



**Brzhezitsky V.,  
Haran Ya.,  
Trotsenko Ye.**

# MATHEMATICAL MODEL OF HIGH-VOLTAGE INSTRUMENT AUTOTRANSFORMER INTENDED FOR USE IN SMART GRID NETWORKS

*Запропонована математична модель активної частини високовольтного автотрансформатора напруги. Модель дозволяє визначити розподіл напруги по витках та групах витків обмотки, враховуючи параметри прикладеної напруги, характеристики магнітного осердя, індуктивність розсіювання. Модель розглядається як засіб для проектування високовольтного вимірювального електрообладнання мереж Smart Grid.*

**Ключові слова:** автотрансформатор, трансформатор напруги, Smart Grid, математична модель, індуктивність розсіювання.

## 1. Introduction

Intelligent power management systems (Smart grid systems) became possible after the electronic components designed for an analog-to-digital conversion (ADC) of electrical signals reached a level allowing, with a minimum error, to reproduce in digital form the electrical signals received from the high-voltage transformers. Since then, the microelectronic element base used in Smart Grid is constantly improved. The sampling frequency of signal, microcontroller performance, immunity to electromagnetic interferences increase while the manufacture cost of such digital equipment decrease. However, primary-side (high-voltage) transformers are still a weak link in Smart Grid, since any high-precision ADCs only convert the signals of the secondary-side circuits of high-voltage electrical measuring equipment. Moreover, an additional measurement error, as well as a decrease in the reliability of Smart Grid, is caused by voltage dividers at the inputs of the ADCs. The need for voltage dividers is caused by the fact that typical high-voltage transformers are usually designed with secondary windings rated for nominal output voltages of about 100 volts.

An alternative solution in this situation can be high-voltage autotransformers, output part of winding of these electrical devices can be designed for several rated output voltages. As high-voltage autotransformers practically were not used in the electric power industry, research of their characteristics and operating conditions close to the no-load conditions (that is typical when the load is digital electronic device connected to their output) were not studied in publications. Also, the effect of the load on the high-voltage instrument autotransformer transformation ratio is not studied in scientific papers.

The combination of these factors determines the relevance of studying the characteristics of high-voltage autotransformers as the primary-side voltage transformers in Smart Grid networks.

## 2. The object of research and its technological audit

*The object of research* is a mathematical model of the active part of a high-voltage instrument autotransformer. In the context of this research, it is necessary to solve the problem of determining the dependence of a high-voltage autotransformer transformation ratio on the parameters of its active part, on the value of applied voltage, and on the load parameters of this autotransformer. At the same time, the feature of this research is that the operating conditions of a high-voltage autotransformer are close to no-load conditions. The considered mathematical model is the further development of the previously performed research devoted to the detailing of transformer equation system [1].

## 3. The aim and objectives of research

*The main aim of the article* is to study the potential for the gradual replacement of high-voltage transformers in Smart Grid networks with high-voltage autotransformers.

The following objectives were set to reach this aim:

1. Create a mathematical model of the active part of a high-voltage autotransformer in the form of equation system that makes it possible to detail the parameters of the winding up to the level of single turn (or groups of turns).
2. Determine the factors affecting the voltage distribution across the winding turns of the high-voltage autotransformer under operating conditions close to the no-load conditions.
3. Compare the results of theoretical computations based on the developed mathematical model of the high-voltage autotransformer with the results of experimental studies.

## 4. Research of existing solutions of the problem

After the importance of an intelligent power management in electric power industry was substantiated, many

publications on this topic covered both the importance of Smart Grid and the operation features of such systems [2–4]. Among the positive factors of Smart Grid application in the electric power industry are: power flow monitoring in the distributed networks [5, 6], commercial metering of electricity [7, 8], and power quality control [9]. A separate problem in Smart Grid research is the question of the accuracy and reliability of the high-voltage transformers. This question was studied, for example, in [6, 10]. However, these papers did not consider the fundamental issue of the possibility of replacing high-voltage instrument transformers with high-voltage autotransformers having a few low (about a few volts) rated output voltages. The modeling of the active part of high-voltage transformers is considered in [11]. In [1], a mathematical model of the active part of a transformer (with a load) is proposed. This model allows detailing the voltage distribution across the turns of winding up to the level of single turn. Nevertheless, there is no similar consideration of modeling the voltage autotransformer with connected load in the literature. In [12], the effect of capacitive currents flowing through the winding of a high-voltage autotransformer on the voltage distribution across the winding turns is studied. It is noted that the higher is the voltage class, the greater is the influence of these currents under conditions close to no-load ones.

The totality of the materials observed points to the relevance of the considered problem in the context of the possible integration of high-voltage autotransformers into Smart Grid networks.

## 5. Methods of research

To achieve objectives that were set such research method was applied: mathematical modeling on a personal computer with a help of software based on the finite element method. The main material in this research is the equation system of high-voltage autotransformer, detailed to the level of single turn (or groups of turns) of its winding.

## 6. Research results

Among the reviewed literature the most close problem is studied in [1] where the equation system of the transformer is offered, allowing to detail voltage distribution across the winding of transformer up to the level of single turn (or groups of turns). Proposed by authors in [1], the system of equations considers the load connected to the secondary winding of the voltage transformer, characteristics of magnetic system, leakage inductance, active resistances, voltage and currents of single turn (or groups of turns) of winding. Such system of the equations can be transformed into similar system of the equations but for the voltage autotransformer. Advantage of the equation system, allowing detailing voltage distribution across the winding of the high-voltage autotransformer is that it allows considering non-uniform distribution of a magnetic linkage of magnetic leakage fluxes with winding turns. It allows choosing at a design stage an appropriate number of autotransformer winding turns to get bigger values of high-voltage autotransformer transformation ratio. Such transformation ratio values take place when digital devices are connected to the

high-voltage autotransformer outputs. These digital devices do not have additional voltage divider at their inputs, thus increasing both accuracy and reliability of such combined high-voltage measuring systems.

In [11] effect of turn arrangement of a high-voltage transformer on value of the resulted equivalent leakage inductance of windings that is used in transformer equivalent circuits at designing was studied. As it shown by authors in [11], it is theoretically possible to present the secondary winding of the voltage transformer as two turns (or groups of turns), convoluted such way that value of the resulted equivalent leakage inductance of primary and secondary windings would be equal to a certain preset value, including zero value.

Similar studies are also important for a high-voltage autotransformer for which the output part of its winding can consist of several turns, and the transformation ratio can attain tens of thousands of points. The system of equations offered in [1], after its modification for the single-phase voltage transformer, which windings have connection group 1/1-0, can be described by formulas (1)–(6) with a use of complex-valued quantities:

$$\dot{U}_{i1} = (j \cdot \omega \cdot \Phi + \dot{I}_1 \cdot (r_{i1} + j \cdot \omega \cdot L_{i1}) - j \cdot \omega \cdot \dot{I}_2 \cdot M_{i2}) \cdot W_{i1}; \quad (1)$$

$$\dot{U}_{k2} = (j \cdot \omega \cdot \Phi - \dot{I}_2 \cdot (r_{k2} + j \cdot \omega \cdot L_{k2}) + j \cdot \omega \cdot \dot{I}_1 \cdot M_{k1}) \cdot W_{k2}; \quad (2)$$

$$\begin{aligned} \dot{U}_1 = & j \cdot \omega \cdot \Phi \cdot W_1 + \\ & + \dot{I}_1 \cdot \sum_1^n (r_{i1} + j \cdot \omega \cdot L_{i1}) \cdot W_{i1} - \dot{I}_2 \cdot j \cdot \omega \cdot \sum_1^n M_{i2} \cdot W_{i1}; \end{aligned} \quad (3)$$

$$\begin{aligned} \dot{U}_2 = & j \cdot \omega \cdot \Phi \cdot W_2 - \\ & - \dot{I}_2 \cdot \sum_1^m (r_{k2} + j \cdot \omega \cdot L_{k2}) \cdot W_{k2} + \dot{I}_1 \cdot j \cdot \omega \cdot \sum_1^m M_{k1} \cdot W_{k2}; \end{aligned} \quad (4)$$

$$\Phi = \frac{\dot{I}_1 \cdot W_1 - \dot{I}_2 \cdot W_2}{R_{mag}}; \quad (5)$$

$$\dot{I}_2 = \frac{\dot{U}_2}{Z_L}, \quad (6)$$

where  $W_1$  – number of turns of transformer primary winding;  
 $W_2$  – number of turns of transformer secondary winding;  
 $n$  – number of groups of turns, primary winding is split up into;

$m$  – number of groups of turns, secondary winding is split up into;

$\Phi$  – main magnetic flux in magnetic core, caused by current  $\dot{I}_1$  in primary winding and current  $\dot{I}_2$  in secondary winding;

$\omega$  – angular frequency of voltage applied to primary winding;

$\dot{U}_1$  – preset value of voltage applied to primary winding;

$\dot{U}_2$  – secondary winding voltage;

$\dot{U}_{i1}$  – voltage drop across  $i$ -group of primary winding turns;

$\dot{U}_{k2}$  – voltage drop across  $k$ -group of secondary winding turns;

$W_{i1}$  – number of turns in  $i$ -group of primary winding turns (wherein  $W_{11} + W_{21} + \dots + W_{i1} + \dots + W_{n1} = W_1$ );

$W_{k2}$  – number of turns in  $k$ -group of secondary winding turns (wherein  $W_{12} + W_{22} + \dots + W_{k2} + \dots + W_{m2} = W_2$ );

$r_{i1}, r_{k2}$  – active resistance of  $i$ -turn of primary winding and  $k$ -turn of secondary winding correspondingly;

$L_{i1}$  – partial leakage inductance corresponding to magnetic leakage flux  $\Phi_{i1}$ , linked with  $i$ -turn of primary winding when current flowing through the primary winding is 1 A;

$L_{k2}$  – partial leakage inductance corresponding to magnetic leakage flux  $\Phi_{k2}$ , linked with  $k$ -turn of secondary winding when current flowing through the secondary winding is 1 A;

$M_{i2}$  – partial mutual leakage inductance corresponding to magnetic leakage flux  $\Phi_{i2}$ , linked with  $i$ -turn of primary winding when current flowing through secondary winding is 1 A;

$M_{k1}$  – partial mutual leakage inductance corresponding to magnetic leakage flux  $\Phi_{k1}$ , linked with  $k$ -turn of secondary winding when current flowing through the primary winding is 1 A;

$\dot{R}_{mag}$  – complex reluctance of magnetic core to the main magnetic flux;

$\dot{Z}_L$  – preset complex impedance of the load connected to the secondary winding of transformer.

Values of  $\Phi_{i1}$ ,  $\Phi_{i2}$ ,  $\Phi_{k2}$ ,  $\Phi_{k1}$  are obtained by a numerical integration that is carried out by means of the personal computer using magnetic leakage field distribution, according to the recommendations stated in [11]. Complex reluctance of magnetic core to the main magnetic flux is defined by existing procedures with use of the given characteristics of a transformer steel, type and size of magnetic core. The same reluctance can be obtained experimentally (with use of a transformer magnetic core), according to valid standards.

The equations (1)–(6) can be modified and used for parameter determination of high-voltage autotransformer. For this purpose let us observe the output part of high-voltage winding in autotransformer model (let us denote it as winding 3). Different low-voltage loading with complex impedance  $\dot{Z}_L$  (for example, voltage analyzers, voltage meters, ADCs) is connected to the output of this winding. Let us consider that this winding contains groups of turns denoted as  $s=1, 2...p$ . Other (input) part of autotransformer high-voltage winding is denoted as winding 1 containing groups of turns, in their turn denoted as  $i=p+1, p+2...n$  (in total  $n$   $p$ ). Considering the concerted directions of current flowing through windings 1 and 3, instead of (1)–(6) we can write down the following system of equations:

$$\dot{U}_{i1} = (j \cdot \omega \cdot \dot{\Phi} + \dot{I}_1 \cdot (r_{i1} + j \cdot \omega \cdot L_{i1}) + j \cdot \omega \cdot \dot{I}_3 \cdot M_{i3}) \cdot W_{i1}; \quad (7)$$

$$\dot{U}_{s3} = (j \cdot \omega \cdot \dot{\Phi} + \dot{I}_3 \cdot (r_{s3} + j \cdot \omega \cdot L_{s3}) + j \cdot \omega \cdot \dot{I}_1 \cdot M_{i3}) \cdot W_{s3}; \quad (8)$$

$$\dot{U}_1 = j \cdot \omega \cdot \dot{\Phi} \cdot W_1 + \dot{I}_1 \cdot \dot{Z}_{11} + \dot{I}_3 \cdot \dot{Z}_{13}; \quad (9)$$

$$\dot{U}_3 = j \cdot \omega \cdot \dot{\Phi} \cdot W_3 + \dot{I}_3 \cdot \dot{Z}_{33} + \dot{I}_1 \cdot \dot{Z}_{31}; \quad (10)$$

$$\dot{\Phi} = \frac{\dot{I}_1 \cdot W_1 + \dot{I}_3 \cdot W_3}{\dot{R}_{mag}}; \quad (11)$$

$$\dot{I}_{3L} = \frac{\dot{U}_3}{\dot{Z}_L}; \quad (12)$$

$$\dot{I}_{3L} + \dot{I}_3 = \dot{I}_1; \quad (13)$$

$$\dot{U}_1 + \dot{U}_3 = \dot{U}_0; \quad (14)$$

where  $\dot{U}_{i1}$  – voltage drop across  $i$ -group of turns of winding 1;

$\dot{U}_{s3}$  – voltage drop across  $s$ -group of turns of winding 3;

$L_{i1}$  – partial leakage inductance corresponding to magnetic leakage flux  $\Phi_{i1}$ , linked with  $i$ -turn of winding 1 when current flowing through the winding 1 is 1 A;

$L_{s3}$  – partial leakage inductance corresponding to magnetic leakage flux  $\Phi_{s3}$ , linked with  $s$ -turn of winding 3 when current flowing through the winding 3 is 1 A;

$M_{i3}$  – partial mutual leakage inductance corresponding to magnetic leakage flux  $\Phi_{i3}$ , linked with  $i$ -turn of winding 1 when current flowing through winding 3 is 1 A;

$M_{s1}$  – partial mutual leakage inductance corresponding to magnetic leakage flux  $\Phi_{s1}$ , linked with  $s$ -turn of winding 3 when current flowing through winding 1 is 1 A;

$W_{i1}$  – number of turns in  $i$ -group of winding 1 turns (wherein  $W_{(p+1)1} + W_{(p+2)1} + \dots + W_{i1} + \dots + W_{n1} = W_1$ , where  $W_1$  – total number of turns in winding 1);

$W_{s3}$  – number of turns in  $s$ -group of winding 3 turns (wherein  $W_{13} + W_{23} + \dots + W_{s3} + \dots + W_{p3} = W_3$ , where  $W_3$  – total number of turns in winding 3);

$\dot{I}_1$ ,  $\dot{I}_3$  – current flowing through the winding 1 and 3 correspondingly;

$r_{i1}$ ,  $r_{s3}$  – active resistance of  $i$ -turn of winding 1 and  $s$ -turn of winding 3 correspondingly;

$\dot{I}_{3L}$  – load current;

$\dot{U}_0$  – preset value of input voltage applied to series connection of high-voltage autotransformer winding 1 and 3, wherein  $\dot{U}_0 = \dot{U}_1 + \dot{U}_3$ , where  $\dot{U}_1$  – voltage drop across winding 1 and  $\dot{U}_3$  – voltage drop across winding 3. Parameters ( $\dot{Z}_{11}$ ,  $\dot{Z}_{13}$ ,  $\dot{Z}_{31}$ ,  $\dot{Z}_{33}$ ) are given by the following expressions:

$$\dot{Z}_{11} = \sum_{i=p+1}^n (r_{i1} + j \cdot \omega \cdot L_{i1}) \cdot W_{i1};$$

$$\dot{Z}_{13} = j \cdot \omega \cdot \sum_{i=p+1}^n M_{i3} \cdot W_{i1};$$

$$\dot{Z}_{31} = j \cdot \omega \cdot \sum_{s=1}^p M_{s1} \cdot W_{s3};$$

$$\dot{Z}_{33} = \sum_{s=1}^p (r_{s3} + j \cdot \omega \cdot L_{s3}) \cdot W_{s3}.$$

Other parameters in (7)–(14) are similar to the parameters in expressions (1)–(6).

System of equation (7)–(14) contains  $n$  unknown variables  $\dot{U}_{i1}$ ,  $\dot{U}_{s3}$ , and also  $\dot{U}_1$ ,  $\dot{U}_3$ ,  $\dot{I}_1$ ,  $\dot{I}_3$ ,  $\dot{I}_{3L}$ ,  $\dot{\Phi}$ . Thus, the total number of equations is  $n+6$ , hence this system is solvable.

Under no-load conditions of high-voltage autotransformer (at  $\dot{Z}_L \rightarrow \infty$ ), the main factors influencing the voltage distribution across its winding turns are: vectors  $j \cdot \omega \cdot \dot{\Phi}$  and also partial and mutual leakage inductances for each group of winding turns. This is a consequence of the fact that usually the diameter of the winding copper wire is chosen to be overestimated for reasons of mechanical strength when convoluting. So, the real part of the impedance of the winding turn groups is small compared to the above-mentioned influencing factors and can be neglected.

In this case, the same no-load current  $\dot{I}_0$  is flowing through the entire winding of high-voltage autotransformer with the number of winding turns  $W_0 = W_1 + W_3$  (as well as

in winding parts 1 and 3). Therefore, equations (9), (10) can be transformed into the following forms:

$$\dot{U}_1 = j \cdot \omega \cdot \Phi \cdot W_1 + j \cdot \omega \cdot \dot{I}_0 \cdot \sum_{i=p+1}^n (L_{i1} + M_{i3}) \cdot W_{i1}; \quad (15)$$

$$\dot{U}_3 = j \cdot \omega \cdot \Phi \cdot W_3 + j \cdot \omega \cdot \dot{I}_0 \cdot \sum_{s=1}^p (L_{s3} + M_{s1}) \cdot W_{s3}. \quad (16)$$

Let's define the transformation ratio as the ratio of the input and output voltages of high-voltage autotransformer:

$$K_{ATV} = \frac{\dot{U}_1 + \dot{U}_3}{\dot{U}_3}, \quad (17)$$

where

$$\dot{U}_1 + \dot{U}_3 = \dot{U}_0 = j \cdot \omega \cdot \Phi \cdot (W_1 + W_3) + j \cdot \omega \cdot \dot{I}_0 \cdot \left[ \sum_{s=1}^p (L_{s3} + M_{s1}) \cdot W_{s3} + \sum_{i=p+1}^n (L_{i1} + M_{i3}) \cdot W_{i1} \right]. \quad (18)$$

According to [11], it can be shown that the expression in square brackets in (18) is equal to the sum:  $\sum_{ji=1}^n L_{ji} \cdot W_{ji}$ , where  $ji$  – generalized numbering of the aforementioned winding groups 3 and 1;  $L_{ji} \cdot W_{ji}$  – partial leakage inductances of these winding turn groups when current flowing through the entire winding of autotransformer is 1 A.

In this case, expression (17) is transformed into the form:

$$K_{AT} = \frac{j \cdot \omega \cdot \Phi \cdot W_0 + j \cdot \omega \cdot \dot{I}_0 \cdot \sum_{i=1}^n L_{ji} \cdot W_{ji}}{j \cdot \omega \cdot \Phi \cdot W_3 + j \cdot \omega \cdot \dot{I}_0 \cdot \sum_{i=1}^p L_{ji} \cdot W_{ji}} = \frac{W_0 \cdot \left( \Phi + \frac{1}{W_0} \cdot \dot{I}_0 \cdot \sum_{i=1}^n L_{ji} \cdot W_{ji} \right)}{W_3 \cdot \left( \Phi + \frac{1}{W_3} \cdot \dot{I}_0 \cdot \sum_{i=1}^p L_{ji} \cdot W_{ji} \right)}. \quad (19)$$

It follows from (19) that the condition for the ideal voltage transformation:

$$K_{ATV} = \frac{W_0}{W_3}$$

by autotransformer is the following equality:

$$\frac{1}{W_0} \cdot \sum_{i=1}^n L_{ji} \cdot W_{ji} = \frac{1}{W_3} \cdot \sum_{i=1}^p L_{ji} \cdot W_{ji}$$

or

$$\begin{aligned} & \frac{1}{W_0} \cdot \sum_{i=1}^n L_{ji} \cdot W_{ji} - \frac{1}{W_3} \cdot \sum_{i=1}^p L_{ji} \cdot W_{ji} = \\ & = \frac{1}{W_0} \cdot \left( \sum_{i=1}^n L_{ji} \cdot W_{ji} - \frac{W_0}{W_3} \cdot \sum_{i=1}^p L_{ji} \cdot W_{ji} \right) = \frac{1}{W_0} \cdot Ls'_{eqv} = 0, \end{aligned}$$

where the relative equivalent leakage inductance of the autotransformer is:

$$Ls'_{eqv} = \sum_{i=1}^n L_{ji} \cdot W_{ji} - \frac{W_0}{W_3} \cdot \sum_{i=1}^p L_{ji} \cdot W_{ji}. \quad (20)$$

Using the system of equations (7)–(14) and the methodology described in [11], it was possible to develop a high-voltage transformer of voltage class 10 kV with a maximum operating voltage of 19 kV and an accuracy class of 0.05.

A feature of this high-voltage autotransformer is the possibility of obtaining 4 rated output voltages (100/3, 100/√3, 100, 300 V) available for its 3 rated input voltages (3, 6, 10 kV). Fig. 1 shows the photo of this high-voltage autotransformer.

The developed and manufactured 10 kV class high-voltage autotransformer is metrologically certified and implemented in the State Enterprise «Ukrmetrteststandard» (Kyiv, Ukraine) as a part of the 110 kV class secondary high-voltage reference unit that uses 50 Hz as alternating current frequency.

The results of experimental studies of high-voltage autotransformer, designed and created on the basis of the above theoretical propositions, allow to conclude that the theoretical positions developed in this study are effective, taking into account, as an additional factor, the effect of capacitive currents, in accordance with [12].



**Fig. 1.** 10 kV class high-voltage autotransformer. View from the side of 3 kV and 6 kV windings

A wide range of input and output voltages and a high accuracy class of a high-voltage autotransformer described above allow us to conclude that it is possible to design and manufacture other high-voltage autotransformers with any range of transformation ratios. According to the above research, the best accuracy of voltage transformation can be achieved under conditions as close as possible to the no-load conditions of high-voltage autotransformer. Since digital electronic devices connected to the output of high-voltage transformers tend to cause minimal load currents, the procedure described in [11], in conjunction with the proposed system of autotransformer equations, will allow to proceed to the design of high-voltage autotransformers intended for use in Smart Grid networks.

## 7. SWOT analysis of research results

*Strengths.* The strengths of this research are the possibility of mathematical modeling of various operating conditions of high-voltage autotransformers, determination of their parameters and characteristics. This feature allows, at the stage of designing expensive high-voltage measuring

electrical equipment to predict its characteristics with high accuracy, taking into account the conditions of its load in operation.

*Weaknesses.* The weaknesses of this research are that at present, practically there is no normative documentation regarding the use of high-voltage autotransformers as primary voltage transformers in high-voltage measuring systems.

*Opportunities.* The opportunities for further research provided by this study include the ability to determine the effect of the magnitude and condition of the load on the transformation ratio of high-voltage instrument autotransformer. Also, for each specific model of voltage autotransformer, this research allows to determine the number and location of winding turn groups, allowing achieving the best matching between real and normative value of transformation ratio.

*Threats.* The threats to the results of the conducted studies are that mathematical model of the active part of high-voltage autotransformer is intended for use at the designing of high-precision and multi-band primary voltage transformers only of autotransformer type, including for use in Smart Grid networks, as well as in various scientific developments.

## 8. Conclusions

1. Mathematical model of the active part of a high-voltage autotransformer in the form of equation system is proposed, making it possible to detail the parameters of the winding up to the level of single turn (or groups of turns). This system of equation allows considering magnitude and type of autotransformer loading, presence of several output parts of its winding.

2. Among the factors influencing the voltage distribution across winding turns of high-voltage autotransformer under conditions close to the no-load conditions are: the electromotive forces induced by the main magnetic flux, as well as the partial leakage inductance of single turn (or groups of turns). At the same time, the paper gives recommendations on minimizing the effect of partial leakage inductances on the voltage distribution across winding turns.

3. The theoretical calculations carried out using the developed mathematical model of high-voltage autotransformer showed a high degree of matching between theoretical and experimental data. All these results allowed designing and manufacturing 10 kV high-voltage autotransformer for the state metrological center. Theoretical propositions of this work, as well as carried out experimental studies of the 10 kV reference autotransformer, allow us to conclude that high-voltage autotransformers can be used in Smart Grid networks as primary voltage transformers.

## References

1. Brzhezitskyi, V. Detailing of the transformer equation to the single winding turns (groups of the winding turns) [Text] / V. Brzhezitskyi, Ya. Haran, I. Masliuchenko // Technology audit and production reserves. – 2016. – Vol. 1, No. 1(27). – P. 32–37. doi:10.15587/2312-8372.2016.59101
2. Xu, Q. Real-Time Generation Dispatch and Communication Architecture of Smart Grid with Renewable Energy [Text] / Q. Xu, C. Deng, L. Chen // Journal of Communications. – 2013. – Vol. 8, No. 8. – P. 497–504. doi:10.12720/jcm.8.8.497-504
3. Yilmaz, C. Smart Grid Architectures and the Multi-Agent System Paradigm [Text] / C. Yilmaz, S. Albayrak, M. Lützenberger // Energy 2014: The Fourth International Conference

on Smart Grids, Green Communications and IT Energy-aware Technologies. – 2014. – P. 90–95.

4. Anderson, K. GridSpice: A Distributed Simulation Platform for the Smart Grid [Text] / K. Anderson, J. Du, A. Narayan, A. E. Gamal // IEEE Transactions on Industrial Informatics. – 2014. – Vol. 10, No. 4. – P. 2354–2363. doi:10.1109/tii.2014.2332115
5. Arya, A. K. Role of Smart Grid to Power System Planning and Operation in India [Text] / A. K. Arya, S. Chanana, A. Kumar // Proceedings of International Conference on Emerging Trends in Engineering and Technology. – 2013. – P. 793–802.
6. Miceli, R. A Perspective on the Future of Distribution: Smart Grids, State of the Art, Benefits and Research Plans [Text] / R. Miceli, S. Favuzza, F. Genduso // Energy and Power Engineering. – 2013. – Vol. 05, No. 01. – P. 36–42. doi:10.4236/epe.2013.51005
7. Vijayapriya, P. Smart Tariff for Smart Meters In Smart Grid [Text] / P. Vijayapriya, G. Bapna, D. P. Kothari // International Journal of Engineering and Technology. – 2010. – Vol. 2, No. 5. – P. 310–315.
8. Khandekar, N. Non-Intrusive Appliance Load Monitoring System Using Zigbee Protocol [Text] / N. Khandekar, K. Thube, N. Patil, P. B. Mane // International Journal of Engineering Research & Technology (IJERT). – 2014. – Vol. 3, No. 4. – P. 2415–2417.
9. Janjic, A. Power Quality Requirements for the Smart Grid Design [Text] / A. Janjic, Z. Stajic, I. Radovic // International Journal of Circuits, Systems and Signal Processing. – 2011. – Vol. 5, No. 6. – P. 643–651.
10. Maitra, S. Smart Energy meter using Power Factor Meter and Instrument Transformer [Text] / S. Maitra // Communications on Applied Electronics. – 2016. – Vol. 4, No. 1. – P. 31–37. doi:10.5120/cae2016652015
11. Brzhezitsky, V. O. Leakage Inductance Calculation of High-Voltage Transformer Windings by Means of the Software using the Finite Elements Method [Text] / V. O. Brzhezitsky, Ja. O. Garan, O. M. Desjatov // Technical Electrodynamics. – 2014. – No. 4. – P. 61–63.
12. Brzhezitskyi, V. Analysis of capacitive currents in the winding of a high voltage measuring autotransformer [Text] / V. Brzhezitskyi, Ya. Haran // Technology audit and production reserves. – 2016. – Vol. 4, No. 1(30). – P. 70–76. doi:10.15587/2312-8372.2016.74694

## **МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ВЫСОКОВОЛЬТНОГО ИЗМЕРИТЕЛЬНОГО АВТОТРАНСФОРМАТОРА, ПРЕДНАЗНАЧЕННОГО ДЛЯ ИСПОЛЬЗОВАНИЯ В СЕТЯХ SMART GRID**

Предложена математическая модель активной части высоковольтного автотрансформатора напряжения. Модель позволяет определять распределение напряжения по виткам и группам витков обмотки, учитывая параметры приложенного напряжения, характеристики магнитного сердечника, индуктивность рассеяния. Модель рассматривается как средство для проектирования высоковольтного измерительного электрооборудования сетей Smart Grid.

**Ключевые слова:** автотрансформатор, трансформатор напряжения, Smart Grid, математическая модель, индуктивность рассеяния.

*Brzhezitsky Volodymyr, Doctor of Technical Sciences, Professor, Department of High Voltage Engineering and Electrophysics, National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute», Ukraine, e-mail: v.brzhezitsky@kpi.ua, ORCID: <http://orcid.org/0000-0002-9768-7544>*

*Haran Yaroslav, Engineer, Department of High Voltage Engineering and Electrophysics, National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute», Ukraine, e-mail: y.garan@kpi.ua, ORCID: <http://orcid.org/0000-0003-3242-9218>*

*Trotsenko Yevgeniy, PhD, Assistant Professor, Department of High Voltage Engineering and Electrophysics, National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute», Ukraine, e-mail: y.trotsenko@kpi.ua, ORCID: <http://orcid.org/0000-0001-9379-0061>*