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**Vorotnikov V.,
Gumenyuk I.,
Pozdniakov P.**

PLANNING THE FLIGHT ROUTES OF THE UNMANNED AERIAL VEHICLE BY SOLVING THE TRAVELLING SALESMAN PROBLEM

Розглянуто методи розв'язання задачі комівояжера для планування маршрутів польоту безпілотних літальних апаратів та проаналізовано результати роботи. Показано, що метод осереднених коефіцієнтів розв'язує задачу найоптимальніше за критерієм відстані, використання якого забезпечує мінімальні експлуатаційні витрати польоту безпілотних літальних апаратів та дає суттєвий виграш порівняно з іншими методами (5–10 %).

Ключові слова: *задача комівояжера, мінімальний маршрут, планування маршрутів, безпілотні літальні апарати.*

1. Introduction

At present, considerable experience has been accumulated in development of unmanned aerial vehicles (UAVs). At the same time, the question of effective application, in accordance with considerable number of publications in this direction in the publications of near and far abroad, is open. UAVs have a number of advantages, namely: low operating costs, low radar and optical visibility, stability and flexibility, simple and affordable creation technology. UAVs can even be used in cases where the use of manned aircraft is impractical, expensive or risky [1].

One of the most important tasks of providing UAVs flights is the task of route planning, consists in determining the set of points in space corresponding to the UAV flight path and determined on the map. The following factors influence the route selection: limited flight time, flight safety, multiplicity of routes. There are a number of methods and algorithms for solving this problem. For example, attempts were made to solve the problem of route planning using the geometric approach. In this case, the optimal trajectory in the solution of the problem may turn out to be such that it is not physically realized. In this case, we have a problem with a restriction on the curvature of the flight, or in principle these approaches are not applicable in the conditions of a large number of reference points along which the route is constructed [2]. However, they do not solve the problem with the desired results.

Most of them are inapplicable to large-dimensional problems, since the complexity of these algorithms grows exponentially. Often it is necessary to choose between the time of the algorithm and the quality of the results.

Such optimization is relevant in the context of limiting the UAV's energy resources and the timing of decision making for flight planning with the corresponding possible operating costs.

2. The object of research and its technological audit

The object of research is the process of planning the UAV flight paths. One of the most problematic issues in this process is the choice of the optimal flight route by the criteria of minimum size and the minimum possible time for planning. This is due to the fact that the existence of a large number of methods for solving this problem does not always give the necessary result. The reason for this is the imperfection of these methods and the exponential growth of algorithmic complexity with an increase in the number of flight reference points. Most of these methods solve the posed problem only on one criterion (the time for planning or the obtained value of the length of the route).

Thus, the main direction of research of the UAV flight planning process is the analysis of possible algorithms for solving the traveling salesman problem for solving this problem and choosing the optimal one based on the time complexity criterion and the obtained route length of the UAV flight planning method. This will reduce the energy costs of the UAV during the flight.

3. The aim and objectives of research

The aim of research is analysis of the existing methods for solving the traveling salesman problem and to choose the optimal solution to the transportation problem for planning the flight route of UAVs of terrestrially territorially separated nodes in real time. Solving this problem will lead to a reduction in operating costs.

For this aim it is necessary to solve the following tasks:

1. To study the work of existing methods or algorithms for solving the traveling salesman problem (Monte-Carlo, reduction of rows and columns, averaged coefficients) for a different number of objects.

2. To determine the optimal method for solving the problem of time complexity and length of the route of solving the traveling salesman problem for planning UAV flight on the basis of analysis of the analytical values of numerical calculations obtained during the investigation.

3. To consider the obtained route as the optimal (suboptimal) route of UAV flight.

4. Research of existing solutions of the problem

A number of scientific papers are devoted to solving the traveling salesman problem, including for planning the flight routes of UAV. In particular, the approach [1, 2] has been implemented to form short UAV flight routes in the constant wind field based on the solution of varieties of traveling salesman problems. At the same time, the complexity of the proposed approaches for solving the effective flight planning of UAVs is noted. The proposed method [3] of laying the optimal UAV flight route for collecting information from remote sensors by the criterion of the minimum of the traversed path provides a significant gain in comparison with existing methods. However, this does not take into account the optimization of the time to solve the UAV flight planning problem while reducing the number of flight reference points. One of the approaches [4] to the solution of the UAV group flight planning problem for observing spatially distributed point, linear and planar targets is considered. This approach is based on the presentation of the task of planning transport routing with loading and its further decomposition into subtasks. The complexity of using such approach consists in an exponential increase in the time to solve the task posed to increase the number of point targets. The proposed algorithm [5] is not physically realized in the conditions of a large number of reference points along which the route is built. In [6, 7], the application of heuristic approaches to the traveling salesman problem for flight planning of UAV groups with minimization of the route length is considered. The use of such approaches does not provide an appropriate optimization solution to the UAV flight planning problem. When using the genetic algorithm [8], the time criterion for solving the problem is not taken into account. In [9], the motion control problem for groups of UAVs and unmanned ground robots as part of a system of unmanned vehicles for performing tasks is considered. It does not take into account the time limit for the UAV flight planning. The considered algorithms [10–12] have problems of similarity and accuracy of the obtained results, which are not always optimal.

The imperfection of the considered approaches necessitates a detailed investigation and selection of the UAV flight planning method that is optimal by the time complexity criterion and the obtained route length.

5. Methods of research

The traveling salesman problem is one of the most famous tasks of combinatorial optimization. It consists in finding the possible optimal route, passing through the given points at least once and then returning to the starting position. Under the conditions of the problem, the criterion for the optimality of the route (the shortest, cheapest, aggregate criterion, etc.) and the corresponding distance (cost) matrix are indicated. As a rule, it is indicated that the route must pass through each point only once – in this case, the choice is made among the Hamiltonian cycles [3].

The task of optimizing the UAV movement can be formulated in the context of the transport task, decision-making tasks, resource management, scheduling, etc. [4]. The formulation of the problem of this class is characterized by many parameters and optimality criteria that determine the greater dimensionality of the solution space, but the main criteria are the minimization of the time for solving the problem and the path of the UAV route [5].

The general formulation of the problem of choosing the optimal route for flying around nodes of the network belongs to the class of NP-complex problems. All the exact algorithms are in fact an optimized full search of variants, in the case of limitations on on-board computers, it is unacceptable, accordingly, it becomes necessary to solve the suboptimal (approximate to the optimal) problem. The task is to minimize the goal function [6]. In this case, certain restrictions must be met, reflecting the condition under which the UAV must fly each object (network node) only once:

$$F(x)=\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}x_{ij}\rightarrow \min;$$

$$\sum_{i=1}^{N}x_{ij}=1,\ \forall j=\overline{1,N};$$

$$\sum_{j=1}^{N}x_{ij}=1,\ \forall i=\overline{1,N}, \qquad (1)$$

where c_{ij} – distance between nodes (i,j);

$$x_{ij}=\begin{cases}1, & \text{the movement of the UAV from node I to node J;}\\ 0, & \text{there is no such movement.}\end{cases}$$

In this case, the trajectory is closed, that is, the starting and ending positions of the UAV are the same.

Condition (1) is an assignment problem, the solution of which contains n variables that are equal to one, and others $n(n-2)$ are zero. The solution can consist of several simple vertex-non-transitional loops (subcycles) passing through the number of objects (nodes of the network) less n [7]. Accordingly, to obtain a route passing through the objects, one should also take into account the cyclicity requirements:

$$u_i-u_j+nx_{ij}\le n-1;\quad j=\overline{1,N};\quad i=\overline{1,N};\quad i\ne j, \qquad (2)$$

where u_i – the number of the stage at which the UAV reached the point i [8].

However, as the number of objects increases, the complexity and solution of the problem increase. To solve this problem, a recursive full search method has been proposed, which allows solving this problem up to 1000 knots for the necessary time, but the solutions obtained by these methods are not always optimal [9]. There is a need to apply certain heuristic approaches. It is advisable to plan the route not for all the objects (nodes of the network), but group by the radius of the UAV coverage zone and move around the centers of attraction of these groups. In this way, the traveling salesman problem is solved, but of a lesser dimension, while saving the solution time and the UAV path [10, 11].

Let's analyze a number of methods for solving the traveling salesman problem.

5.1. The solution of the traveling salesman problem by the Monte Carlo method. Monte Carlo methods call any procedure that uses a statistical sample, including to find the solution of the traveling salesman problem (a random

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number generator is used). At the first stage among the reference points with numbers (point No. 1 – initial) is randomly determined by one point, forming a sequence. Let's note that in the future this sequence is considered the optimal route. For the obtained route, the goal function is calculated, after which the procedure is repeated. In the case that the goal function has not changed or has the worst value, the result is not taken into account. Otherwise, this value is the solution of the traveling salesman problem. Let's note that this algorithm allows for a short period of time to calculate a significant number of routes and choose among them suboptimal, but the best.

5.2. The solution of the traveling salesman problem by the method of reduction of rows and columns. The algorithm for solving the traveling salesman problem by this method consists of the following stages and rules:

1. Let's find the upper possible boundary of the goal function, for which it is necessary to select an arbitrary route and calculate the value of this function $F_{\max}(x)$.

2. Let's find the minimal element $A_i = \min_j(c_{ij})$ in each row of the matrix $C = \|c_{ij}\|$ and subtract it from all the elements of the corresponding row and write it down to the last column (reduction of rows) and get the matrix:

$$c'_{ij} = c_{ij} - \min_j c_{ij}.$$

3. If there are columns appearing in the matrix C' represented by rows, do not contain zero values, let's perform column reduction. For this, in each column of the matrix C' let's select the minimal element B_j, $j = \overline{1,n}$ and subtract it from all the elements of the corresponding column and obtain the matrix:

$$C'' = \left\| c_{ij} - \min_j c_{ij} - \min_i c'_{ij} \right\|,$$

each row and column of which contains at least one zero value. Such matrix is called reduced in rows and columns.

4. Let's summarize the elements A_i and B_j, and get the lowest goal function:

$$F_{\min}(x) = \sum_{i=1}^{n} A_i + \sum_{j=1}^{n} B_j.$$

5. Let's check the condition of having one zero value in each row or column. When the condition of interchanges is satisfied, the goal function has the form:

$$F_{\min}(x) \le F(x) \le F_{\max}(x),$$

if this condition is not fulfilled, let's proceed to determine one of the steps of the optimal path. To do this, let's define the line penalties a_i (the smallest value of the i-th line after the first zero value) and the column penalties b_j (the smallest value of the i-th column after the first zero value).

6. For non-zero cells, let's determine the secondary penalties $a_{ij} = a_i + a_j$ and find among them the maximum value with the numbers: i_0-row and j_0-column, which, in accordance with the cross out of the matrix $C = \|c_{ij}\|$, and the arc $(i_0; j_0)$ is entered into the route.

7. Performing the same steps (10–6) of $(n-2)$ steps, let's obtain a matrix $C^n = \|c_{ij}^{n-1}\|$ with two rows and columns, and the arc $(i_n; j_n)$ is the final part of the route. This completes the solution of the problem.

5.3. The solution of the traveling salesman problem by the averaged coefficient method. The traveling salesman problem is solved by the method of averaged coefficients by the $(n-2)$ stage.

1. Let's find in each row and column of the matrix $C = \|c_{ij}\|$ the average value of the row $\overline{c}_i$ and column $\overline{c}_j$. For each cell, let's obtain the averaged coefficients, which are calculated by the difference in the elements of the matrix $C = \|c_{ij}\|$ and the sum of the average values of the row and column are recorded in the matrix:

$$c'_{ij} = c_{ij} - \left(\overline{c}_i + \overline{c}_j\right).$$

2. Among the averaged coefficients, let's determine the smallest $U_i = \min_j(c_i)$ or $U_j = \min_i(c_j)$, and the arc $(i_0; j_0)$ is recorded in the UAV flight route. Cross out the i_0-row and j_0-column of the matrix C'.

3. In the matrix C' is similar (paragraph 1 of this method) let's find in each row and column of the matrix $C' = \|c_{ij}\|$ the average value of the row $\overline{c}_i$ and column $\overline{c}_j$. Let's obtain the averaged coefficients for each cell, which are calculated by the difference of the matrix elements $C' = \|c_{ij}\|$ and the sum of the average values of the row.

4. Among the averaged coefficients, let's determine the smallest $U_i = \min_j(c'_i)$ or $U_j = \min_i(c'_j)$, and the arc $(i_1; j_1)$ is recorded in the UAV flight route. Cross out the i_1-row and j_1-column of the matrix C'.

5. Performing the same steps (1–2) of $(n-2)$ steps, let's obtain a matrix $C^n = \|c_{ij}^{n-1}\|$ with two rows and columns, and the arc $(i_n; j_n)$ is the final part of the route. This completes the solution of the problem [12].

6. Research results

Investigation of efficiency of the considered algorithms for solving the traveling salesman problem for UAV flight planning will be carried out with the help of a software implementation developed by the authors. For the input data, let's use geographically distributed objects in the amount of $n = 50$ (Fig. 1), which are the reference points for the UAV flight route. These objects are located on a plane (area of terrain) of a given size. The UAV moves a straight path in space at a constant height $H = \text{const}$ with a constant speed $v = \text{const}$ along a certain route, characterized by a set of reference points of space with the coordinates of the projection to the earth's surface.

The reference points in this paper are depicted by these objects, and the initial and final positions of the UAV are in the first object.

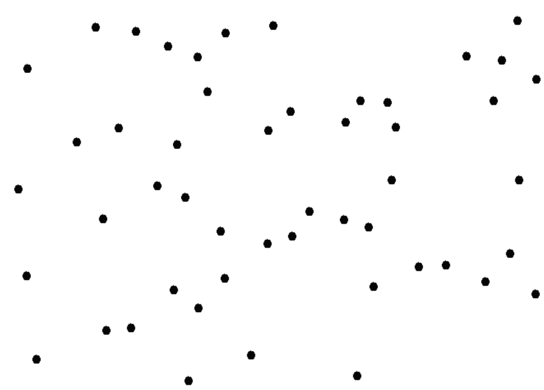


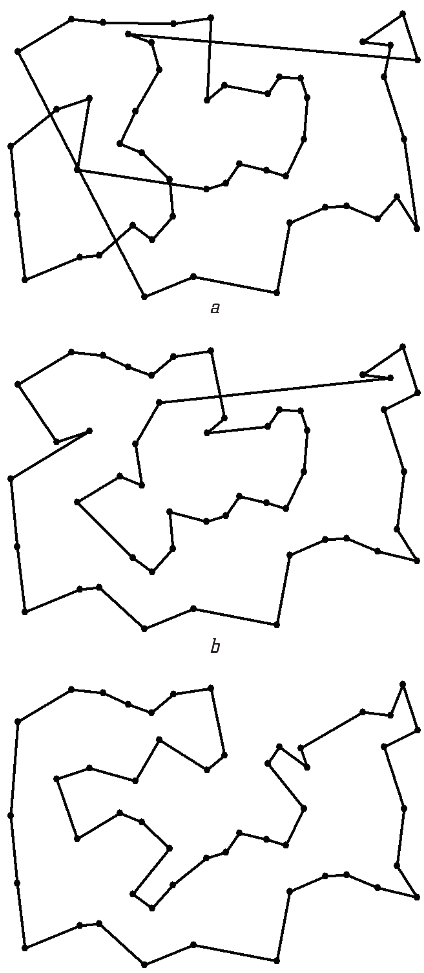
Fig. 1. Reference points for planning the flight path ($n = 50$ objects)

Table 1 shows the analytical values of numerical calculations, and Fig. 2 geometrically reflects the results of the work of algorithms for solving the traveling salesman problem: Monte Carlo, reduction of rows and columns, averaged coefficients.

Table 1

Results of algorithms for solving the traveling salesman problem ($n = 50$ objects)

Algorithm for solving the traveling salesman problem	Solution time (*s*)	Route length (*m*)
Monte Carlo	0.1	3714
Reduction of rows and columns	2.1	3621
Averaged coefficients	1.2	3484



a b c

Fig. 2. Optimal UAV flight routes based on the results of solving the traveling salesman problem by methods ($n = 50$ objects): *a* – Monte Carlo; *b* – reduction of rows and columns; *c* – averaged coefficients

For input data, let's use geographically distributed objects in the amount of $n = 100$ (Fig. 3), which are the reference points for the UAV flight route.

Table 2 shows the analytical values of numerical calculations, and Fig. 4 geometrically reflects the results of the work of algorithms for solving the problem traveling salesman: Monte Carlo, reduction of rows and columns, averaged coefficients.

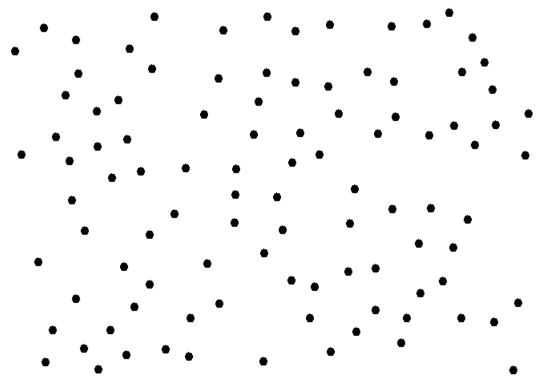
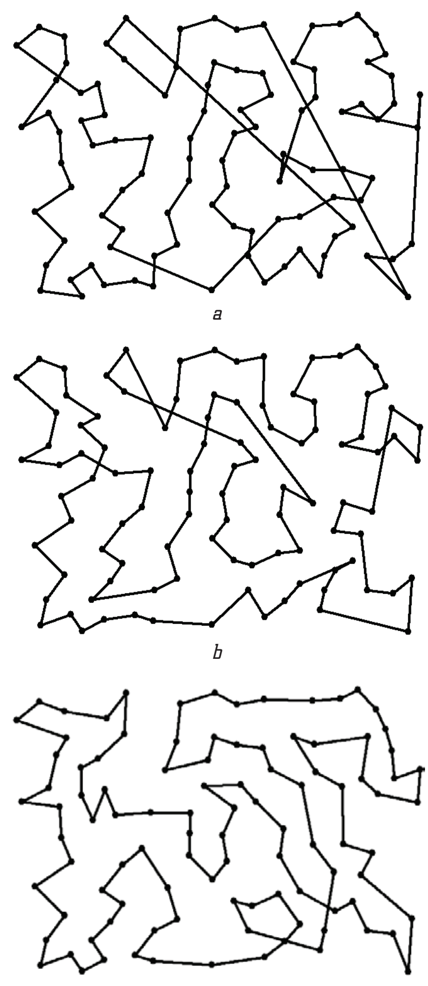


Fig. 3. Reference points for planning the flight path ($n = 100$ objects)

Table 2

Results of algorithms for solving the traveling salesman problem ($n = 100$ objects)

Algorithm for solving the traveling salesman problem	Solution time (*s*)	Route length (*m*)
Monte Carlo	0.5	6000
Reduction of rows and columns	3.9	5531
Averaged coefficients	2.5	5224



a b c

Fig. 4. Optimal UAV flight routes based on the results of solving the traveling salesman problem by methods ($n = 100$ objects): *a* – Monte Carlo; *b* – reduction of rows and columns; *c* – averaged coefficients

Fig. 4 clearly shows that with an increase in the number of objects in the work of Monte Carlo algorithms, reduction of rows and columns there are errors that affect the minimization of the route.

For the input data, let's use geographically distributed objects in the amount of $n=200$ (Fig. 5), which are the reference points for the UAV flight route.

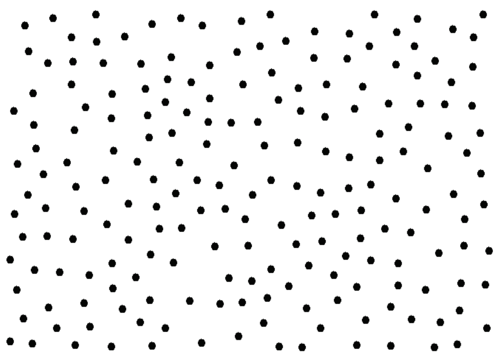


Fig. 5. Reference points for planning the flight path ($n=200$ objects)

Table 3 shows the analytical values of numerical calculations, and Fig. 6 geometrically reflects the results of the work of algorithms for solving the problem traveling salesman: Monte Carlo, reduction of rows and columns, averaged coefficients.

Table 3

Results of algorithms for solving the traveling salesman problem ($n=200$ objects)

Algorithm for solving the traveling salesman problem	Solution time (s)	Route length (m)
Monte Carlo	1.2	8760
Reduction of rows and columns	12.4	8105
Averaged coefficients	7.5	7977

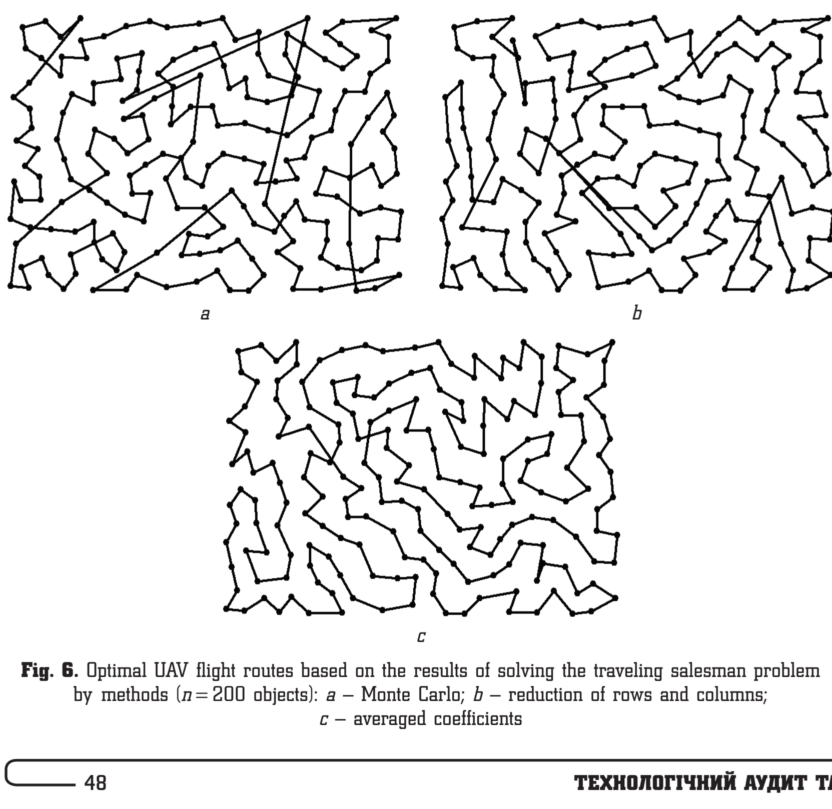


Fig. 6. Optimal UAV flight routes based on the results of solving the traveling salesman problem by methods ($n=200$ objects): *a* – Monte Carlo; *b* – reduction of rows and columns; *c* – averaged coefficients

Research results show that for different values of the number of reference points of the UAV flight route, the methods: reduction of rows and columns, averaged coefficients solve the traveling salesman problem with almost identical results of the length of the route. However, according to the time criterion for the solution, the latter has a significant advantage, which is especially noticeable with an increase in the number of objects.

7. SWOT analysis of research results

Strengths. The method of averaged coefficients is optimal for UAV flight planning according to the solution time criteria and route lengths in comparison with other methods. The use of this method ensures the minimum operating costs of a UAV flight. From a practical point of view, this will make it possible to make operative decisions on various types of operations involving the use of UAVs for both military and civilian purposes with minimal operating costs, while limiting the time for decision-making.

Weaknesses. During the research it is established that inaccuracies appear in the solution of the traveling salesman problem in Monte Carlo algorithms, reduction of rows and columns. In particular, the first algorithm produces a result with a high level of error, and the second with an increase in the number of objects minimizes these errors.

Opportunities. In future studies, it is planned to use this method for the UAV group, taking into account the breakdown of the wireless network into clusters to ensure the connectivity of mobile subscribers.

Threats. The complexity of implementing the results is that there are no modifications to existing UAV flight planning software that don't take into account software limitations in the number of reference points.

8. Conclusions

1. The effectiveness of the methods of solving the traveling salesman problem (Monte Carlo, reduction of rows and columns, averaged coefficients) is analyzed for planning the UAV flight paths using software developed by the authors. As reference points of the route, objects of different numbers are territorially spaced. The theoretical basis of these methods is given.

2. It is established that among the considered methods optimal for solution of the problem and path length is the algorithm of averaged coefficients. From the obtained results, it is clearly visible that this method yields a significant gain (5–10 %) in comparison with other methods, and an increase in the gain is rectilinear to an increase in the number of reference points.

3. UAV flight path obtained by solving the traveling salesman problem by the method of averaged coefficients is optimal (suboptimal) on the basis of the

numerical values of the analytical calculations obtained during the research.

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ПЛАНИРОВАНИЕ МАРШРУТОВ ПОЛЕТА БЕСПИЛОТНЫХ ЛЕТАТЕЛЬНЫХ АППАРАТОВ ПУТЕМ РЕШЕНИЯ ЗАДАЧИ КОММИВОЯЖЕРА

Рассмотрены методы решения задачи коммивояжера для планирования маршрутов полета беспилотных летательных аппаратов и проанализированы результаты работы. Показано, что метод усредненных коэффициентов решает задачу оптимально по критерию расстояния, использование которого обеспечивает минимальные эксплуатационные расходы полета беспилотных летательных аппаратов и дает существенный выигрыш (5–10 %) в сравнении с другими методами.

Ключевые слова: задача коммивояжера, минимальный маршрут, планирование маршрутов, беспилотные летательные аппараты.

Vorotnikov Vladimir, *Doctor of Technical Science, Associate Professor, Department of Computer Integrated Technologies and Cyber Security, Korolyov Zhytomyr Military Institute, Ukraine, e-mail: vvvorotnik@ukr.net, ORCID: http://orcid.org/0000-0001-8584-3901*

Gumenyuk Igor, *Adjunct, Department of Computer Integrated Technologies and Cyber Security, Korolyov Zhytomyr Military Institute, Ukraine, e-mail: ig_gum@ukr.net, ORCID: http://orcid.org/0000-0001-5853-3238*

Pozdniakov Pavel, *PhD, Chief of Research Laboratory, Research Centre, Korolyov Zhytomyr Military Institute, Ukraine, e-mail: pozdner86@gmail.com, ORCID: http://orcid.org/0000-0003-4188-9486*

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Riznyk V., Solomko M.

MINIMIZATION OF BOOLEAN FUNCTIONS BY COMBINATORIAL METHOD

Розглянуто поширення принципу мінімізації за допомогою алгебричних перетворень на метод мінімізації з використанням комбінаторної блок-схеми з повторенням. Математичний апарат блок-схеми з повторенням дає більше інформації стосовно ортогональності, суміжності, однозначності блоків комбінаторної системи, якою є власне таблиця істинності заданої функції, тому застосування такої системи мінімізації функції є більш ефективним.

Ключові слова: *булева функція, метод мінімізації, мінімізація логічної функції, блок-схема з повторенням, мінтерм.*

1. Introduction

The problems and shortcomings of the known methods for minimizing Boolean functions are associated with a rapid growth in the amount of computation, which results in an increase in the number of computational operations, and, consequently, in the increase in the number of variables of the logical function.