

14. Postan, M. Ya. Analiz vliyaniya urovnya nadezhnosti tekhnologicheskoy linii na ee proizvoditel'nost' [Text] / M. Ya. Postan, L. V. Shiryaeva // Metodi ta zasobi upravlinnya rozvitkom transportnikh sistem. – 2007. – Vol. 12. – P. 5–24.
15. Postan, M. Ya. Application of Semi-Markov Drift Processes to Logistic Systems Modeling and Optimization [Text]: Proceedings of the 4th International Conference, LDIC 2014, Bremen, Germany / M. Ya. Postan // Dynamics in Logistics. – Berlin: Springer, 2016. – P. 227–237. doi:10.1007/978-3-319-23512-7
16. Postan, M. Ya. Ob odnoi stokhasticheskoy modeli optymalnoho upravleniya zapasamy pry sluchainom sprose na produktsiyu [Text]: Tezy dopovidei II mizhnarodnoi naukovo-metodychnoi konferentsii / M. Ya. Postan // Problemy ekonomichnoi kibernetiky. – Poltava, 2014. – P. 103–104.
17. Postan, M. Ya. Model' optimal'nogo upravleniya mnogonomenklaturnymi zapasami pri sluchainom sprose na produktsiyu [Text]: Abstracts of XXXVI International Conference / M. Ya. Postan // Problems of decision making under uncertainties (PDMU-2015). – Odessa, 2015. – P. 124–126.
18. Avramchuk, E. F. Tekhnologiya sistemnogo modelirovaniya [Text] / E. F. Avramchuk, A. A. Vavilov, S. V. Emelyanov et al. – Moscow: Mashinostroeniye; Berlin: Tekhnik, 1988. – 520 p.

РАЗРАБОТКА МЕТОДА ОПТИМАЛЬНОГО УПРАВЛЕНИЯ ЗАПАСАМИ ПРОДУКЦИИ ПРИ НЕПРЕРЫВНОМ ИХ ПОПОЛНЕНИИ И СЛУЧАЙНОМ СПРОСЕ

Сформулирована задача стохастической оптимизации для нахождения интенсивности поставок однородной продукции на склад снабженческой фирмы при случайном колебании интенсивности спроса на нее. Для формализованного описания работы склада использован аппарат полумарковских процессов со сносом. Выведена система интегральных уравнений для нахождения предельного распределения вероятностей этого процесса и найдено решение ее для частного случая. Рассмотрено обобщение модели стохастической оптимизации на случай нескольких видов продукции.

Ключевые слова: снабженческая фирма, запас продукции, непрерывная поставка продукции, полумарковский процесс со сносом, оптимизация интенсивности поставок.

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DEVELOPMENT OF BINOMIAL PRICING MODEL OF SHARES AND BONDS FOR A LIFE INSURANCE COMPANY

Проведено аналіз інвестиційної діяльності страхових компаній накопичувального типу. Також проаналізовано процес еволюції цін акцій та створена мультиплікативна модель еволюції ціни облигації із заданим часом погашення та нормальною вартістю, яка описує випадковий процес із характеристиками, близькими до броунівського мосту.

Ключові слова: страховий ринок, страхування життя, накопичення, броунівський міст, випадковий процес, акцій, облигації, біноміальна модель.

1. Introduction

With the development of the insurance market, more people are using life insurance to create capital. Thus, such companies get large capital at their disposal, which they then invest in different sources. Today, the issue of the investment activities of insurance companies that have significant differences from other facilities in the market has not yet been fully explored. In this regard, it is relevant to study the features of the investment activities of life insurance companies, to improve their reliability and stability.

This topic is relevant, because insurance companies accumulate significant capital, which can then be increased through various investment tools. Thus, the availability of reliable instruments for calculating the expected profitability of investment instruments can ensure the stability of the insurance company at the market.

2. The object of research and its technological audit

The object of research is the investment activity of life insurance company. Since insurance companies have

a lot of long-term obligations, and dispose a large reserve capital, all of its activities depend on the effectiveness of investment policy in general.

One of the most serious problems in this topic is the issue of forecasting profitability from investment activities and minimizing risks. Since the insurance company cannot afford to invest in risky sources, because it has obligations on the insurance poles.

The largest market for life insurance is the United States, where is the largest number of insurance companies, as well as insurance coverage in the country. In the second place is China, which is actively developing in recent years, and Japan is in the third place.

3. The aim and objectives of research

The main aim of the paper is assess of the probabilistic distribution of changes in the price of shares and bonds of a life insurance company to increase the effectiveness of the management of the financial flows of the insurance company. This will allow the insurance company assesses more accurately the investment yield, which will lead to increased stability of the company in the future.

To achieve this aim it is necessary:

1. To study the processes of conducting the investment activity of the life insurance company.
2. To define and study the properties of this process.
3. To develop a model that will help predict the price changes the debt market.

4. Research of existing solutions of the problem

Today, the issue of investment activities of insurance companies is not fully disclosed. So in Ukraine this sector has just begun to develop [2]. Most scientists pay attention to the issues of statistical trends in the life insurance market, such as: increasing insurance premiums [3] and insurance payments, covering the insurance market and so on. At the same time, there is no clear methodology for managing the investment activity of an insurance company, which includes all the features of this type of activity. The same questions were raised about the safety of investment activities of insurance companies [1] and the factors that determine it, but such studies are more conducted abroad than in the Ukrainian market.

Abroad, scientists pay attention to such issues as data envelopment analysis in insurance companies, Intellectual capital and performance, productivity and efficiency of the life insurance companies [4–8].

Also, asset management [7] or external financing in financial crisis [6].

5. Methods of research

The methodological basis of the article is the fundamental positions of the general economic theory, insurance theory, statistics and econometrics and also methods of mathematical modeling

6. Research results

Let's consider the time series of share market prices. Schedules of share prices are chaotic in nature. Therefore, the value of the price of the share S_t at time t is assumed to be a random variable, which also determines some random process in time $t \in [0, +\infty)$.

Let's break the time interval $[0, t]$ on n intervals of the same length Δt : $t = n \cdot \Delta t$. Let's consider the model of the price evolution of a particular share, when we will be interested only in the price of the opening and the price of closing. Assuming that the price of the opening of the next period is equal to the price of the closing of the previous one, in the framework of our model we will be interested only $n+1$ value of the share.

Let's assume that the price of a share within each time interval can only either increase with the coefficient u , or fall with the coefficient d . Then the share price $S_{(t+\Delta t)}$ at the end is either Su or Sd . Let's assume also that from any investor in each interval of time it is possible to get a guaranteed interest $r_f = r_f(\Delta t)$. The growth factor when investing in a risk-free asset is denoted by $r = 1 + r_f$. To exclude the possibility of arbitrage, let's assume that:

$$d < 1 < r < u. \tag{1}$$

Let's denote by p the probability that the price will increase. Then $1-p$ – the probability that the price will decrease:

$$\begin{aligned} P\{S_{t+\Delta t} = Su | S_t = S\} &= p, \\ P\{S_{t+\Delta t} = Sd | S_t = S\} &= 1-p. \end{aligned} \tag{2}$$

Let's assume that a hypothesis about the risk neutrality of investors with a given current value S and possible future prices Su , taking into account the risk-free coefficient r , is fulfilled. Then the purchase of a share at the price S is equivalent to the operation of investing the sum S into a risk-free asset. This means that the equality is fulfilled:

$$S = \frac{M(S_{t+\Delta t} | S_t = S)}{r} \text{ or } S = \frac{Su \cdot p + Sd \cdot (1-p)}{r}. \tag{3}$$

From this let's obtain that $r = up + d(1-p)$ Expressing p from this equality, we finally find that the probabilities p and $(1-p)$ are equal:

$$p = \frac{r-d}{u-d}, \quad 1-p = \frac{u-r}{u-d}. \tag{4}$$

Let ξ_i – an indicator of a random event, consisting of the fact that the price increased during the period $[(i-1)\Delta t, i\Delta t]$, $i = 1, \dots, n$. Then $S_{i\Delta t} = S_{(i-1)\Delta t} u^{\xi_i} d^{1-\xi_i}$ and:

$$\ln \frac{S_{i\Delta t}}{S_{(i-1)\Delta t}} = \xi_i \cdot \ln \frac{u}{d} + \ln d. \tag{5}$$

Considering the value of the current share price is not a random variable equal to $S_0 = S$, let's find from the obtained formulas that:

$$S_t = S_{n\Delta t} = S_0 \prod_{i=1}^n (u^{\xi_i} d^{1-\xi_i}) = S \cdot u^{\sum_{i=1}^n \xi_i} \cdot d^{\sum_{i=1}^n (1-\xi_i)}. \tag{6}$$

Let's denote by v_n the number of periods until the time t , in which the price was moving up. Then $(n-v_n)$ will be the number of single periods by the time t , in which the price went down. Where v_n is the sum of Bernoulli random variables ξ_1, \dots, ξ_n . As a result:

$$S_t = Su^{v_n} d^{n-v_n}. \tag{7}$$

Since the random variable $v_n \in [0; n]$, then the possible values of the share price at the end of the i -th period are equal to $Su^k d^{n-k}$, $k = 0, \dots, n$.

To obtain the probabilistic distribution of the random variable S_t , let's use Bernoulli's scheme. Imagine that every single unit of time is an experiment, the «success» of which is the movement of prices up, and «failure» – the movement of prices down. Then we have n independent experiments with the probability of «success» equal to p . The entered random variable v_n will be the number of «successes» in the Bernoulli scheme with the given parameters.

From the formula (7) implies that:

$$P\{S_t = Su^k d^{n-k}\} = C_n^k \cdot p^k (1-p)^{n-k}. \tag{8}$$

The mathematical expectation of a random variable S_n equals:

$$MS_t = S(Up + d(1-p))^n. \quad (9)$$

Thus, the probability p correlates with the values u , d , and r such that the expected value of the share at time n equals the investment in the risk-free asset.

Let's find the logarithm of equality (7):

$$\ln \frac{S_t}{S} = v_n \ln u + (n - v_n) \ln d = v_n \ln \frac{u}{d} + n \ln d. \quad (10)$$

From the properties of mathematical expectation and dispersion it follows that:

$$\begin{aligned} M \ln \frac{S_t}{S} &= np \ln \frac{u}{d} + n \ln d, \\ D \ln \frac{S_t}{S} &= np(1-p) \cdot \left(\ln \frac{u}{d} \right)^2. \end{aligned} \quad (11)$$

Let's consider a certain level of possible values of the price of the share X . Such magnitude is a probabilistic characteristic of the random variable S_t , which is called the distribution function. Since the condition $S_t < X$ is equivalent to the inequality $\ln \frac{S_t}{S} \leq \ln \frac{X}{S}$, from formula (10) and the positivity of the quantity $\ln \frac{u}{d}$ (due to $u > d$) it follows that:

$$P\{S_t \leq X\} = P\left\{v_n \ln \frac{u}{d} + n \ln d \leq \ln \frac{X}{S}\right\} = P\{v_n \leq x\}, \quad (12)$$

where

$$x = \frac{\ln\left(\frac{X}{S}\right) - n \ln d}{\ln(u/d)}. \quad (13)$$

Let's denote by $B(x, n, p)$ the function of a binomial distribution, which is equal to the probability that the number of «successes» in the Bernoulli scheme will not exceed x . So:

$$B(x, n, p) = \{v_n \leq x\} = \sum_{k=0}^{[x]} C_n^k \cdot p^k (1-p)^{n-k}, \quad (14)$$

where $[x]$ – means the largest integer that does not exceed x . Then the desired probability $P\{S_t \leq X\}$ is based on the formula:

$$P\{S_t \leq X\} = B(x, n, p). \quad (15)$$

Consequently, by means of formulas (13) and (15) the share price is determined at time t , using the value of the binomial distribution. Therefore, such model is called binomial. Parameters of this model are: n – number of periods, i and d – coefficients of possible growth and fall in period, respectively, r – coefficient of increase of risk-free asset. All these parameters depend on the value of Δt .

As practice shows, the so-called logarithmic mean a and the volatility σ^2 , which show the average changes in

mathematical expectation and the variance of the random process per unit time, are the most statistically significant values in the study of the evolution of a particular financial series. More strictly, these parameters are determined by the following formulas:

$$a = \frac{1}{T} \left[\ln \frac{S_T}{S} \right], \quad \sigma^2 = \frac{1}{T} D \left[\ln \frac{S_T}{S} \right]. \quad (16)$$

Knowing the distribution of the random variable S_T value of the price of the financial series at the time T , it is possible to obtain a numerical estimate of the parameters a and σ^2 of the given series. Then, according to formula (15), knowing the parameters of the binomial model u , d , p , n it is possible to construct the share price distribution at time t . There is a simple problem, how the prescribed model parameters should depend on the calculated characteristics a and σ^2 .

Even more important is the dependence of the model parameters on the choice of the length Δt of the time interval. Thus, there is the question of assessing the variability of prices of a particular share. This parameter is precisely the volatility of the action σ^2 . Let's determine the dependence of the coefficients u and d on it:

$$u = \exp\{\sigma\sqrt{\Delta t}\}, \quad d = \exp\{-\sigma\sqrt{\Delta t}\}. \quad (17)$$

In this case, the binomial model of risk-free growth rate r in one interval is more convenient to express, using continuously accrued by the interest rate μ . By the definition of this value, in time Δt , the risk-free asset increases in $\exp\{\mu \cdot \Delta t\}$ times. Consequently, $r = \exp\{\mu \cdot \Delta t\}$. The probabilities p and $1-p$ in this case can be calculated as risk-neutral using formulas (8). In practice, they usually use more convenient formulas:

$$\begin{aligned} p &= \frac{1}{2} \left(1 + \left(\frac{\mu}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \right), \\ 1-p &= \frac{1}{2} \left(1 - \left(\frac{\mu}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \right). \end{aligned} \quad (18)$$

The constructed model makes it possible to estimate the probable characteristics of the random variable S_t . For example, for a given level of risk capital X from the formula (15) let's obtain that the probability of risk is $P\{S_t \leq X\} = B(x, n, p)$, where x is determined according to (13). By the definition of the quantities u and d we find that $\ln\left(\frac{u}{d}\right) = 2\sigma\sqrt{\Delta t}$. And so

$$\begin{aligned} x &= \frac{\ln(X/S) - n \ln d}{\ln(u/d)} = \\ &= \frac{\ln(X/S) - \sigma\sqrt{\Delta t}}{2\sigma\sqrt{\Delta t}} = \frac{1}{2\sigma\sqrt{\Delta t}} \ln\left(\frac{X}{S}\right) \frac{n}{2}. \end{aligned} \quad (19)$$

Let's formulate a binomial model more strictly. Let, as before, the time interval $[0, t]$ is divided into n equal intervals of length Δt . For simplicity, let $\Delta t = 1$. This means

that $\Delta t = n$. Let's assume that the price of an asset within each single time interval can only either increase with the coefficient u or fall with the coefficient d . Let the choice of direction – *up or down* – in the first interval is made with a probability of $1/2$. Further dynamics depends on the direction of movement on the previous interval. Let's assume that the probability of continuing the price movement in the same direction as in the previous step is equal to $1-c$, $c \in [0, 1]$. The probability of changing the direction of motion relative to the previous step is equal to c . If $c = 1/2$, then we have a simple classical binomial model whose probabilistic characteristics are determined only by the parameters u i d . If $c = 0$ or $c = 1$, then this random process will be degenerate, since in the next step the previous motion will continue or be necessarily changed. In this case, the behavior of the process will be fully determined by the first step taken. Thus, the dynamics of the probabilistic characteristics of the generalized model is given by the parameters u , d i c .

6.1. Model pricing of shares using the geometric Brownian motion. Let's be interested in the dynamics of price changes of some financial asset, which is described by the random process S_t , $t \in [0, +\infty]$. Let's call the random process S_t a geometric Brownian motion with parameters a and σ^2 if for any non-negative numbers t and s the following conditions are fulfilled: the random variable S_{t+s}/S_s is statistically independent of the values of the random process to the time moment s , and $\ln(S_{t+s}/S_s)$ has a Gaussian distribution with mean at and a dispersion $\sigma^2 t$. In other words, the financial time series is modeled by the geometric Brownian motion, if the price correlation in the future time t to the price in the present time does not depend on the past price dynamics and has a log-normal distribution with the parameters at and $\sigma^2 t$.

The process of geometric Brownian motion can be obtained from a binomial model if $\Delta t \rightarrow 0$, for $n \rightarrow \infty$. To prove this, one must use the central boundary theorem. Let's formulate a partial case of the central limit theorem (Muavra-Laplace theorem).

Let's assume that we conduct n independent experiments, each of which with probability p ends with «success» and with the probability of $1-p$ ends, respectively, «failure».

Then, as before, the number of successful experiments v_n has a binomial distribution with the parameters n and p . In the Muavra-Laplace theorem states that for any real number y there is a limiting distribution:

$$\lim_{n \rightarrow \infty} \left\{ \frac{v_n - np}{\sqrt{np(1-p)}} \leq y \right\} = \Phi(y), \quad (20)$$

where the Laplace function $F(y)$ is a function of the standard Gaussian distribution:

$$\Phi(y) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^y e^{-u^2/2} du. \quad (21)$$

By virtue of the properties of the Gaussian distribution, the Muavre-Laplace theorem can be interpreted as follows: for sufficiently large n , the random variable v_n

has a distribution that is close to Gaussian with parameters np and $\sqrt{np(1-p)}$. But due to the fact that:

$$\ln \frac{S_t}{S} = v_n \ln u + (n - v_n) \ln d = v_n \ln \frac{u}{d} + n \ln d, \quad (22)$$

where $\ln(S_t/S)$ is a linear function of v_n , which means that it also has an asymptotic Gaussian distribution. Moreover, it is possible to state that:

$$M \left[\ln \frac{S_t}{S} \right] = at, D \left[\ln \frac{S_t}{S} \right] = \sigma^2 t \cdot \left(1 - \frac{a^2}{\sigma^2} \Delta t \right) \approx \sigma^2 t. \quad (23)$$

Thus, if $\Delta t \rightarrow 0$, then the distribution $\ln(S_t/S)$ converges to the Gaussian distribution with the parameters at and $\sigma^2 t$. Accordingly, when $\Delta t \rightarrow 0$, the binomial model leads us to a geometric Brownian motion.

The evolutionary model of the risk asset constructed in this way is often called the lognormal model, since one of the important conclusions of the geometric Brownian motion is that the relation of the future price S_t to the present price S is a random variable that has a lognormal distribution with the parameters at and $\sigma^2 t$. The latter means that:

$$\frac{S_t}{S} = \exp \{ at + \sigma \sqrt{t} \cdot \eta \}, \quad (24)$$

where η is a standard Gaussian random variable. The resulting formula gives an opportunity to estimate the probabilistic distribution using the Laplace function. In particular, the probability of a price risk not exceeding a given price level X can be estimated by the following formula: $P\{S_t \leq X\} = P\{\eta \leq z\} = \Phi(z)$, where, taking into account the balance equation $a = \mu - \sigma^2 / 2$:

$$z = \frac{1}{\sigma \sqrt{t}} \ln \left(\frac{X}{S} \right) - \frac{a \sqrt{t}}{\sigma} = \frac{1}{\sigma \sqrt{t}} \ln \left(\frac{X e^{-\mu t}}{S} \right) + \frac{\sigma \sqrt{t}}{2}. \quad (25)$$

One consequence of the log-normal model is the fact that the expected value of the price increases as a non-risk asset. Indeed, by virtue of the lognormal distribution properties, the mathematical expectation of the right-hand side in formula (24) is equal to:

$$\begin{aligned} M \left[\frac{S_t}{S} \right] &= M \left[\exp \{ at + \sigma \sqrt{t} \cdot \eta \} \right] = \\ &= \exp \left\{ t \left(a + \frac{\sigma^2}{2} \right) \right\} = e^{\mu t}. \end{aligned} \quad (26)$$

6.2. Model of bond price evolution with geometric Brownian motion. Changes in bonds market prices are more stable than in the share market. Therefore, a model that describes the dynamics of bond prices B_t is implemented by a differential equation that does not contain a stochastic component:

$$dB_t = r(t) B_t dt. \quad (27)$$

The value $r(t)$ is the interest rate that is continuously calculated and may be random. As a rule, all risks associated with interest rate changes are modeled directly by the random process $r(t)$. In this case, the model of price dynamics of the bond becomes rather complex and difficult to implement. This section proposes a model for pricing bonds, which is a generalization of the Samuelson model, which can be used to market the bonds for an insurance company.

The main difference between share bonds is that the bond has a limited period of validity, and its price at the time of repayment of T equals its nominal value: $B_T = F$. If it is considered that the change in bond prices is something like the geometric Brownian motion, then it is worth considering an arbitrary process with given conditions. Such random processes are known in the theory of probability. For example, the Brownian Bridge belongs to them.

According to the source [2], under the Brownian bridge Y_t it is customary to call the Gaussian random process with the function of the mathematical expectation and autocovariance function, which respectively equal:

$$MY_t = \alpha \left(1 - \frac{t}{T}\right) + \beta \frac{t}{T}, \quad \text{Cov}(Y_t, T_s) = \min\{s, t\} - \frac{st}{T}. \quad (28)$$

Since, at time T , the mathematical expectation is equal to β , and the dispersion is zero, then with probability 1, the random process Y_t takes value β , which in principle corresponds to the dynamics of the price of the risk bonds. However, the use of the Brownian Bridge to simulate the evolution of the price of the insurance company's bonds is quite complicated, since it may take values less than zero, which is impossible for the price. In addition, the Brownian Bridge implements an additive, rather than a multiplicative model. This is evident from the corresponding stochastic differential equation:

$$dY_t = \frac{\beta - Y_t}{T - t} dt + \sigma dW_t, \quad t \in (0, T). \quad (29)$$

Since, at time T , the mathematical expectation is equal to β , and the dispersion is zero, then with probability 1, the random process Y_t takes value β , which in principle corresponds to the dynamics of the price of the risk bonds. However, the use of the Brownian Bridge to simulate the evolution of the price of the insurance company's bonds is quite complicated, since it may take values less than zero, which is impossible for the price. In addition, the Brownian Bridge implements an additive, rather than a multiplicative model. This is evident from the corresponding stochastic differential equation:

$$dY_t = \frac{\beta - Y_t}{T - t} dt + \sigma dW_t, \quad t \in (0, T). \quad (30)$$

Indeed, the increment dY_t depends on the value of Y_t only in the coefficient with the value of the increment of the argument dt and, consequently, is adaptive. This drawback can be eliminated if instead of the Brownian bridge taking its stochastic exponent.

Let's use the formula of a stochastic differential equation:

$$dY_t = \frac{\beta - Y_t}{T - t} dt + \sigma dW_t, \quad t \in (0, T). \quad (31)$$

whereas $F(t, X)$ we take the following function:

$$dF(t, X) = \tilde{a} dt = \tilde{b} dW_t, \quad (32)$$

but as an X - Brownian bridge, which is subject to the following differential equation:

$$DY_t^0 = \frac{\beta - Y_t^0}{T - t} dt + dB_t, \quad 0 < t < T, \quad (33)$$

which has the form: $Y_t^0 = W_t^0$.

Then, if use the ITO formula for the coefficients:

$$\tilde{a} = \frac{\partial F(t, X)}{\partial t} + a \cdot \frac{\partial F(t, X)}{\partial X} + \frac{1}{2} b^2 \frac{\partial^2 F(t, X)}{\partial X^2}, \quad (34)$$

$$\tilde{b} = b \frac{\partial F(t, X)}{\partial t}, \quad (35)$$

where

$$a = \frac{\beta - Y_t^0}{T - t}, \quad b = 1,$$

$$\frac{\partial F(t, Y_t^0)}{\partial t} = f'(t) \exp\{f(t) + BY_t^0\}, \quad (36)$$

$$\frac{\partial F(t, Y_t^0)}{\partial Y_t^0} = B \exp\{f(t) + BY_t^0\}, \quad (37)$$

$$\frac{\partial^2 F(t, Y_t^0)}{\partial (Y_t^0)^2} = B^2 \exp\{f(t) + BY_t^0\}. \quad (38)$$

From here:

$$dF(t, Y_t^0) = \exp\{f(t) + BY_t^0\} \left(f'(t) + \frac{\beta - Y_t^0}{T - t} \right) dt + \exp\{f(t) + BY_t^0\} B dW_t. \quad (39)$$

In addition, Y_t^0 can be expressed from equation (32):

$$\ln F(t, Y_t^0) = f(t) + BY_t^0, \quad (40)$$

$$Y_t^0 = \frac{1}{B} \ln F(t, Y_t^0) - \frac{f(t)}{B}. \quad (41)$$

If substitute the resulting expression in equation (39):

$$dF(t, Y_t^0) = \exp\{f(t) + BY_t^0\} \times \left(f'(t) + \frac{\beta \cdot B - \ln F(t, Y_t^0)}{T - t} + \frac{f(t)}{T - t} + \frac{1}{2} B^2 \right) dt + \exp\{f(t) + BY_t^0\} B dW_t. \quad (42)$$

Choose $f(t)$ and B such that:

$$f'(t) + \frac{f(t)}{T-t} + \frac{1}{2}B^2 = 0.$$

With the aid of mathematical transformations, let's find that the function $f(t)$ takes the following form:

$$f(t) = (T-t) \frac{B^2}{2} \ln(T-t). \quad (43)$$

Substituting (43) and $B = \sigma$ in (32), let's find that the desired function takes the form:

$$F(t, Y_t^0) = \exp \left\{ (T-t) \frac{\sigma^2}{2} \ln(T-t) + \sigma Y_t^0 \right\}. \quad (44)$$

The stochastic equation for this function is as follows:

$$dF(t, Y_t^0) = F(t, Y_t^0) \left(\frac{\beta \cdot \sigma - \ln F(t, Y_t^0)}{T-t} dt + \sigma dW_t \right). \quad (45)$$

As a result, we have a stochastic differential equation that defines this random process B_t :

$$dB_t = B_t \left(\frac{\ln F - \ln B_t}{T-t} dt + \sigma dW_t \right), \quad t \in (0, T). \quad (46)$$

The solution of the resulting equation will be the following random process, which is determined by the standard Brownian bridge Y_t^0 (for it $a = \beta = 0$):

$$B_t = \exp \left\{ \left(1 - \frac{t}{T} \right) \ln B_0 + \frac{t}{T} \ln F + \left(T-t \right) \frac{\sigma^2}{2} \ln(T-t) + \sigma Y_t^0 \right\}, \quad (47)$$

where B_0 is the current bond price. Thus, a constructed model describing a random process that has characteristics close to the Brownian bridge, and implements a multiplicative model of the evolution of the bond price with the time of repayment T and the nominal value F . The obtained stochastic differential equation (40) makes it possible to estimate the probability distribution of the growth of dB_t the bond price at the time t , as the known stochastic value of the volatility σ^2 .

7. SWOT analysis of research results

Strengths. The strength of the research is the analysis of the share price evolution process, as well as the creation of a multiplicative model of the evolution of the bond price with a given maturity and a normal value that describes a random process with characteristics close to the Brownian Bridge.

Weaknesses. The weak point is that the model can't include all the risks that may arise in the process of investment and business activities of the insurance company.

Opportunities. Opportunities for further research are the further improvement of the model, the inclusion of

higher risk factors for it. And also the use of this model for a real insurance company

Threats. Threats to the results are difficulty in using the model in real terms.

8. Conclusions

As a result of the studies, the following results are obtained:

1. The analysis of the evolution of the price of shares as a random process. The scheme for calculating the expected share price as an investment in a risk-free asset.

2. It is reviewed and summarized lognormal and binomial model to calculate the expected stock price. The constructed binomial model makes it possible to estimate the probabilistic characteristics of the expected share price. The definition of the random process of formation of the price of shares through the geometric Brownian bridge is given.

3. A model is developed that describes a random process that has characteristics close to the Brownian Bridge and implements a multiplicative model of the evolution of the bond price with the time of repayment of T and the nominal value of F . The resulting stochastic differential equation gives an opportunity to estimate the probabilistic distribution of the bond price rise at a given time point, as the known stochastic value of volatility.

This model can be used to effectively manage the investment activity of an insurance company to prevent loss-making from activities, and maximize the company's returns.

References

- Shiryayev, A. N. Fundamentals of stochastic financial mathematics [Text] / A. N. Shiryayev. – Moscow: Fazis, 1998. – 512 p.
- Boyko, A. O. Theoretical bases and practical experience of providing financial stability of the insurance company [Text] / A. O. Boyko // Economic sciences. Series Accounting and Finance. – 2010. – Vol. 7 (25), Part 4. – P. 36–50.
- Ivanyuk, V. A. Methods of Optimization of Investment Strategy [Text] / V. A. Ivanyuk, K. N. Andropov, N. E. Egorova // Fundamental research. – 2016 – Vol. 3, Part 1. – P. 159–163.
- Kao, C. Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan [Text] / C. Kao, S.-N. Hwang // European Journal of Operational Research. – 2008. – Vol. 185, No. 1. – P. 418–429. doi:10.1016/j.ejor.2006.11.041
- Lu, W.-M. Intellectual capital and performance in the Chinese life insurance industry [Text] / W.-M. Lu, W.-K. Wang, Q. L. Kweh // Omega. – 2014. – Vol. 42, No. 1. – P. 65–74. doi:10.1016/j.omega.2013.03.002
- Berry-Stolzle, T. R. External Financing in the Life Insurance Industry: Evidence From the Financial Crisis [Text] / T. R. Berry-Stolzle, G. P. Nini, S. Wende // Journal of Risk and Insurance. – 2014. – Vol. 81, No. 3. – P. 529–562. doi:10.1111/jori.12042
- Bawa, N. An analytical study oa asset under management of life insurance companies in India [Text] / N. Bawa, N. Dhanda // Journal of Services Research. – April–September, 2016. – Vol. 16.1. – P. 61–80.
- Eling, M. Under pressure: how the business environment affects productivity and efficiency of European life insurance companies [Text] / M. Eling, P. Schaper // European Journal of Operational Research. – 2017. – Vol. 258, No. 3. – P. 1082–1094. doi:10.1016/j.ejor.2016.08.070
- Kamat, M. An application of Fixed-Effects Model (FEM) to evaluate financial performance of the Indian Life Insurance Industry [Text] / M. Kamat, B. Ramesh, P. Dhume // SAARJ Journal on Banking & Insurance Research. – 2016. – Vol. 5, No. 5. – P. 14–31. doi:10.5958/2319-1422.2016.00006.0

10. Adelman, M. A Large-Scale Optimization Model for Replicating Portfolios in the Life Insurance Industry [Text] / M. Adelman, L. F. Arjona, J. Mayer, K. Schmiedders // SSRN Electronic Journal. – 2016. – No. 16–04. – P. 1–44. doi:10.2139/ssrn.2727281

РАЗРАБОТКА БИНОМИАЛЬНОЙ МОДЕЛИ ЦЕНООБРАЗОВАНИЯ АКЦИЙ И ОБЛИГАЦИЙ ДЛЯ СТРАХОВОЙ КОМПАНИИ НАКОПИТЕЛЬНОГО ТИПА

Проведен анализ инвестиционной деятельности страховых компаний накопительного типа. Также проанализирован процесс эволюции цен акций и создана мультипликативная модель эволюции цены облигации с заданным временем погашения и нормальной стоимости, которая описывает случайный процесс с характеристиками, близкими к броуновскому мосту.

Ключевые слова: страховой рынок, страхование жизни, накопления, броуновский мост, случайный процесс, акции, облигации, биномиальная модель.

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FINANCIAL EQUILIBRIUM AS BASIS FOR ENTERPRISE'S SUSTAINABLE DEVELOPMENT: ECONOMIC AND MATHEMATICAL FOUNDATION

Досліджено вплив фінансової рівноваги на забезпечення сталого розвитку підприємства. Розроблена економіко-математична модель, яка описує взаємозв'язок між коефіцієнтом внутрішнього зростання фінансового потенціалу, який виступає індикатором сталого розвитку підприємства, та чотирма показниками, що характеризують стан фінансової рівноваги. Отримана економіко-математична модель дозволяє прогнозувати тенденцію сталого розвитку залежно від фактичного встановлення на підприємстві фінансової рівноваги, а також моделювати ймовірні зміни його фінансового стану.

Ключові слова: фінансова рівновага, сталий розвиток, фінансовий потенціал, фінансовий важіль, економіко-математична модель.

1. Introduction

Ensuring the enterprise's sustainable development is an urgent problem, the solution of which will help to raise the level of management to a higher level. Smoothing of objectively conditioned cyclical downturns in the process of vital activity of an enterprise is not enough for an integrated and systemic solution of this problem. To date, the management of changes in the enterprise and the formation of conditions for sustainable development are the features of modern professional management.

One of the main conditions for ensuring enterprise's sustainable development is financial equilibrium.

2. The object of research and its technological audit

The object of research is the process of forming an enterprise's sustainable development for achieving its financial equilibrium. The mechanism of this relationship is realized through financial potential, which is a consequence of financial equilibrium and the cause of enterprise's sustainable development. However, building financial capacity should not be an end in itself [1]. Therefore, the key is the ques-

tion of the need to ensure the conditions for building the financial capacity of the enterprise to ensure sustainable development. This basic condition is the financial equilibrium.

The study of financial equilibrium of the enterprise by foreign scientists corresponds to the term «financial health», which in translation means «financial health» [2, 3]. For its evaluation by foreign economic science, the discriminative models have been developed [4, 5]. In the practice of domestic enterprises, they can be used to predict bankruptcy, that is, a diametrically opposite state of sustainable development. In the absence of signs of bankruptcy, a conclusion is made about the success of the enterprise.

The originality of our study lies in the fact that our model has not only to state the absence of bankruptcy. It provides for the establishment of the interdependence of sustainable development on the state of financial equilibrium of the enterprise. A certain object of research is studied on the example of machine-building enterprises of Ukraine.

3. The aim and objectives of research

The aim of research is development of an economic and mathematical model that allows to assess the impact