]$ from the contour of the first multiplier to the first column $[c^2_{4,2}]=1$ as a result of matrix multiplication

Lemma 4. The connections between the vertices of a graph through 1...n arcs reflect the degrees of the adjacency matrix from 1 to n, respectively.

Proof. As defined in Lemma 2, the elements of all columns of the contour, except the last, of the adjacency matrix of degree n are shifted to the degree of $n+1$ by one column in the direction of digraph edges. That is, each $n+1$ stage reflects the connections from the i-th to $n+1$ vertices of the graph. So, the connections are obtained on the basis of the 2nd degree of the adjacency matrix, reflect the connections in the graph through one transit vertex (dotted line, Fig. 5, *a*).

As we can see, new connections connect those vertices, which were connected by two arcs in the initial

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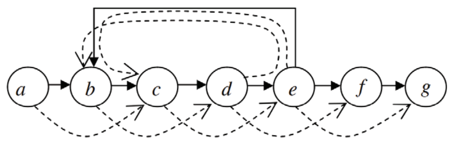
matrix (Fig. 4, Fig. 5). These conclusions are also true for the third degree of the adjacency matrix, with the difference that the detected connections already pass through three arcs and two transit vertices of the graph (Fig. 6).



a

2nd degree		To the top						
		a	*b*	*c*	*d*	*e*	*f*	*g*
From the top	*a*	0	0	1	0	0	0	0
	b	0	0	0	1	0	0	0
	c	0	0	0	0	1	0	0
	d	0	1	0	0	0	1	0
	e	0	0	1	0	0	0	1
	f	0	0	0	0	0	0	0
	g	0	0	0	0	0	0	0

b

Fig. 5. Reflection of the connections in the second-degree adjacency matrix through one transit vertex of the graph: *a* – connections on the directed graph; *b* – second-degree adjacency matrix

As can be seen from Fig. 6, there is a definite regularity in the variation of the connections, characteristic for different degrees of the adjacency matrix.

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	1	0	0	0	0	0
b	0	0	1	0	0	0	0
c	0	0	0	1	0	0	0
d	0	0	0	0	1	0	0
e	0	1	0	0	0	1	0
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

a

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	0	1	0	0	0	0
b	0	0	0	1	0	0	0
c	0	0	0	0	1	0	0
d	0	1	0	0	0	1	0
e	0	0	1	0	0	0	1
f	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0

b

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	0	0	1	0	0	0
b	0	0	0	0	1	0	0
c	0	1	0	0	0	1	0
d	0	0	1	0	0	0	1
e	0	0	0	1	0	0	0
f	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0

c

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	0	0	0	1	0	0
b	0	1	0	0	0	1	0
c	0	0	1	0	0	0	1
d	0	0	0	1	0	0	0
e	0	0	0	0	1	0	0
f	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0

d

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	1	0	0	0	1	0
b	0	0	1	0	0	0	1
c	0	0	0	1	0	0	0
d	0	0	0	0	1	0	0
e	0	1	0	0	0	1	0
f	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0

e

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	0	1	0	0	0	1
b	0	0	0	1	0	0	0
c	0	0	0	0	1	0	0
d	0	1	0	0	0	1	0
e	0	0	1	0	0	0	1
f	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0

f

Fig. 6. Shift of the elements of the adjacency matrix in degrees from n=1 to n=6: *a* – n=1; *b* – n=2; *c* – n=3; *d* – n=4; *e* – n=5; *f* – n=6

Elements of the adjacency matrix move from right to left (in the direction of the digraphs). At the same time the penultimate elements of the cycle passes the specific path (with a jump). The indicated properties of the degrees of adjacency matrices make it possible to put forward a hypothesis about the possibility of a computational determination of contours in the directed graph.

6. Research results

Let's assume that the Boolean sum of adjacency matrices of degrees from 1 to m is an reachability matrix that forms the graph of all paths of the scheme, including a closed contour.

Proof. Let's use the conclusions of Lemma 4. To obtain the $\mathbf{R}^{n}$ matrix of all paths of directed graph or reachability matrix, let's create the Boolean sum of all degrees of the adjacency matrix, are shown in Fig. 6. The elements $[r_{ij}]$ of the reachability matrix are determined by the use of the disjunction operations ($\vee$) or the conjunction ($\wedge$). The reachability matrix of the first rank $\mathbf{R}^{(1)}$ is the adjacency matrix $\boldsymbol{C}^{1}$ of the first degree:

$$[r_{ij}^{(1)}]=[c_{ij}^{1}],\ \forall\ i,\ j\ \in\ \{1,\ 2,\ ...,\ m\}. \quad (13)$$

The reachability matrices of the following ranks $\mathbf{R}^{(n)}$ for the values $n>1$ are determined using the reachability matrices of ranks $(n-1)$ and adjacency matrices of the corresponding degrees:

$$[r_{ij}^{(2)}]=\begin{cases}1, & \text{if}\quad (r_{ij}^{(1)}=1)\vee(c_{ij}^{2}=1),\\ 0, & \text{if}\quad (r_{ij}^{(1)}=0)\wedge(c_{ij}^{2}=0);\end{cases}$$

$$[r_{ij}^{(3)}]=\begin{cases}1, & \text{if}\quad (r_{ij}^{(2)}=1)\vee(c_{ij}^{3}=1),\\ 0, & \text{if}\quad (r_{ij}^{(2)}=0)\wedge(c_{ij}^{3}=0);\end{cases}$$

$$[r_{ij}^{(n)}]=\begin{cases}1, & \text{if}\quad (r_{ij}^{(n-1)}=1)\vee(c_{ij}^{n}=1),\\ 0, & \text{if}\quad (r_{ij}^{(n-1)}=0)\wedge(c_{ij}^{n}=0).\end{cases} \quad (14)$$

The reachability matrix $\mathbf{R}^{n}$ contains all the links from vertex i to vertex j in terms through of n arcs of the graph (Fig. 7).

As the degree n of adjacency matrices grows, the reachability matrix $\mathbf{R}^{n}$ becomes a filled with 1 because of the validity of Lemma 2. Submatrix filled with 1 shows that all its vertices have a connection in direction of the arcs of the graph. And this is the description of all possible paths in the digraph in direction of the arcs of the graph. In some rows, the elements of the main diagonal (MD) of the reachability matrix have the value $[r_{ii}]$=1. This is a sign that this line contains a description of the path in the digraph from the element and → and. The existence of such path from the element i to i is possible in the cycle of the directed graph. It should also be pointed out that some elements of the row i and in which there is exists a connection $i \to i$ do not enter a closed loop. Since the direction of the arcs of the graph from the vertex i is the path to the terminal vertices of the graph, for example, to the vertices f and g in Fig. 2.

To determine all subsystems that exist in the graph and enter the contour, let's replace the directions by the inverse of all arcs of the graph by transposing the reachability matrix $\mathbf{R}^{n}\to(\mathbf{R}^{n})^{\mathrm{T}}$ with the subsequent superposition $\mathbf{W}=$ $=\mathbf{R}\cap\mathbf{R}^{\mathbf{T}}$. Elements of the superposition matrix $\mathbf{W}=\mathbf{R}\cap\mathbf{R}^{\mathbf{T}}$ are formed by using disjunction operations ($\vee$ – logical «OR») or conjunction ($\wedge$ – logical «AND») as follows:

$$w_{ij} = \begin{cases} 1, & if \quad (r_{ij}=1) \wedge (r_{ij}^{\mathrm{T}}=1), \\ 0, & if \quad (r_{ij}=0) \vee (r_{ij}^{\mathrm{T}}=0). \end{cases} \qquad (15)$$

	a	b	c	d	e	f	g
a	0	1	1	1	1	1	1
b	0	1	1	1	1	1	1
c	0	1	1	1	1	1	1
d	0	1	1	1	1	1	1
e	0	1	1	1	1	1	1
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

a

	a	b	c	d	e	f	g
a	0	0	0	0	0	0	0
b	1	1	1	1	1	0	0
c	1	1	1	1	1	0	0
d	1	1	1	1	1	0	0
e	1	1	1	1	1	0	0
f	1	1	1	1	1	0	0
g	1	1	1	1	1	1	0

b

	a	b	c	d	e	f	g
a	0	1	1	1	0	0	0
b	0	0	1	1	1	0	0
c	0	1	0	1	1	1	0
d	0	1	1	0	1	1	1
e	0	1	1	1	0	1	1
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

c

	a	b	c	d	e	f	g
a	0	1	1	1	1	0	0
b	0	1	1	1	1	1	0
c	0	1	1	1	1	1	1
d	0	1	1	1	1	1	1
e	0	1	1	1	1	1	1
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

d

	a	b	c	d	e	f	g
a	0	1	1	1	1	1	0
b	0	1	1	1	1	1	1
c	0	1	1	1	1	1	1
d	0	1	1	1	1	1	1
e	0	1	1	1	1	1	1
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

e

	a	b	c	d	e	f	g
a	0	1	1	1	1	1	1
b	0	1	1	1	1	1	1
c	0	1	1	1	1	1	1
d	0	1	1	1	1	1	1
e	0	1	1	1	1	1	1
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

f

Fig. 7. The reachability matrix R^n for different n: *a* – n=1; *b* – n=2; *c* – n=3; *d* – n=4; *e* – n=5; *f* – n=6

Non-zero elements of MD of the matrix W indicate a line containing all paths of the contour. Dedicated contours in which all elements are connected with all other elements constitute the basis of ergodic subset of the directed graph. In this case, not only the final result of the superposition matrix $\mathbf{W}^n$ is informative, but also the results that show the formation of closed cycles.

Let's transpose the reachability matrix $\mathbf{R}^6 \to (\mathbf{R}^6)^{\mathrm{T}}$, which is shown in Fig. 7, *f*, with the subsequent superposition $\mathbf{W}^6 = \mathbf{R}^6 \cap (\mathbf{R}^6)^{\mathrm{T}}$ (Fig. 8).

	a	b	c	d	e	f	g
a	0	1	1	1	1	1	1
b	0	1	1	1	1	1	1
c	0	1	1	1	1	1	1
d	0	1	1	1	1	1	1
e	0	1	1	1	1	1	1
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

a

	a	b	c	d	e	f	g
a	0	0	0	0	0	0	0
b	1	1	1	1	1	0	0
c	1	1	1	1	1	0	0
d	1	1	1	1	1	0	0
e	1	1	1	1	1	0	0
f	1	1	1	1	1	0	0
g	1	1	1	1	1	1	0

b

	a	b	c	d	e	f	g
a	0	0	0	0	0	0	0
b	0	1	1	1	1	0	0
c	0	1	1	1	1	0	0
d	0	1	1	1	1	0	0
e	0	1	1	1	1	0	0
f	0	0	0	0	0	0	0
g	0	0	0	0	0	0	0

c

Fig. 8. The superposition matrix $W^6 = R^6 \cap (R^6)^{\mathrm{T}}$, obtained on the basis of the reachability matrix R^6 of the example in Fig. 6: *a* – R^6; *b* – $R^6)^{\mathrm{T}}$; *c* – $W^6 = R^6 \cap (R^6)$

As can be seen from the results of superposition (Fig. 8, *c*), the developed method makes it possible to determine the presence of a closed contour in the directed graph. The contour includes such vertices connected by constraints: $b \to c \to d \to e \to b$.

Let's show the application of theoretical research provisions on the example of structural analysis of a fragment of contextual competences in the sphere of professional project management. As is known, the field of knowledge of project management covers three main areas of competence: technical – 20 elements, behavioral – 15 elements and contextual – 11 elements [3]. In addition, NCB (ver 3.1) also defines additional competencies (national and industry) – 6 elements [3]. These 52 elements of competences have complex interrelations, which together form the area of knowledge of project management. Considering the essential interdependence of these elements of competences, a hypothesis has been put forward on the existence of certain sets of competences in this area of knowledge that are connected by strong ties, which allows them to be identified as the «core» of knowledge (competences). All elements of the «core» of knowledge form a complete subgraph of many competences.

The set of contextual competencies and the adjacency matrix, which reflects the relations between the elements in the group of contextual competencies without taking into account the relations with other groups, are given in Table 1.

Table 1

Adjacency matrix of the group of contextual competencies

Contextual competencies	Relations of contextual competencies (3.xx)										
	3.01	3.02	3.03	3.04	3.05	3.06	3.07	3.08	3.09	3.10	3.11
3.01. Project-oriented management	0	0	1	1	0	0	0	0	0	0	0
3.02. Program-oriented management	0	0	1	0	0	0	0	0	0	0	0
3.03. Portfolio-oriented management	1	1	0	1	0	0	0	0	0	0	0
3.04. Implementation of programs/portfolios/ projects	1	1	1	0	0	1	0	1	0	0	0
3.05. Sustainable organization	0	0	0	0	0	1	0	1	0	0	0
3.06. Business activity	0	0	0	0	1	0	0	0	0	1	1
3.07. Systems, products and technologies	0	0	0	0	0	1	0	0	1	1	1
3.08. Personnel management	0	0	0	0	1	0	0	0	0	0	1
3.09. Health, safety, labor protection	0	0	0	0	0	0	0	0	0	0	1
3.10. Finance	0	0	0	0	0	1	0	0	0	0	0
3.11. Legal aspects	0	0	0	0	0	0	0	0	0	0	0

Let's define the cores of knowledge based on analysis. For simplicity, we will not specify the number of the third group of contextual competencies, as is customary in NCB version 3.1 [3]. According to the method developed by the analytical analysis of digraphs, let's calculate successively the second, third, and subsequent degrees of the adjacency matrix. Next, let's define reachability and superposition matrices for all degrees. The obtained results make it possible to conclude that in the subsystem of contextual competencies, there are

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contours of connections that unite knowledge-related competences.

Let's perform the transpose of the reachability matrix $\mathbf{R}^3 \rightarrow (\mathbf{R}^3)^T$, obtained from the data of Table 1, followed by a superposition $\mathbf{W}^6 = \mathbf{R}^6 \cap (\mathbf{R}^6)^T$ (Fig. 9).

	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0	0	0	0	0
3	1	1	1	1	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0	0	0	0
5	0	0	0	0	1	1	0	1	0	1	0
6	0	0	0	0	1	1	0	1	0	1	0
7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	1	0	1	0	1	0
9	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	1	1	0	1	0	1	0
11	0	0	0	0	0	0	0	0	0	0	0

Fig. 9. The superposition matrix $W^3 = R^3 \cap (R^3)^T$, obtained on the basis of the reachability matrix R^3 according to the data of Table 1

The cores of knowledge of contextual competences in the superposition matrix $\mathbf{W}^3 = \mathbf{R}^3 \cap (\mathbf{R}^3)^T$ form the elements: 3.1...3.4; and also a set of elements 3.5, 3.6, 3.8, 3.10. These two cores of knowledge are combined into separate complexes, highlighted for clarity by color.

Special attention should be paid to the elements: 3.7 (Personnel management) 3.9 (Health, safety, labor protection) and 3.11 (Legal aspects). These elements are related to the competences of other groups – technical and behavioral competencies.

Thus, based on the analysis of the fragment of relations in the group of contextual competences in the field of professional project management, it is shown that in this group there are knowledge kernels of a set of competences that are connected together by strong relations and form a system of interrelated elements. Such conclusion allows to formulate the content of educational disciplines.

7. SWOT analysis of research results

Strengths. The strengths of the approach presented in the work are:

1. Visibility in the presentation of results – both the final results of the analysis (reachability matrix) and its intermediate steps (adjacency matrices).

2. The mathematical apparatus necessary for calculations is not heavy in the sense. The steps of the model formation are understandable, well amenable to algorithmization.

3. The basic mathematical operations (matrix actions), necessary for the model formation and its processing, are presented to date in most table editors, members of the standard office software family.

4. The software necessary for calculations is distributed, both on a fee basis, for example Microsoft Excel, and on a free or shareware basis, for example, as part of LibreOffice. There are versions for both the Windows operating system, and for MacOS and Linux.

5. Considering the points 2–4, to use the proposed approach, there is no need to develop special software. The necessary actions can be performed by any qualified user (not a programmer).

6. The strengths, perhaps even more important, of the presented approach can be topology analysis for complex competency models with a large number of interrelated elements. Such analysis will make it possible to simplify the model and identify «nodes» that exert maximum influence on the entire system.

Weaknesses. The weaknesses of the approach presented in the work are:

1. In the case of analysis of complex competency models (such as, for example, NCB 3.0), involving a large number of elements, visibility may be lost.

2. When constructing industry models, an additional considerable amount of labor may be required to create an adjacency matrix on the basis of peer review. In this case, there will also be a need for a correlation analysis (which, however, with a serious approach, is more likely to be the inevitable factor).

Opportunities. The opportunities for the approach presented in the work are:

1. Ease of introduction and use in the activities of specific enterprises of various fields of activity (on the basis of a list of strengths). This minimizes the need for special training, licensing the right to use this approach, the lack of the need to purchase additional software.

2. Possibility of use in the activity of personnel services and departments of human resources management of enterprises. As for the formation of requirements for specialists and managers, and for assessing competencies. As for employment, and in the process of production activities. For example, when forming project teams, developing requirements for educational programs, etc.

3. The use of this approach can be recommended for calculating such indicator as a «return to knowledge» in the extended Kirkpatrick model.

4. It is possible to create, using the presented approach, a whole family of specialized software products from templates for the most common table editors in mobile applications.

5. It is possible to create a specialized Internet resource that provides the possibility of forming, analyzing and further adjusting the model for enterprises of different industry focus with the goal of creating a database for further refinement of the model (under impersonal conditions).

Threats. Threats to the approach presented in the work are:

1. Neglect of the need to adapt this approach to the needs (specificity) of a particular enterprise, which can lead to incorrect interpretation of the results made on the basis of the «universal basic model». Even if it is based on the latest version of the international standard.

2. It is possible to counteract the use of this approach by individuals and organizations interested in promoting paid services in the design, evaluation and development of staff competencies.

3. Complexity in the possible procedure of patent protection, both the method itself and possible «derivatives». This is due to the simplicity of the basic principles of its operation and the availability for users of computational tools for creating and processing of the models.

8. Conclusions

1. A method for investigation of the adjacency matrix properties of directed graphs and its degrees is developed. It is shown that the degrees of the adjacency matrix follow the general structure of the directed graph with certain

regularities of reflection of the arcs of the graph. This makes it possible to construct the reachability matrix of investigated topological structure with the outlining of contours in the digraph.

2. A methodology for identifying cycles in graphs is developed on the basis of the formation of a zero sum of the reachability matrix degrees with its subsequent transposition and superposition. This makes it possible to obtain the contour reflection in the graph in the form of a square submatrix filled with 1.

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АНАЛИЗ СВОЙСТВ СТРУКТУРНЫХ МОДЕЛЕЙ КОМПЕТЕНЦИЙ В ПРОЕКТНОМ УПРАВЛЕНИИ

Выполнен анализ структурных моделей, которые отражают топологию процессов управления проектами с помощью ориентированных графов. Показано, что сущность анализа ориентированных графов связана с определением замкнутых циклов. Доказана возможность структурного анализа ориентированных графов благодаря специфическим свойствам матриц смежности и матриц досягаемости, что позволяет автоматизировать структурный анализ схем управления на основе компетентностного подхода.

Ключевые слова: компетентностный подход, ориентированный граф, матрица смежности, замкнутые циклы, аналитический поиск.

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