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DEVELOPMENT OF METHOD FOR OPTIMAL INVENTORY CONTROL UNDER CONTINUOUS SUPPLY OF PRODUCT AND RANDOM DEMAND

Сформульовано завдання стохастичної оптимізації для знаходження інтенсивності поставок однорідної продукції на склад постачальної фірми в умовах випадкового коливання інтенсивності попиту на неї. Для формалізованого опису роботи складу використано апарат напівмарковських процесів зі знесенням. Виведено систему інтегральних рівнянь для знаходження граничного розподілу ймовірностей цього процесу та знайдено її рішення для окремого випадку. Розглянуто узагальнення моделі стохастичної оптимізації на випадок декількох видів продукції.

Ключові слова: постачальницька фірма, запас продукції, безперервне постачання продукції, напівмарковський процес зі знесенням, оптимізація інтенсивності постачання.

1. Introduction

At the present time in the theory of inventory control and logistics a number of various models have been developed to optimize the replenishment processes based on certain strategies [1–3]. In the actual practice of the work of various supply or logistics companies, decisions about the time of replenishment and the size of the lot of the replenishment must be made under conditions of a random fluctuation in the demand for products, the restricted reliability of suppliers, the behavior of competitors. In such situations, stochastic models describing the external uncertainty of the market and the factors of internal uncertainty in the activities of the company itself (equipment failures, human factor) are naturally used to develop a replenishment strategy. Such models are known to be developed and investigated in such sections of operations research as queuing theory, stochastic storage theory, game theory and statistical decisions making, etc. In these theories, in particular, various types of the Markov and semi-Markov processes play an important role [4–7], allowing to describe the processes of replenishment and consumption of inventories in a variety of production systems and processes. However, the specificity of some production and logistics processes requires an expansion of the variety of types of Markov processes. For example, for a simultaneous description of the continuous nature of production in an industrial enterprise and the discrete nature of the operation of the main types of surface/water transport, it is expedient to introduce and study a special class of the Markov processes that include, along with a discrete component (chain), one or more continuous components of the random walk type with boundaries. In this case, continuous components can describe random fluctuations in the levels of inventories of various material stocks: raw materials, work in process, finished products. Such Markov processes in the special literature are usually called Markov processes with drift (MPD) [5]. At present, the theory of Markov processes of this type and its applications to logistical problems are in the process

of formation. Therefore, it is of theoretical and practical interest to analyze the possibilities of its application for optimizing inventory control in various production and logistics systems operating under uncertainty and risk.

2. The object of research and its technological audit

Managers of the supply firm or procurement department at an industrial enterprise often face the situation when it is necessary to make decisions regarding the volume of purchases of goods or raw materials under conditions of an uncertain volume of expected demand for products. Therefore, they need to use modern methods to predict the time series or methods of the theory of random processes in order to obtain a more or less reliable estimate of the magnitude of demand. Only after the fulfillment of such predictive assessments it is possible to formulate and solve the tasks of optimizing the timing and volumes of deliveries of goods purchased from suppliers. Scientific methods of solving these two problems are the object of research of experts in the field of logistics management in any enterprise where modern management methods are used.

3. The aim and objectives of research

The aim of this article is demonstration of the possibility of using the theory of semi-Markov processes with drift to construct and analyze the model of stochastic optimization of replenishment intensities of the stock of production in the warehouse of supply or logistics company, taking into account the random fluctuations in demand for products.

To achieve this aim, the following objectives are set:

1. Formalization of the description of the functioning of the warehouse of the supply firm, in which the intensity of stock replenishment and consumption is controlled by the Markov chain in terms of MPD.
2. Derivation of the system of integral equations for finding the limit probabilistic distribution of the quantity

of products stored in the warehouse at an arbitrary time, and the state of the Markov chain.

3. Finding the solution of the above system of integral equations.

4. Formulation and solution of the problem of finding the optimum values of the stock replenishment intensities of production in the warehouse with random fluctuations in the intensity of demand for this product.

4. Research of existing solutions of the problem

Development and analysis of mathematical models of logistics systems operating under conditions of uncertainty and risk has recently received considerable attention in the literature [7–16].

One of the first works on the MPD application to modeling and optimizing production and technical systems is the works [6, 7]. In the cited works, applications of these processes to the organization of industrial production and the design of information and computing systems are considered. In [8], the coordination between the strategies of the product supplier and the retailer is examined, taking into account the random fluctuation in demand for products. To model such interaction, it is proposed to use the theory of Markov decision processes. As a result, the authors propose to use not only two widely used delivery policies, namely a policy based on the delivery time of goods and a policy based on determining the quantity of delivered goods, and a mixed policy. In [9], in order to optimize the relationship between supplier and retailer in conditions of random demand with the known distribution law, it is proposed to use the method of stochastic optimization of the sizes of deliveries during small time intervals in order to minimize the average total costs including losses from unsatisfied demand and from surplus stocks.

In [10], it is proposed to use Markov chains with continuous time for a supply chain consisting of a raw material supplier, a producer of products and a distribution center. At the same time, the demand for products located in the distribution center is assumed to be subordinate to the distribution of the Poisson, and the producer places orders for production and ensures the delivery of finished products to the center at random intervals, distributed according to exponential laws.

The Markov chain describing the operation of this chain is used to estimate the expected values of its performance indicators and maximize the overall profit.

In the works [7, 11–16], for the modeling, analysis and optimization of various logistic and transport systems, the use of the MPD apparatus is proposed. The MPD was used to find the optimal lot size in the conditions of accidental delay in the replenishment of the stock of production in the supply shop warehouse and in the case of an occasional fluctuation in demand for products located in the retail stores is considered in [15–17]. At the same time, the field of possible applications of the theory of Markov/semi-Markov processes with drift is far from being limited to the examples listed above. For example, it is possible to study systems with continuous replenishment of stocks, but taking into account the possibility of changing the intensity of stock replenishment in a warehouse, taking into account random variations in the intensity of use of the stock. This kind of flexible, adaptive inventory control is typical for logistics

systems. This work is devoted to the consideration of such problems of optimal inventory control.

5. Methods of research

To model many types of logistics systems that operate under uncertainty and risk conditions, the so-called Markov and semi-Markov processes with drift are a convenient mathematical tool. In addition to the discrete component $Z(t)$, these random processes also contain one or more continuous components $\xi(t)$ of the random walk type. In [1–4], a number of logistical systems are investigated using this class stochastic processes. The study of the impact of a random fluctuation in demand for products on the economic performance of logistics systems is of particular interest. For example, the intensity and volume of deliveries of finished goods (or semi-finished products) to a wholesale warehouse may vary depending on random fluctuations in demand, which can be considered as adaptive supply management.

6. Research results

To demonstrate the possibility of using MPD in logistics management, let's consider the simplest case of a single warehouse where products with intensity $W_{Z(t)}$ arrive, where $Z(t)$ is a semi-Markov process with two possible states, describing the change in demand: $Z(t)$ if the intensity of demand is U_1 , and $Z(t)=2$, if this intensity is equal to $U_2 > U_1$. Let $W_1=0$, $W_2=W > U_2$ (that is, if demand falls, supplies to the warehouse stop). If $\xi(t)$ is the inventory level in the warehouse at time moment t , then the following equation is used to describe the fluctuation of the inventory level (with probability 1):

$$\xi'(t) = W_2 I(Z(t)=2) - U_1 I(Z(t)=1) - U_2 I(Z(t)=2) + U_1 I(Z(t)=1, \xi(t)=0), \quad (1)$$

where $I(A)$ is the indicator of an event A . Let $B_k(t)$ is the distribution function (d. f.) of the time of the process $Z(t)$ is in the state k and let $\{t_n\}$ is the sequence of the moments of time at which the component $Z(t)$ changes its state. Let's denote:

$$\begin{aligned} \Phi_{nk}(x) &= \mathbf{P}\{Z(t) = k, \xi(t_n) \leq x\}, \\ F_k(x, t) &= \mathbf{P}\{Z(t) = k, \xi(t) \leq x\}, \\ \Phi_k(x) &= \lim_{n \rightarrow \infty} \Phi_{nk}(x), \\ F_k(x) &= \lim_{t \rightarrow \infty} F_k(x, t), \quad k = 1, 2; x \geq 0, \end{aligned} \quad (2)$$

under the assumption of the existence of these limits.

By methods of the theory of semi-Markov processes with drift and with allowance for (1) it can be shown [5, 15] that the functions (2) satisfy the following system of convolution type integral equations on the half-axis:

$$\begin{aligned} \Phi_1(x) &= \int_0^{x/V} \Phi_2(x-Vt) dB_2(t), \\ \Phi_2(x) &= \Phi_1(0) + \int_0^\infty \Phi_1(x+U_1 t) dB_1(t), \quad x \geq 0, \end{aligned} \quad (3)$$

where $V=W-U_2$. Equations (3) are a consequence of the theorem of total probability. From the theory of semi-Markov

processes it follows [5, 15] that functions $F_k(x)$, $k=1,2$, can be expressed in terms of functions $\Phi_k(x)$, $k=1,2$, according to formulas:

$$F_1(x) = \frac{1}{\beta_1 + \beta_2} \int_0^\infty (1 - B_1(\tau)) \Phi_1(x + U_1\tau) d\tau,$$

$$F_2(x) = \frac{1}{\beta_1 + \beta_2} \int_0^\infty (1 - B_2(\tau)) \Phi_2(x - V\tau) d\tau, \quad x \geq 0, \quad (4)$$

where

$$\beta_k = \int_0^\infty (1 - B_k(\tau)) d\tau < \infty, \quad k = 1, 2.$$

For arbitrary d. f. $B_k(t)$ the solution of the system of equations (3) is a complex mathematical problem. The general method for its solving it is based on reducing it by means of a two-sided Laplace transform to the Riemann boundary-value problem of the theory of functions of a complex variable [11]. However, in a number of particular cases the solution can be obtained in closed form in terms of the Laplace transform. For example, for

$$B_k(t) = 1 - e^{-\lambda t}, \quad t \geq 0,$$

from (4) one can obtain an explicit expression for the Laplace-Stieltjes transform of the limit d. f. $F(x) = F_1(x) + F_2(x)$ of quantity of products in the warehouse at an arbitrary moment of time, which is given by the following formula:

$$f(s) = F(0) \left[1 + \lambda \frac{1 - \beta_2(sV)}{sV} \right] \left[1 - \lambda \frac{1 - \beta_2(sV)}{sU_1} \right]^{-1},$$

Re $s \geq 0$, (5)

where $\beta_2(s)$ is the Laplace-Stieltjes transform for d. f. $B_2(t)$. The constant $F(0)$ (the limit probability that the warehouse at any time is empty) is determined from the condition $f(0) = F(\infty) = 1$ and is equal to:

$$F(0) = \left(1 - \frac{\lambda \beta_2 V}{U_1} \right) / (1 + \lambda \beta_2). \quad (6)$$

It follows from (6) that the necessary condition for the stable operation of the warehouse is inequality:

$$\lambda \beta_2 V < U_1.$$

With the help of expressions (5), (6) one can find the stationary mathematical expectation of the quantity of products in the warehouse:

$$M\xi = -f'(0) = \frac{\lambda \beta_2^{(2)} V (V + U_1)}{2(1 + \lambda \beta_2)(U_1 - \lambda \beta_2 V)}, \quad (7)$$

where

$$\beta_2^{(2)} = \int_0^\infty t^2 dB_2(t) < \infty.$$

Using (6), (7), let's estimate the average cost per unit of time for the supply of products to the warehouse, its

storage, as well as market losses due to the emptying of the warehouse:

$$\bar{C}(W) = c_1 W + p F(0) U_1 + c_{st} M \xi, \quad (8)$$

where c_1 is the unit price of the goods purchased by the warehouse; p is sale price of a unit of production; c_{st} is daily cost for storage of a unit of production in a warehouse. Thus, taking into account (6), (7), it is possible to find a value W minimizing expression (8). Elementary analysis shows that the equation $\bar{C}'(W) = 0$ does indeed have a single positive root.

This approach allows us analyze a more general case when demand varies on a finite set of different states of a semi-Markov process.

Let's now consider the case of a single warehouse where the production of M species comes with the intensities $W_{m, Z_m(t)}$, $m=1, 2, \dots, M$, where $Z_1(t), Z_2(t), \dots, Z_M(t)$ are the semi-Markov processes that are stochastically independent from each other and have two possible states describing the change in demand: $Z_m(t) = 1$ if the intensity of demand for products of the m th species is equal to U_{m1} and $Z(t) = 2$ if this intensity is equal to U_{m2} , $U_{m2} > U_{m1} > 0$. Let $W_{m1} = 0$ (that is, if the demand for products of the m th type falls, its supply to the warehouse stops) $W_{m2} \equiv W_m > U_{m2}$. Let's denote $B_{mk}(t)$ by the distribution function (d. f.) of the process time $Z_m(t)$ sojourn time in the state k . By methods of the theory of semi-Markov processes with drift [1], one can find an explicit expression for the Laplace-Stieltjes transform of the limit d. f. $F_m(x)$ of quantity of products $\xi_m(t)$ of the m -th type in the warehouse at any time. For example, for $B_{m1}(t) = 1 - \exp(-\lambda_m t)$, $t \geq 0$, this expression is given by the following formula:

$$\phi_m(s) = \frac{F_m(0) \left[1 + \lambda_m \frac{1 - \beta_{2m}(sV_m)}{sV_m} \right]}{\left[1 - \lambda_m \frac{1 - \beta_{2m}(sV_m)}{sU_{m1}} \right]}, \quad \text{Re } s \geq 0, \quad (9)$$

where $\beta_{2m}(s)$ is the Laplace-Stieltjes transform of d. f. $B_{2m}(t)$; $V_m = W_m - U_{m2}$. The limiting probability that there is no product of the m th type in the stock at any time (assuming that $Z_m(t) = 1$) is equal to:

$$F_m(0) = (U_{m1} - \lambda_m \beta_m V_m) / U_{m1} (1 + \lambda_m \beta_m),$$

$$\beta_m = \int_0^\infty (1 - B_{2m}(t)) dt < \infty. \quad (10)$$

The necessary conditions for the stable operation of the warehouse are inequalities $\lambda_m \beta_m V_m < U_{m1}$, $m=1, 2, \dots, M$. Using expressions (9), (10), one can find stationary mathematical expectations of the quantity of products of each kind in the warehouse:

$$M\xi_m = -\phi'_m(0) = \frac{\lambda \beta_m^{(2)} V_m (V_m + U_{m1})}{2(1 + \lambda \beta_m^{(2)})(U_{m1} - \lambda_m \beta_m^{(2)} V_m)},$$

$$\beta_m^{(2)} = \int_0^\infty t^2 dB_{2m}(t) < \infty. \quad (11)$$

Using (10), (11), one can estimate the average costs per unit of time for the supply of products of all types

to the warehouse, its storage, including also market losses due to the emptying of the warehouse:

$$\bar{C}(W_1, \dots, W_M) = \sum_{m=1}^M [c_{1m} W_m + p_m F_m(0) U_{m1} + c_m^{st} M \xi_m], \quad (12)$$

where c_{1m} is the price of a unit of the m th product purchased by a warehouse; p_m is the selling price of a unit of this type of product; c_m^{st} is holding cost for storage of a unit of production of the m th type in a warehouse. Thus, taking into account (10), (11), it is possible to find the values W_1, \dots, W_M , minimizing expression (12) with the storage capacity restriction E , i. e.

$$\sum_{m=1}^M M \xi_m \leq E.$$

7. SWOT analysis of research results

Strengths. The proposed method takes into account at the same time a random abrupt change in demand for products and fluctuations in its stock level and allows to determine the value of the replenishment intensity that minimizes the average current costs of the firm, including costs for the purchase of goods and their delivery, as well as possible market losses due to inadequate production in the warehouse with a sudden increase in demand;

Weaknesses. Practical use of the proposed method of optimal inventory management is complicated by the need in the general case to solve a complex system of integral equations in order to find the explicit analytical dependencies of the components of these total average current costs from the required control parameters (replenishment rates).

Opportunities. The above method, based on the MPD use, is quite universal. It allows to set and solve many other tasks of optimizing inventory control in a variety of logistics systems operating under uncertainty and risk. For example, in echeloned systems, a variety of transport and logistics systems [11–15].

Threats. The above analytical approach to the study and optimization of logistics systems operating under conditions of uncertainty and risk, has natural limits of practical use because it is impossible to take into account in all foreseeable manner all the factors that determine the operation of these complex systems. The most effective approach to the study of the work of such systems is the combination of analytical methods (similar to the one proposed here) and simulation in the spirit of the concept of directed simulation experiments proposed in [18].

8. Conclusions

As a result of research, the following conclusions can be drawn:

1. In terms of a semi-Markov process with drift, a formal description of the work of the warehouse of the supply firm is made, taking into account random fluctuations in demand and the corresponding replenishment of the stock, which allowed finding explicit expressions for the main performance indicators of the warehouse, as well as finding the condition for its stable operation in the steady state.

2. To find the limit joint probabilistic distribution of the quantity of products in the warehouse and the state of

the market environment (demand for products), a system of corresponding integral equations of convolution type on the semi-axis is derived and a method for its solution (for the case of two possible states of the medium) is proposed.

3. A solution is found in a closed form for solving the system of integral equations for the case of the exponential distribution of the time interval when there is no demand.

4. The problem of stochastic optimization of the intensity of supply of homogeneous products to the warehouse (in periods of availability of demand) is formulated and solved in order to minimize the average full cost of replenishing the stock of the product, storing it in a warehouse, as well as losses from a shortage of products in a warehouse during periods of demand for it. At the same time, the obtained results also allow to formulate a similar optimization problem for the case of several types of products. In addition, they allow us investigate a more general case when the state of the market environment is described by a finite number of possible states.

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РАЗРАБОТКА МЕТОДА ОПТИМАЛЬНОГО УПРАВЛЕНИЯ ЗАПАСАМИ ПРОДУКЦИИ ПРИ НЕПРЕРЫВНОМ ИХ ПОПОЛНЕНИИ И СЛУЧАЙНОМ СПРОСЕ

Сформулирована задача стохастической оптимизации для нахождения интенсивности поставок однородной продукции на склад снабженческой фирмы при случайном колебании интенсивности спроса на нее. Для формализованного описания работы склада использован аппарат полумарковских процессов со сносом. Выведена система интегральных уравнений для нахождения предельного распределения вероятностей этого процесса и найдено решение ее для частного случая. Рассмотрено обобщение модели стохастической оптимизации на случай нескольких видов продукции.

Ключевые слова: снабженческая фирма, запас продукции, непрерывная поставка продукции, полумарковский процесс со сносом, оптимизация интенсивности поставок.

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DEVELOPMENT OF BINOMIAL PRICING MODEL OF SHARES AND BONDS FOR A LIFE INSURANCE COMPANY

Проведено аналіз інвестиційної діяльності страхових компаній накопичувального типу. Також проаналізовано процес еволюції цін акцій та створена мультиплікативна модель еволюції ціни облигації із заданим часом погашення та нормальною вартістю, яка описує випадковий процес із характеристиками, близькими до броунівського мосту.

Ключові слова: страховий ринок, страхування життя, накопичення, броунівський міст, випадковий процес, акції, облигації, біноміальна модель.

1. Introduction

With the development of the insurance market, more people are using life insurance to create capital. Thus, such companies get large capital at their disposal, which they then invest in different sources. Today, the issue of the investment activities of insurance companies that have significant differences from other facilities in the market has not yet been fully explored. In this regard, it is relevant to study the features of the investment activities of life insurance companies, to improve their reliability and stability.

This topic is relevant, because insurance companies accumulate significant capital, which can then be increased through various investment tools. Thus, the availability of reliable instruments for calculating the expected profitability of investment instruments can ensure the stability of the insurance company at the market.

2. The object of research and its technological audit

The object of research is the investment activity of life insurance company. Since insurance companies have

a lot of long-term obligations, and dispose a large reserve capital, all of its activities depend on the effectiveness of investment policy in general.

One of the most serious problems in this topic is the issue of forecasting profitability from investment activities and minimizing risks. Since the insurance company cannot afford to invest in risky sources, because it has obligations on the insurance poles.

The largest market for life insurance is the United States, where is the largest number of insurance companies, as well as insurance coverage in the country. In the second place is China, which is actively developing in recent years, and Japan is in the third place.

3. The aim and objectives of research

The main aim of the paper is assess of the probabilistic distribution of changes in the price of shares and bonds of a life insurance company to increase the effectiveness of the management of the financial flows of the insurance company. This will allow the insurance company assesses more accurately the investment yield, which will lead to increased stability of the company in the future.